LABORATORY MEASUREMENT OF STIFFNESS AND DAMPING OF RUBBER ELEMENT

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A sample of rubber element used for reduction of noise and vibrations of tram and railway wheels was loaded by harmonic force at different frequencies (10, 20, 30 Hz) and amplitudes in the range 0.25–2.25 kN and by a constant preload 6 kN. Measurements of forces and deformation were carried out by two separate apparatus sets. Two methods of data evaluation are explained. Results of this evaluation is presented in the form of diagrams stiffness versus amplitude and phase angle versus amplitude of harmonic loading.

Key words: *damping element, harmonic loading, measurement, evaluation, rheological properties*

1. Introduction

More strict standards for noise and vibrations in many industrial and transport applications make necessary to introduce various damping elements into new machines and their subsystems. These new trends are very important for railway and tram transport means, where the higher passenger comfort is one of the required properties and very often decides of the marketability of the product. Very intensive source of noise and vibration are tram or railway wheels especially at high speeds of transport. Therefore the modern types of steel railway wheels contain the visco-elastic damping cushioning, which reduce the transfer of noise and vibrations.

Various rubber-like materials are used as resilient machine elements, with different elastic, damping and also thermo-mechanical properties. Therefore our investigation is aimed on the ascertaining of rheologic characteristic of rubber material used in Czech Republic for damping of railway wheels.

The first step of research work was experimental investigation of force-deformation characteristics in laboratories of Institute of Thermomechanics ASCR at different levels of loads and at different frequencies of harmonic force [1]. The second step was elaboration of evaluation methods of experimentally gained data and summarize the results into the form of diagrams applicable in industrial practice.

Mechanical properties are presented in this paper. The interaction of thermal and mechanical properties is briefly mentioned in [2, 3] and is in more detailed form included into the complex rheological model of rubber properties [8].

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2. Studied specimen and its measurement

The experimental investigation was carried out in the branch laboratory of our Institute in Pilzen on the specimen drawn in Fig. 1. Material of the specimen is isopren-butadien rubber 42-809 produced by Rubena Hradec Králové. Mechanical properties are density 1357 kg/m^3, dynamic small deformation Young modulus 52 MPa, Poisson number cca 0.5, hardness Shore 80. It has length 50 mm, height 25 mm and width 80 mm. The specimen was loaded by controlled force with amplitudes $F(t) = F_{\mathrm{st}} + F_0 \cos \omega t$ consisting of the static preload $F_{\mathrm{st}} = 6$ kN and of harmonic force

$$\begin{aligned} F_0 = 0.25\,,\ 0.5\,,\ 0.75 \quad &\text{or} \\ 0.5\,,\ 1\,,\ 1.5 \quad &\text{or} \\ 0.75\,,\ 1.5\,,\ 2.25 \quad &\text{or} \\ 1.2\,,\ 2.4\,,\ 3.6\,\mathrm{kN} & \end{aligned} \tag{1}$$

and with frequencies

$$f = \frac{\omega}{2\pi} = 10\,,\ 20\,,\ 30\,\mathrm{Hz}\,. \tag{2}$$

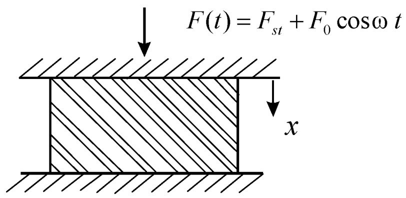

Fig.1: Specimen of rubber-like material

The forces were generated by servo-cylinder PL 250 N in the loading stand Schenck-Hydropuls PSB 250. The exciting forces $F(t) = F_{\mathrm{st}} + F_0 \cos \omega t$ and deformations $x(t) = x_{\mathrm{st}} + x_0 \cos(\omega t - \psi)$ consisting of the static F_{st}, x_{st} and of dynamic $F_0 \cos \omega t$, $x_0 \cos(\omega t - \psi)$ components were measured both by the measuring apparatus installed in the vibrostand Schenck (apparatus I) and by the piezo-resistance force transducer and laser vibrometer IT AS CR (apparatus II).

