LINEAR AND NONLINEAR DAMPING IN DYNAMICS OF GEAR MESH OF THE PARAMETRIC SYSTEMS WITH IMPACTS

Milan Hortel, Alena Škuderová*

The aim of this contribution is the analysis of damping properties both the material of gear mechanism in the mesh and the lubricating oil film in tooth space at tooth profile contact bounce into the area of technological gear backlash. The damping influence over gear mesh stability is pursued on the special case of simulation model of system with split power flow for selected frequency area of resonance characteristic.

Key words: *nonlinear dynamics, parametric systems, motions with impacts, gearing systems, numerical solution*

1. Introduction

Present and future developments in the world of modern machines generally more and more lead with their requirements to symbiosis of parameters in two extreme areas. It is on the one hand the requirements for maximum power outputs of machines at high revolutions and on the other hand the requirements for their minimum dimensions and mass, while retaining operational reliability, safety, and service lifetime or durability.

The above mentioned, apparently contradictory extreme requirements, are at the present time fulfilled most of all by systems with kinematic couplings – gears with split or branched power flow, of all mechanical transmission systems. Their designing has to be based on perfect knowledge of dynamic phenomena, which can occur in these weakly and strongly nonlinear parametric systems.

Nonlinear dynamics of parametric, i.e. heteronomous systems forms in recent some decade special, highly topical branch, which dominates especially in planetary transmission systems with kinematic couplings in aeronautical turbopropelled units. High revolution of turbines (order of tens of thousands rpm) are connected with the requirements for light elastic design and take effect especially in the dynamics of kinematic couplings – gear mesh. Similar structures are applied both in mobile machines – for instance Wilson's transmission systems containing planetary differential mechanisms and in stationary driving units namely with turbined units etc.

The important problem here performs from internal dynamics e.g. the impact effects in gear mesh due to existence technological tooth backlash, eventually mounting tolerances when dynamic deformations are greater than static-elastic deformations. The impact effects caused by influence of teeth bounce are respected in the mathematical description by so-called strongly nonlinearities.

* Ing. M. Hortel, DrSc., Ing. A. Škuderová, Ph.D., Institute of Thermodynamics AS CR, v.v.i., Dolejškova 5, Praha 8

Theme of contribution, which was presented at conference 'Dynamics of Machines 2006' and which referring to works subject [1–3], is the analysis of damping properties both the material of gear mechanism in the mesh and the lubricating oil film in tooth backlash at tooth profile contact bounce into the area of technological gear backlash. The influence of damping over dynamic properties of nonlinear parametric system is analysed in all phases of gear mesh, i.e. in the phase of normal gear mesh, in the phase of teeth profile contact loss and in the phase of inverse gear mesh. The damping whether material – in the gear mesh, or viscous – in the area of gear backlash is assumed partly linear, partly nonlinear – quadratic and cubic. The damping influence over gear mesh stability is pursued on the special case of simulation model of system with split power flow for selected frequency area of resonance characteristic.

For the motion equations composition of mathematical physical models of planetary systems, alternatively their special cases with kinematic couplings – gears has been applied Lagrange's method. Lagrange (1736–1813) summarizes in his last work 'Mechanique analytique' (1788) whole then progress in mechanics as integrated scientific subject of knowledge. There is concerned systematically processing of then pieces of knowledge, which consist in three fundamentals:

– principle of virtual working*,
– d'Alambert's principle**,
– Lagrange's principle of release***.

The solution of that way created motion equations of mechanic discrete system in the form of weakly and strongly nonlinear parametric ordinary differential equations of second order is carried out by means of

1) analytic: transformation of boundary differential problem onto equivalent system of integrodifferential equations with solving kernel in form of Green's resolvents and E. Schmidt's method of kernel splitting [4],
2) numerical: on the simulation model of system in MATLAB/Simulink [5].

Problems of gear conditions of teeth profiles of wheels is by the light shell structures of transmission housing in comparison with stiff those, more complicated. Gear profiles of kinematic pairs pursue complicated motion rolling – sliding namely sliding resulting as from gear mesh geometry so from relative motions of elastic bearings, i.e. motions caused as by elastic bearings so by wheel run-out. Gear profiles of kinematic pairs pursue complicated motion rolling – sliding namely sliding resulting as from gear mesh geometry so from relative motions of elastic bearings, i.e. motions caused as by elastic bearings so by wheel run-out. The velocity of creation and quality (carrying capacity) of oily film after recontact of teeth profiles in normal or inverse mesh depend on

– the contact distance from central point (line) of gear mesh,
– the size of relative motion of meshing teeth profiles from motions of elastic bearings.