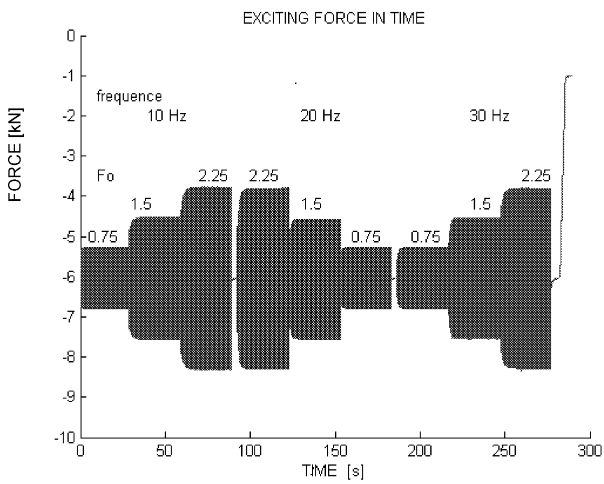

Fig.2: Loading procedure

The single ended bead-type thermistor of the type SEMI833 ET Hydrotec Messtechnic measures the inner temperature during the loading, which lasts approximately 5 minutes. The increase of temperature at the end of measurements with the highest loading force (amplitudes up to 3.6 kN) was more than 20 °C, at the lowest loading (up to 0.75 kN) was only 2 °C.

An example of the force $F(t)$ versus time t plot of the exciting amplitudes set $F_0 = 0.75, 1.5, 2.25$ kN is shown in the Fig. 2, where this sequence of amplitudes is, repeated with the frequencies 10, 20, 30 Hz.

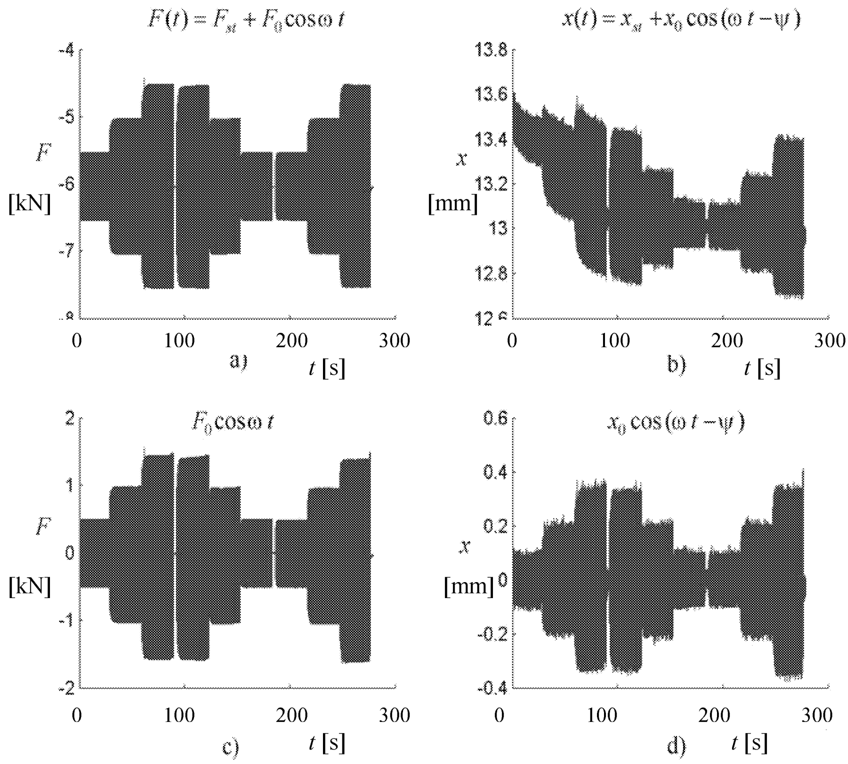

Fig.3: Records of a) total force, b) total displacement, c) oscillating force component, d) oscillating motion, – all versus time

Response of rubber element measured in of vertical deformation was sampled with the frequency 500 samples per second in 5 channels (displacement I, force I, displacement II, force II, temperature of specimen) and recorded into digital memory. The rheologic properties of the rubber material were very sensitive on the loading time and on the temperature that varies according to the lost energy in the element and according to the flow of heat into the air and into the steel parts of loading device. The temperature of the loading stand Hydropuls varies also during the tests due to the large energy consumption with small efficiency and in spite of insulation of the specimen, its static deformation from preload 6 kN presents the creep shift. It is seen on the record deformation-time in right upper corner b) of Fig. 3. Before the evaluation of oscillating motion, the suppression of this shift together with elimination of constant preload $F_{st} = 6$ kN has to be done. The separation of oscillating components of force ($F_0 = 0.5, 1, 1.5$ kN, $f = 10, 20, 30$ Hz) and of specimen deformations

$x(t) \doteq x_0 \cos(\omega\, t - \psi)$ was realized numerically by exponential moving averaging and is shown in Fig. 3c) and d).

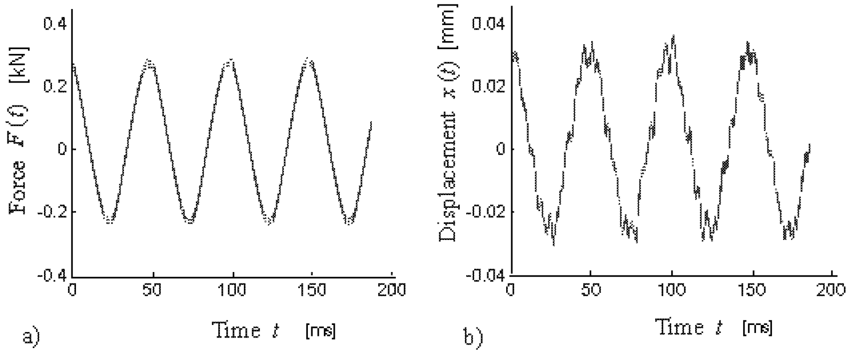

Fig.4: Measured force and displacement at $f = 10\,Hz$, $F_0 = 0.25\,kN$

The bottom records c) and d) were used for further evaluation of dynamical response of investigated specimen. The time history of force is very near to harmonic as shown in Fig. 4a, where the ten periods of exciting force with amplitude $F_0 = 0.25\,\mathrm{kN}$ and frequency $f = 10\,\mathrm{Hz}$ is shown. The response x has more complicated form (Fig. 4b). This record is the example of motion at the worst loading conditions – very low amplitude of force at the lowest frequency. All the other displacement records are much better.

3. Evaluation of measurement

Several methods can be used for determination of linear model parameters. The so-called wattmetric method and FFT method was applied. The first one is based on the assumption that the excitation force is pure harmonic or very near to it, which is here adequately fulfilled.

The ascertaining first harmonic of components of response is then based on the orthogonality of harmonic functions and gives

$$\frac{a\,\cos\psi}{F_0} = \frac{\int\limits_0^{2\pi} F_0\,\sin(\omega\,t)\,x(t)\,\mathrm{d}(\omega\,t)}{\int\limits_0^{2\pi} (F_0\,\sin\omega\,t)^2\,\mathrm{d}(\omega\,t)}\;,$$

$$\text{and} \qquad \frac{a\,\sin\psi}{F_0} = \frac{\int\limits_0^{2\pi} F_0\,\cos(\omega\,t)\,x(t)\,\mathrm{d}(\omega\,t)}{\int\limits_0^{2\pi} (F_0\,\sin\omega\,t)^2\,\mathrm{d}(\omega\,t)}\;, \tag{3}$$

where $x(t)$ is the measured displacement

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k\,\cos(k\,\omega\,t - \psi_k)\;. \tag{4}$$

If the system is linear (or lightly nonlinear) then

$$a = a_1 \gg a_k \ , \quad k \neq 1 \ .$$

At the digital measurement, the integrals must be replaced by sums:

$$\begin{aligned} \frac{a}{F_0}\cos\psi &= \frac{\sum\limits_{i=1}^{n}(Y_i\,X_i)}{\sum\limits_{i=1}^{n}Y_i^2} \ , \\ \frac{a}{F_0}\sin\psi &= \frac{\sum\limits_{i=1}^{n}(Y_i'\,X_i)}{\sum\limits_{i=1}^{n}Y_i^2} \ , \end{aligned} \tag{5}$$

where n is the number of digital samples corresponding to the integer l of motion periods:

$$l\,\frac{2\pi}{\omega} = n\,\Delta T \ , \qquad \Delta T = \text{sampling interval} \ .$$

X_i, Y_i are measured, digitalized and centered values of displacements $x(t)$ and forces $F(t)$ in the chosen interval of time

$$t \div t + n\,\Delta t \ , \quad i = 1 \div n+1 \ . \tag{6}$$

$Y' = Y_{i+n'}$ are values of force read in the same set of n values, but shifted by a number n' corresponding to a quarter of exciting period

$$n'\,\Delta t = \frac{T}{4} = \frac{\pi}{2\,\omega} \ . \tag{7}$$

Amplitude a and phase shift ψ of response can be then calculated from (5)

$$\frac{a}{F_0} = \frac{\left[\left(\sum\limits_{i=1}^{n}(Y_i\,X_i)\right)^2 + \left(\sum\limits_{i=1}^{n}(Y_i'\,X_i)\right)^2\right]^{1/2}}{\sum\limits_{i=1}^{n}Y_i^2} \ , \tag{8a}$$

$$\tan\psi = \frac{\sum\limits_{i=1}^{n}(Y_i'\,X_i)}{\sum\limits_{i=1}^{n}(Y_i\,X_i)} \ . \tag{8b}$$