In accordance with brief consideration is evident that creation of carrier oily film in gear mesh is complicated matter of tribology and dynamic forces here create conditions for point of issue medium- dry or dry friction, and conditions for issue self-excited vibrations. This

* Johann Bernoulli (1667–1747) formulates principle of virtual working 1712.

** D'Alambert (1718–1783) presents universal principle of mechanics, when transforms problems of kinetics on problems of statics, s. Traité de dynamique, 1743.

*** A. A. Andronov, M. A. Leontovič: O kolabanijach sistěmy s periodičeski menjajuščimsja parametrami. Ž. Russ. Fiz. Chim. Obšč. (fiz) 559, 429–443.

phenomenon is so very important in dynamics of gearing by analysis of motion in mechanical systems with kinematic couplings.

Parametric excited vibration constitutes partial area of heteronomous vibration, which is quickly exploited by reason of physical trend of light structures. The vibration can be called heteronomous, when can be described by differential equations of second order in form

$$y'' = f(y, y', t) ,$$

autonomous afterwards vibration, where no explicit dependence on time occur. If there are coefficients – parameters on y dependent terms with time periodic, is vibration called parametric excited (after A. A. Andronov and M. A. Leontovič).

In the given model is parametric function represented by time variable resulting stiffness function in gear mesh.

2. Mathematical physical model of spur gear mesh

The influence of parametric vibration is below pursued on the model of one gear of one branch of power – forces flow of pseudoplanetary transmissive reducer with three double satellites, whose substitute mechanical system is presented in Fig. 1. It is the elastic supported spur gear unit with six degrees of freedom.

The cog wheels ($j = 3, 2$, m_j … the mass, J_j … inertia torque, φ_j … angular displacement, $R_{\mathrm{b}j}$ … radius of basic circle, M_j … external momentum, z_j … number of teeth of j^{th} wheel, α' pressure angle) are connected each other parallel with the spring $C(t)$ and the damper, which model elasticity in gear mesh. The time variable function $s(t)$ is the teeth backlash. This is influenced by possible eccentricities e_j of wheels and their relative angular displacement for the phase angle Δ and in the case of elastic mounting gear even by the motions in bearing y_j, z_j of wheels. ${}^{1,2}f(t)$ is the deflect function, or the deviation of the cog side form from the ideal involute, ${}^{1,2}F_{\mathrm{T}}(t)$ is the friction force in gear mesh. The index number 1 denotes the normal mesh, 2 the inverse mesh. The bearings of wheels are modelled by the stiffness $C_{j\mathrm{y}}$, $C_{j\mathrm{z}}$ and the damping coefficients $k_{j\mathrm{y}}$, $k_{j\mathrm{z}}$ in two orthogonal directions of the coordinate system, which is oriented in the direction of the mesh line c.

The resulting stiffness function $C(t)$ changes periodically at mesh line in course of change generally k and $k+1$ pairs of teeth in gear mesh.

Analytical form of the resulting stiffness function of spur gearing in mesh can be expressed by Fourier's series in form [6]

$$C(t) = C_{\mathrm{s}} + \frac{C_{\max}\,(1-\kappa)}{2} \sum_{n=1}^{\infty} \frac{4}{\pi\, n}\, (-1)^n\, \sin n\,[(\varepsilon - 2)\,\pi]\, \cos n\,\omega_{\mathrm{c}}\, t \ , \tag{1}$$

where the mean stiffness is defined by

$$C_{\mathrm{s}} = \kappa\, C_{\max} + \frac{C_{\max}\,(1-\kappa)}{2}\, [1 + (2\,\varepsilon - 3)] \ . \tag{2}$$

The symbol $\kappa = C_{\min}\, C_{\max}^{-1}$ represent the amplitude modulation of resulting stiffness function in gear mesh, $C_{\min}$, $C_{\max}$ are minimal and maximal values of stiffness in gear mesh and ε is coefficient of mesh duration, which indicates how many teeth pairs is at any one

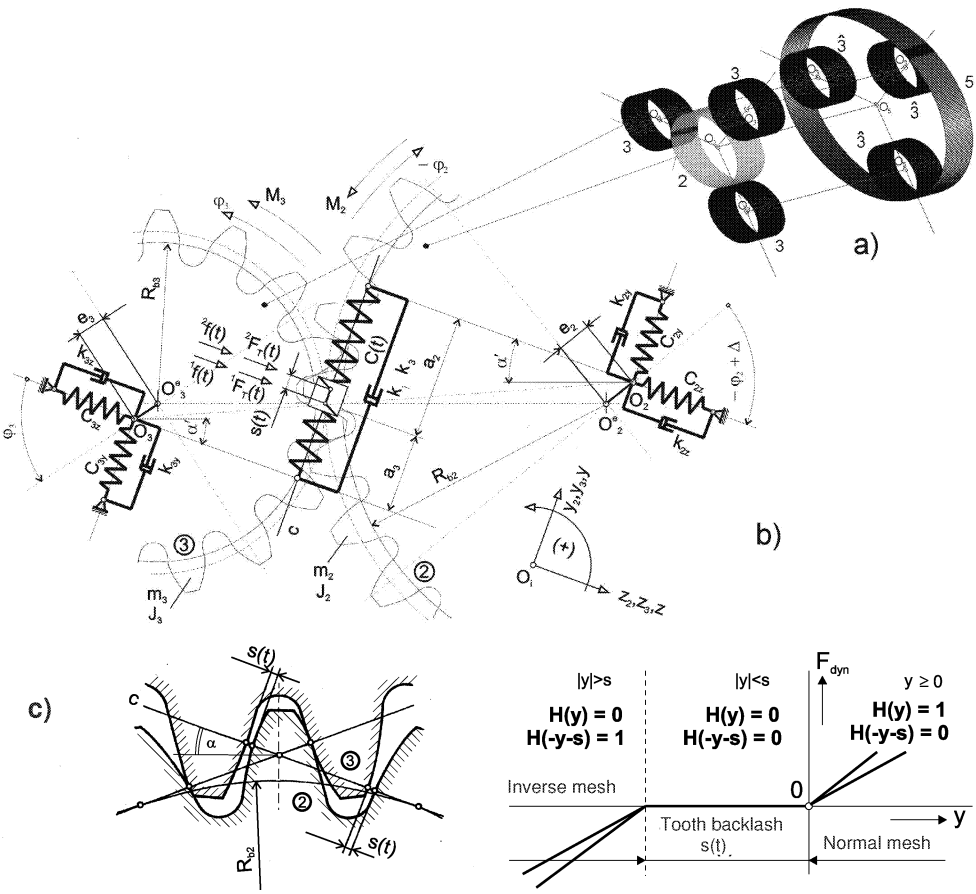


Fig.1: The substitutive mathematical – physical model of kinematic pair of gears (b) of pseudoplanetary system with double satellites – (a), the technological teeth backlash and the values of Heaviside's functions H in the areas of gear mesh with clearances (c)

time in mesh at mesh line. In extreme cases, for example $\varepsilon = 1$, is during the mesh time at mesh line only one teeth pair, in the case $\varepsilon = 2$ are two pairs of teeth whole time in mesh. Intermediate values ε determine relationship of the alternation of teeth pair number in mesh at mesh line. In the Fourier's series (1) ε determines the time relationship of alternation minimal and maximal resulting stiffness $C_{\min}$, $C_{\max}$ during the gear mesh. This fact indicates markedly in dynamics of system in size of amplitude of relative motion in gear mesh. The amplitude is impacted by time duration of gear mesh at the given potential level of stiffness of relevant reversible force. In the stiffness of teeth will in the next application respected only the stiffness of separate cogs and fixation into a solid half-space, discs are considered absolutely solid.

The motion equations are composed by means of Lagrange's methodology in the form [6]

$$\begin{aligned} &\mathbf{M}\,\mathbf{v}'' + {}_1\mathbf{K}(\beta,\delta_i,H)\,\mathbf{v}' + \sum_{K_1>1} {}_{K_1}\mathbf{K}(D,D_i,H)\,|\mathbf{w}'(\mathbf{v}')|^{K_1}\,\mathrm{sgn}(\mathbf{w}'(\mathbf{v}')) + \\ &+ {}_1\mathbf{C}(\varepsilon,\kappa,Y_n,U_n,V_n,H,\tau)\,\mathbf{v} + \sum_{K>1} {}_K\mathbf{C}(\varepsilon,\kappa,I_n,H,\tau)\,\mathbf{w}^K(\mathbf{v}) = \mathbf{F}(a_n,b_n,\bar{\varphi},H,\tau)\ . \end{aligned} \tag{3}$$