These values are sufficiently accurate if the exciting force $F(t)$ is harmonic, without any distortion. The distortion of displacement $x(t)$ has no influence on the accuracy. The equation (8a) gives the dynamic stiffness $k = F_0/a$, the coefficient of linear damping of the rubber element is proportional to the angle ψ: $b \approx a/F_0 \sin\psi$ and can be ascertained by (8b).

The amplitude of response can be calculated also from the simpler formula

$$\frac{a}{F_0} = \sqrt{\frac{\sum\limits_{i=1}^{n} X_i^2}{\sum\limits_{i=1}^{n} Y_i^2}} \ , \tag{9}$$

but it gives over-estimated values, due to the greater influence of higher harmonics.

The second method is based on the Discrete Fourier Transformation (DFT) of both signals. Let $x(i\,\Delta T)$, $y(i\,\Delta T)$ $(i = 0 \div n)$ represent the synchronous discrete time functions of deformation and force signals. Then using the DFT we receive their corresponding discrete frequency images $X(k\,\Delta f)$, $Y(k\,\Delta f)$ $(k = 0 \div n)$ given by

$$\begin{aligned} X(k\,\Delta f) &= \sum_{i=0}^{n} x(i\,\Delta T)\ \exp(-\mathrm{j}\,i\,\Delta T\,2\pi\,k\,\Delta f)\ , \\ Y(k\,\Delta f) &= \sum_{i=0}^{n} y(i\,\Delta T)\ \exp(-\mathrm{j}\,i\,\Delta T\,2\pi\,k\,\Delta f)\ , \end{aligned} \tag{10}$$

where $\Delta f = 1/(n+1)/\Delta T$ is sampling frequency step.

The ratio of the functions of (10) gives us the complex dynamic stiffness K for the selected items $k\,\Delta f$ of frequencies

$$K(k\,\Delta f) = \frac{Y(k\,\Delta f)}{X(k\,\Delta f)} = K' + \mathrm{j}\,K''\ . \tag{11}$$

The part K' corresponds directly to stiffness of the specimen and its loss angle ψ can be evaluated from the relation

$$\tan(\psi) = \frac{K'}{K''}\ . \tag{12}$$

4. Example

The measurement of investigated specimen was evaluated first by the wattmetric method using the formulae (5) and (8) which are successively applied on each of the 36 loading states given by the combinations of force amplitudes (1) and frequencies (2) of excitation

The digital sampling rate was 500 samples per second. The appropriate intervals of stationary oscillations were selected and evaluated in each state of loading. So we get the sets of 36 values of absolute compliance a/F_0 or stiffness $k = F_0/a$ and 36 values of corresponding phase angles ψ. These values obtained from measurements by apparatus I are drawn in Fig. 5a (stiffness $k = F_0/a$ versus F_0) and in Fig. 5b (angle ψ versus F_0). Measurements at frequency $f = 10\,\mathrm{Hz}$ are distinguished by dotted lines, those at $f = 20\,\mathrm{Hz}$ by dashed lines and those at 30 Hz by full lines. The dependence of both stiffness and damping on exciting frequency is negligible, but both change with increasing amplitudes of loading force.