Here $\mathbf{v}$ means generally the m-dimensional vector of displacement of system vibration, $\mathbf{w}^K(\mathbf{v})$ K-th power of vector $\mathbf{v}$, which is defined by expression $\mathbf{w}^K(\mathbf{v}) = \mathbf{D}(\mathbf{w}(\mathbf{v})\,\mathbf{w}^{K-1}(\mathbf{v}))$. $\mathbf{D}(\mathbf{w}(\mathbf{v}))$ denotes the diagonal matrix, whose elements at the main diagonal are comprised by elements of vector $\mathbf{w}(\mathbf{v}) \equiv \mathbf{v}$. Furthermore $\mathbf{M}$ is the matrix of mass and inertia forces, ${}_1\mathbf{K}$ and ${}_{K_1}\mathbf{K}$ are the matrix of linear and nonlinear damping forces, ${}_1\mathbf{C}$ and ${}_K\mathbf{C}$ are the matrix of linear and nonlinear reversible forces and $\mathbf{F}(\tau)$ is the vector of non-potential external excitation with components a_n, b_n and with the phase angle $\bar{\varphi}$. H is the Heaviside's function, which allows to describe the motions – contact bounces – due to strongly non-analytical nonlinearities, for example due to technological tooth backlash $s(\tau)$. Corresponding linear and nonlinear coefficients of damping are denoted by β, δ_i, D, D_i linear parametric stiffness function by the symbols Y_n, U_n, V_n and nonlinear parametric functions, so-called parametric nonlinearities, by the symbol I_n. ε and κ are the coefficients of mesh duration and amplitude modulation of stiffness function ${}_1\mathbf{C}$. Derivative by non-dimensional time τ are denoted by dashes, $\tau = \omega_\mathrm{c}\,t$, ω_c ... mesh frequency, t ... time.

The relative motion as the measure of dynamic loading in the gear mesh, i.e. in the course of mesh line, can be described for the generally elastic supported system with bearing motions $\{y_{3,2}; z_{3,2}\}$ of the gear pairs 3,2 by respecting so-called run-out of pitch circles, which are modelled by eccentricities $e_{3,2}$, in the form

$$y(\tau) = R_{\mathrm{b}3}\,\varphi_3 + R_{\mathrm{b}2}\,\varphi_2 + y_3 - y_2 + e_3\,\sin\varphi_3 - e_2\,\sin(\Delta - \varphi_2) + {}^{1,2}f(\tau)\ . \tag{4}$$

The dynamic force in gear mesh can be expressed in the form $F_\mathrm{dyn} = {}_1C(\tau)\,y(\tau)$. The technological tooth backlash s is not in elastic supported systems constant s_k, but is time function $s(t)$, respectively $s(\tau)$

$$s(\tau) = -s_\mathrm{k} + y_3 - y_2 + e_3\,\sin\varphi_3 - e_2\,\sin(\Delta - \varphi_2)\ ,$$

see Fig. 1.

3. The analysis of special cases of damping

The resonance characteristics of linear as well as nonlinear systems with constant coefficients with typical overhangs of solidifying or softening resonance characteristics has been enough known. The same cannot be declared in full about the resonance characteristics of linear or nonlinear parametric systems, where can occur the all phenomena of linear or nonlinear systems including the influences of time variation of stiffness level and phase shift of amplitude of relative motions y by reason of damping forces action against the parametric exciting function $C(t)$, see Fig. 2. The phase shift of relative motions $y(t)$ is here by reason of clearness evident from two courses of $y(t)$ with hundredfold different coefficient of linear material damping k_1 of cogs in gear mesh. In figure mentioned values k_1 conform to proportional damping $\beta = 0.0624$ for $k_1 = 4\,\mathrm{N\,mm^{-1}\,s}$, $\beta = 0.000624$ for $k_1 = 0.04\,\mathrm{N\,mm^{-1}\,s}$.

The resonance characteristics for the fifth revolution of gear wheels with strong nonlinearities – tooth backlash are shown in the coordinates of relative motion y depending on the tuning ν_s, that is related to mean value of resulting stiffness C_s of gear in mesh, see equation (2)

$$\nu_\mathrm{s} = \omega_\mathrm{c}\,\Omega_\mathrm{s}^{-1}\ ,$$

where ω_c is mesh frequency of given gearing and $\Omega_\mathrm{s}^2 = C_\mathrm{s}\,m_\mathrm{red}^{-1}$ is mean eigenfrequency, m_red is reduced mass of solved model [6].

In given parametric, i.e. heteronomous, systems but the stiffness functions, see eq. (1), and consequently also the eigenfrequencies in gear mesh vary between $\Omega_{\rm max}$ (for $C_{\rm max}$) and $\Omega_{\rm min}$ (for $C_{\rm min}$) in certain time distances t. These time distances t, i.e. time duration generally k and $k+1$ cogs pairs in mesh are functions of coefficient of mesh duration ε and the amplitude modulation κ. For every ε and κ has resonance characteristic the different course. The time distances t in them the gear mesh inhered in that which frequency tuning $\nu_{\rm max}$ or $\nu_{\rm min}$ are deciding about extension of amplitude course at that which stiffness level of gear mesh. For example for $\varepsilon = 1.1$, see Fig. 4a, prevails the time distance of gear mesh at the minimal stiffness level $C_{\rm min}$, conversely for $\varepsilon = 1.9$, see Fig. 4b, prevails gear mesh at the maximal stiffness level $C_{\rm max}$. The course of relative motion y, which is plotted by thin line, is for the system with linear material damping in gear mesh $k_1 = 0$, i.e. for conservative system in gear mesh, the course of relative motion y which is marked by the thick line is for the non-conservative system with damping $k_1 \neq 0$. The values of linear material damping k_1 and linear viscous damping in tooth backlash $k_{\rm 1m}$ are considered in all next given examples of solution identical, i.e. $k_1 = k_{\rm 1m} = 3.95\,{\rm N\,mm^{-1}\,s}$, which corresponds to proportional damping $\beta = \beta_{\rm m} = 0.062$.

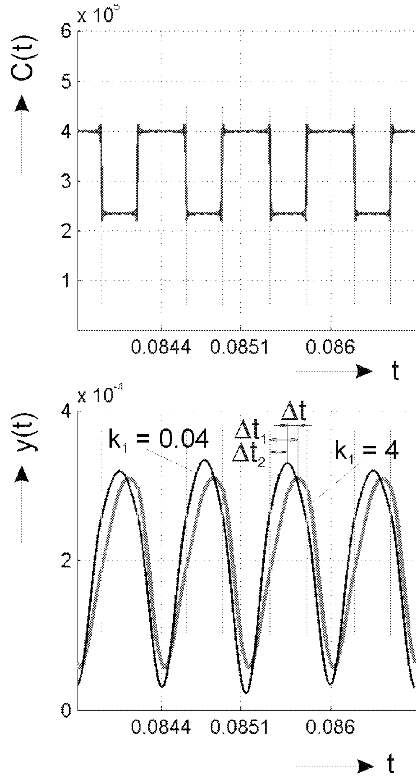


Fig.2: The exemplification of phase shift in time courses of relative motion as a result of the size of coefficients of the linear material damping k_1 towards the parametric excited function $C(t)$

In Fig. 3 are given by reason of comparison the scale of tuning $\nu_{\rm s}$ towards mean value of resulting stiffness $C_{\rm s}$ in gear mesh and scales of tuning $\nu_{\rm max} = \omega_{\rm c}\,\Omega_{\rm max}^{-1} = \nu_{\rm s}\,\Omega_{\rm s}\,\Omega_{\rm max}^{-1}$ and $\nu_{\rm min} = \omega_{\rm c}\,\Omega_{\rm min}^{-1} = \nu_{\rm s}\,\Omega_{\rm s}\,\Omega_{\rm min}^{-1}$ towards maximal $C_{\rm max}$ alternatively minimal $C_{\rm min}$ stiffness values in gear mesh, with $\Omega_{\rm max}^2 = C_{\rm max}\,m_{\rm red}$ and $\Omega_{\rm min}^2 = C_{\rm min}\,m_{\rm red}^{-1}$.

In the graph are determined the area of normal gear mesh for $y \geq 0$ – by white coloured area, phase with contact bounces due to tooth backlash $s(t)$, where $|y| < s(t)$ – light grey coloured area and phase of inverse gear mesh, where $|y| > s(t)$ – dark grey coloured area. The resonance characteristic of relative motion y of cogs in mesh are given in Fig. 3 for different combination of linear damping both material k_1 and viscous $k_{\rm 1m}$ in lubricating oil film, alternatively in lubricating medium in the phase of contact bounce.

The courses $y_{\rm ma}$, $y_{\rm min}$ for purely conservative system, i.e. for $k_1 = k_{\rm 1m} = 0$, are marked by circlets. The courses have in the frequency area $\nu_{\rm s} \in \langle 0.6; 0.8)$ smooth, i.e. continuous character excepting the small area in vicinity $\nu_{\rm s} = 0.66$. With exception of this singularity is the kinematic pair of teeth in the normal mesh $y \geq 0$. In the area $\nu_{\rm s} \approx 0.8$ dawn the contact bounces $(y < 0)$, the motion through tooth backlash, even inverse gear mesh $(|y| > s)$ and