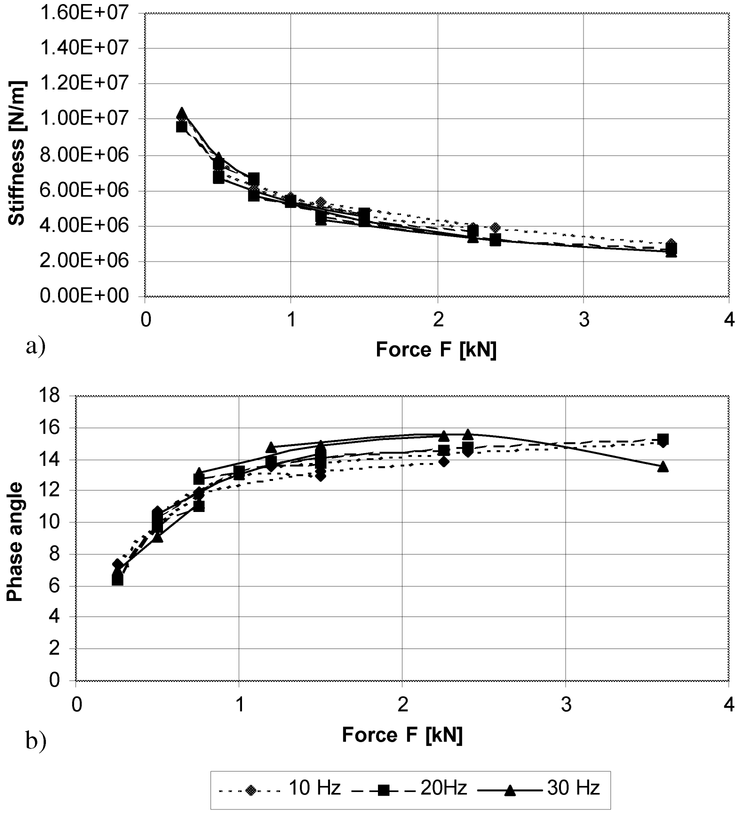

Fig.5: a) stiffness $k = F_0/a$, b) damping phase angle ψ, versus amplitude F_0 loading – apparatus I

The results of measurement by apparatus II are presented in Fig. 6a and 6b. By comparison of Figures 5 and 6 it is shown that both types of apparatus give the very close values.

Because the influence of exciting frequency proved to be very small in the range of 10 to 30 Hz, the measured sets of data can be mathematically described by a simple algebraic function with only one variable – loading force F_0. We chose logarithmic (for stiffness k) and polynomial (for damping angle ψ) regression functions, which fitted these data in least-square sense. For stiffness $k = F_0/a$ and for data gained by apparatus I we get

$$k = -2.025\times10^6 \ln(F_0 - 0.19) + 4.92\times10^6 \,, \tag{13}$$

where $[k]$ = kN/mm, $[F_0]$ = kN.

The phase shift ψ is expressed by following function of

$$\psi = 15.96 - \frac{2.496}{F_0} - 0.056\, F_0^2 \,, \tag{14}$$

where ψ is in degrees.

Averaging values given by equations (13) and (14) enter in the diagrams Fig. 7a,b by bolt-solid curves.

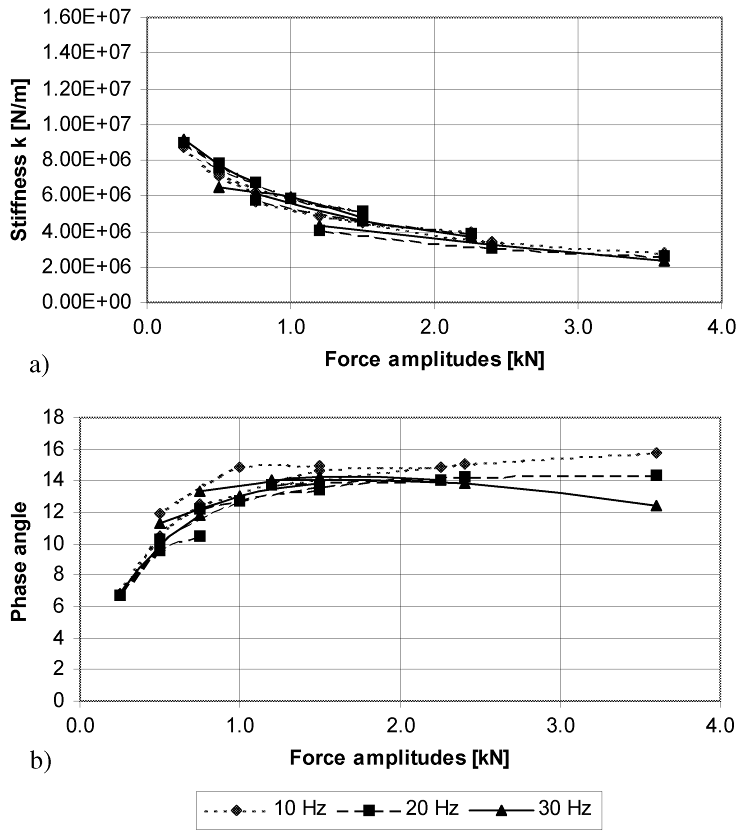

Fig.6: a) stiffness $k = F_0/a$, b) damping phase angle ψ, versus amplitude F_0 loading – apparatus II

Similar measured data and curves of evaluations were gained from the measurement by the apparatus II. The results of evaluation based on equations (5) and (8) are plotted in the form of rhombuses in Fig. 8a,b, where the regression curves are drawn as well. These regression curves are expressed by

$$k = -2.01\times10^6 \ln(F_0 - 0.15) + 4.92\times10^6 \tag{15}$$

for stiffness (Fig. 8a) and by equation

$$\psi = 15.79 - \frac{2.489}{F_0} - 0.046\, F_0^2 \ , \tag{16}$$

for damping angle (Fig. 8b).