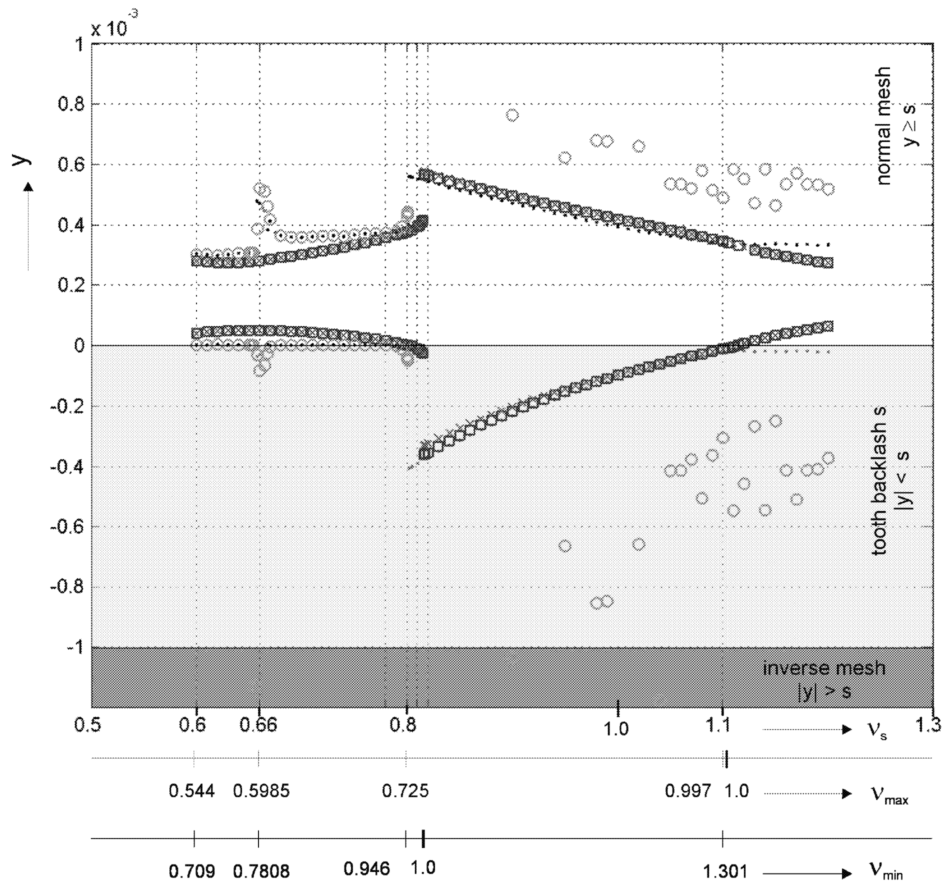


Fig.3: The resonance characteristics $\{y;\nu\}$ of parametric – heteronomous system for $\varepsilon = 1.569$; $\kappa = 0.5879$; $C_{\max} = 0.4{\times}10^6\,N\,mm^{-1}$; $m_{\mathrm{red}} = 0.003123\,kg$, for the linear damping: $\square$ – material $k_1 \neq 0$; $k_{1\mathrm{m}} = 0$; $\bullet$ – in the tooth backlash (viscous) $k_1 = 0$; $k_{1\mathrm{m}} \neq 0$; $\times$ – material and viscous $k_1 \neq 0$; $k_{1\mathrm{m}} \neq 0$; $\circ$ – conservative system in gear mesh $k_1 = 0$; $k_{1\mathrm{m}} = 0$

impact effects. For $\nu_{\mathrm{s}} > 0.8$ appear in the whole area the similar phenomena with contact bounces and inverse gear mesh. The course of relative motion y has the chaotic character.

The other courses y in Fig. 3 for the non-conservative system with different combination of damping have continuous convergent character excepting the area $\nu_{\mathrm{s}} \approx 0.8$. In the interval $\nu_{\mathrm{s}} \in \langle 0.6; 0.8\rangle$ has the solved system continuous divergent character with the normal gear mesh. The amplitudes of relative motion y increase in the area $\nu_{\mathrm{s}} \in \langle 0.8; 1.12\rangle$, the contact bounces come up with consequential impact effects. There is not inverse gear mesh. The gear mesh in area $\nu_{\mathrm{s}} > 1.12$ is in the phase of purely normal mesh excepting the damping variant $k_1 = 0$; $k_{1\mathrm{m}} \neq 0$. The course of relative motion of this damping variant is marked by dots.

The phase planes $\{y'; y\}$ of relative motion in gear mesh are given in Fig. 5 for six values of tuning ν_{s} and four combinations of linear damping k_1, $k_{1\mathrm{m}}$ by reason of the better physical explanation of some quantitative and qualitative phenomena in resonance characteristics from Fig. 3. The values of tuning ν_{s} were selected from resonance characteristics what are

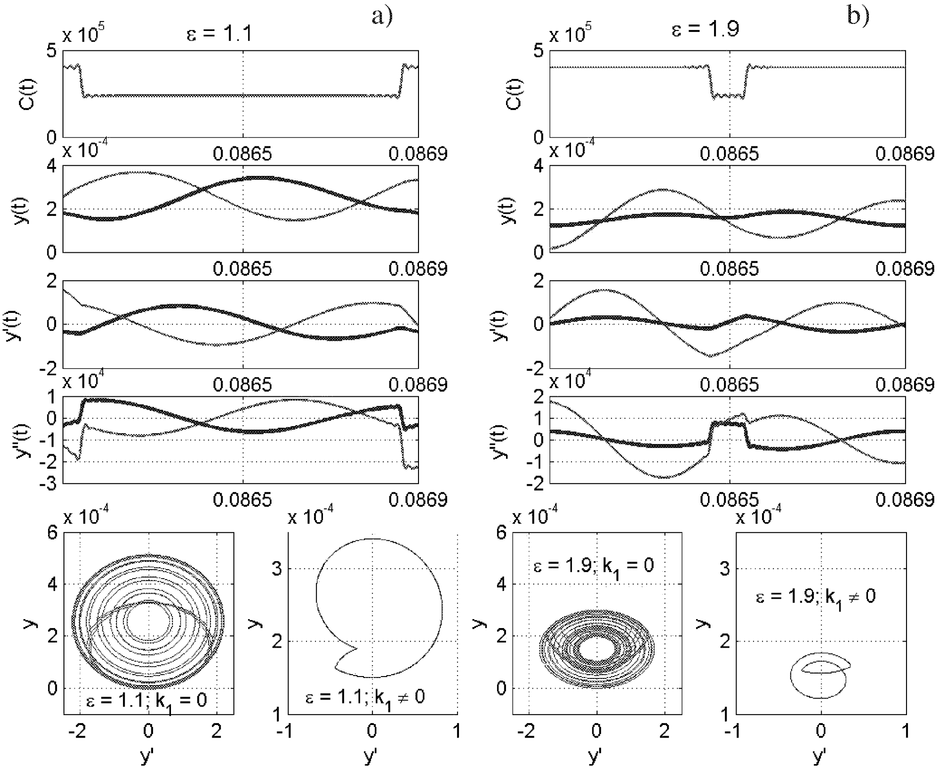


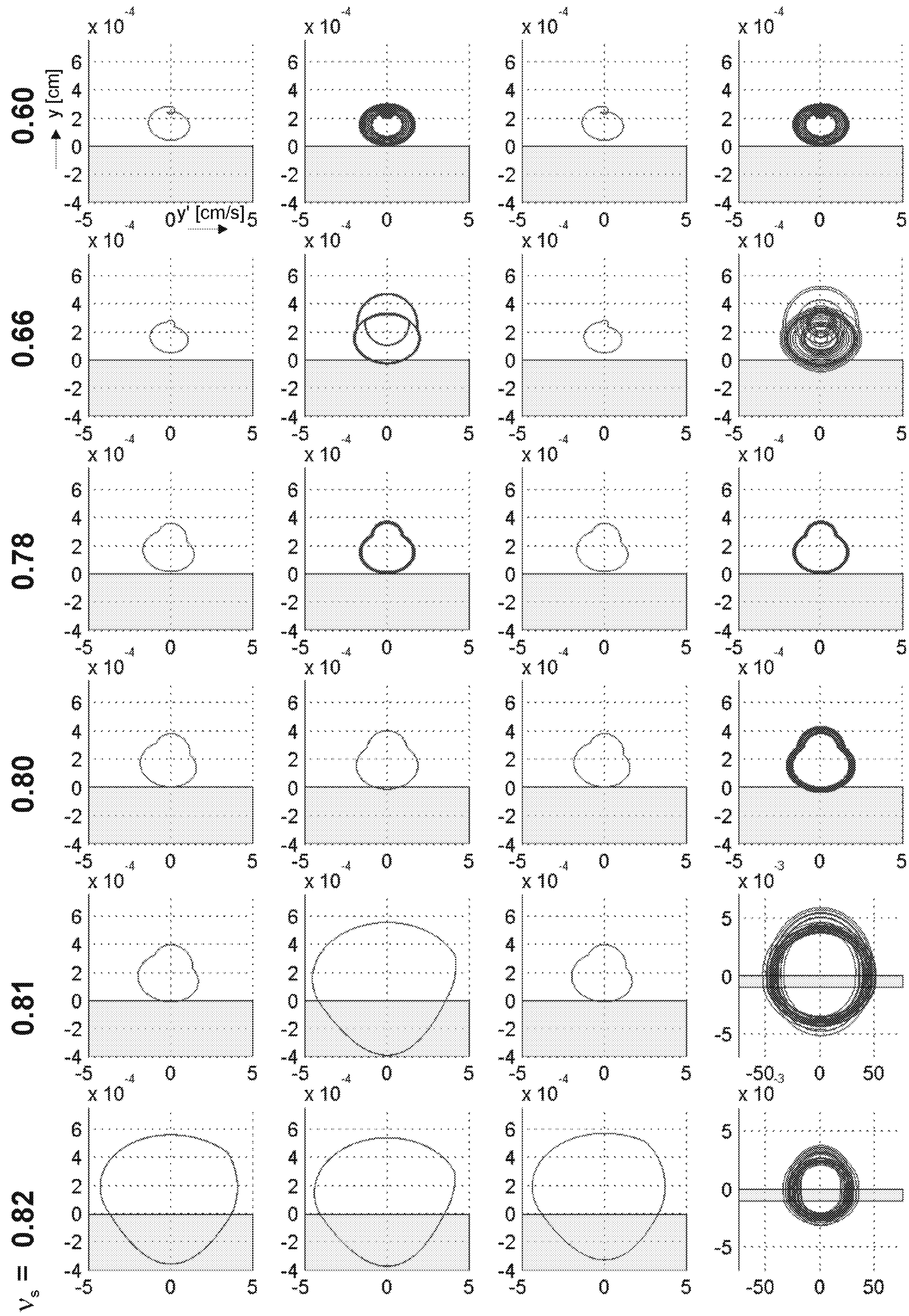
Fig.4: The time course of stiffness function $C(t)$, relative motions $y(t)$, their velocity $y'(t)$, acceleration $y''(t)$ and the phase planes $\{y';y\}$ for a) $\varepsilon = 1.1$ and $k_1 = 0$, $k_{1\mathrm{m}} > 0$; b) $\varepsilon = 1.9$ and $k_1 = 0$, $k_{1\mathrm{m}} > 0$

given in Fig. 3. They are data record of fifth revolution of gear wheels with transmission $i = 1$. The area which are marked by light grey colour represent the area of tooth backlash.

From Fig. 5 is evident that all phase planes $\{y';y\}$ of fourth column for damping values $k_1 = k_{1\mathrm{m}} = 0$ present unsteady solution of analysed system of gears in mesh and have for tuning values $\nu_{\mathrm{s}} = 0.66, 0.80, 0.81, 0.82$ unstable gear mesh with contact bounces, i.e. with impact effects. For the tuning values $\nu_{\mathrm{s}} = 0.81, 0.82$ arise even the inverse gear mesh.