The evaluation of all measured time histories of excitation forces and stationary vibrations were carried out also by the second evaluation procedure based on DFT transformations (see equations 11, 12). Gained values are plotted in Fig. 8a,b and marked by great squares. It is seen that both evaluation methods give very close results.

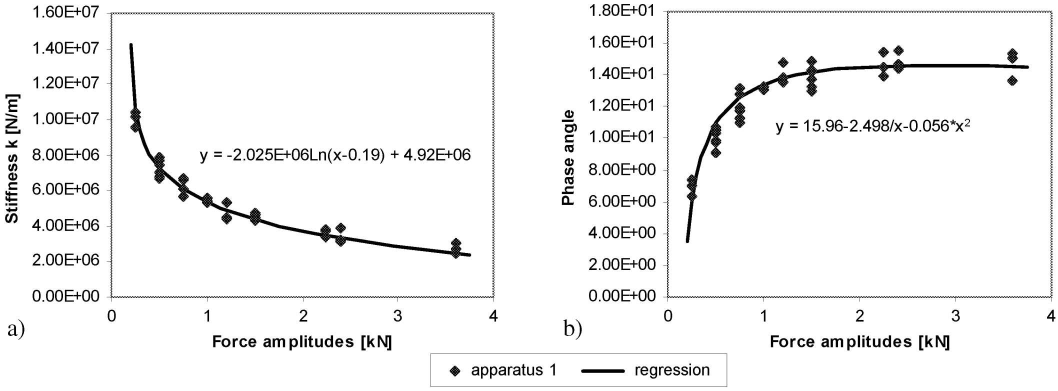

Fig.7: a) logarithmic regression function of stiffness k, b) polynomial regression function of damping angle ψ

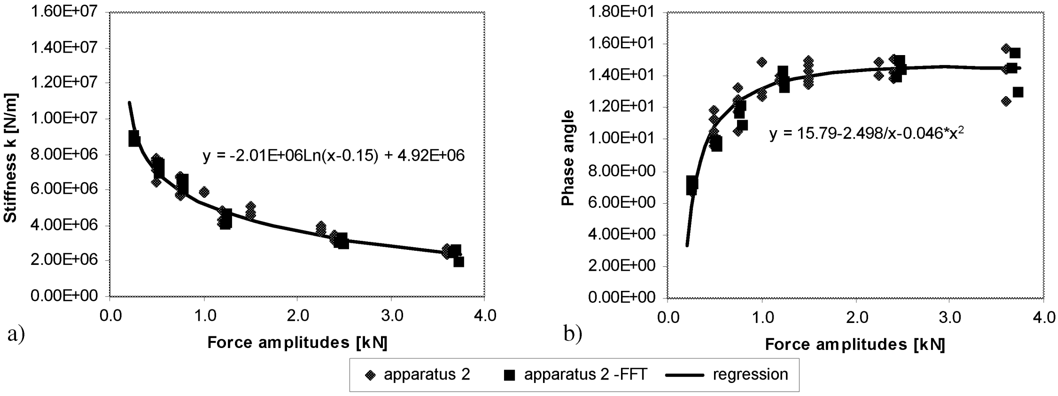

Fig.8: a) logarithmic regression function of stiffness k, b) polynomial regression function of damping angle ψ, – comparison with DFT evaluation

5. Conclusion

Our investigation may briefly be reviewed in the following conclusion:

– The double measurements of dynamical properties of rubber element together with the application of two different methods of data evaluation increase the reliability of gained results.

– The dynamical properties - stiffness and phase angle – of the investigated rubber element are not sensitive on frequency of excitation in the range 10–30 Hz.

– The amplitudes of loading harmonic exciting forces in the range $F_0 = 0.25$–3.6 kN, ($\sigma = 0.062$–0.9 MPa) influence very significantly the stiffness (decrease in the ratio 3:1) and damping angle of the investigated material.

– The influence of temperature increase caused by the lost energy due to the inner material damping was not considered in this contribution. The more precise measurement of temperature during the mechanical loading is now prepared and it will be used for creation of thermo-mechanical rheological model of damping element.

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