Similarly unsteady motions of different intensities demonstrate the phase planes in the second column, i.e. for $k_1 = 0$, $k_{1\mathrm{m}} \neq 0$ and the tuning $\nu_{\mathrm{s}} = 0.60, 0.66, 0.78$. In all others cases are the phase planes of relative motion in gear mesh during the fifth revolution of gears for given tuning steady.

As has already been noted on beginning of this section, the resonance characteristics of here analysed parametric systems with impact effects in the gear mesh are sensitive function especially of parameters of amplitude modulation κ and coefficients of mesh duration ε of resulting stiffness function $C(t)$ in gear mesh, see eq. (1), (2). The greater is the number κ, the greater is the amplitude of potential stiffness level $C_{\max}$, $C_{\min}$ of resulting stiffness function $C(t)$ in gear mesh with corresponding possible dynamic – resonance effect on the course of relative motion $y(t)$, as is for example evident from Fig. 5 for the phase planes $\{y';y\}$ in the range of tuning $\nu_{\mathrm{s}} = 0.81$ and $\nu_{\mathrm{s}} = 0.82$ and for combination of linear material damping k_1 of cogs in the gear mesh and the viscous mediums $k_{1\mathrm{m}}$ in the tooth backlash $s(t)$.



k_1	$\neq 0$	0	$\neq 0$	0
k_{1m}	0	$\neq 0$	$\neq 0$	0

Fig.5: The phase planes $\{y'; y\}$ of relative motions y in gear mesh for the combinations of linear material damping k_1, viscous those $k_{1\mathrm{m}}$ and the tuning ν_s

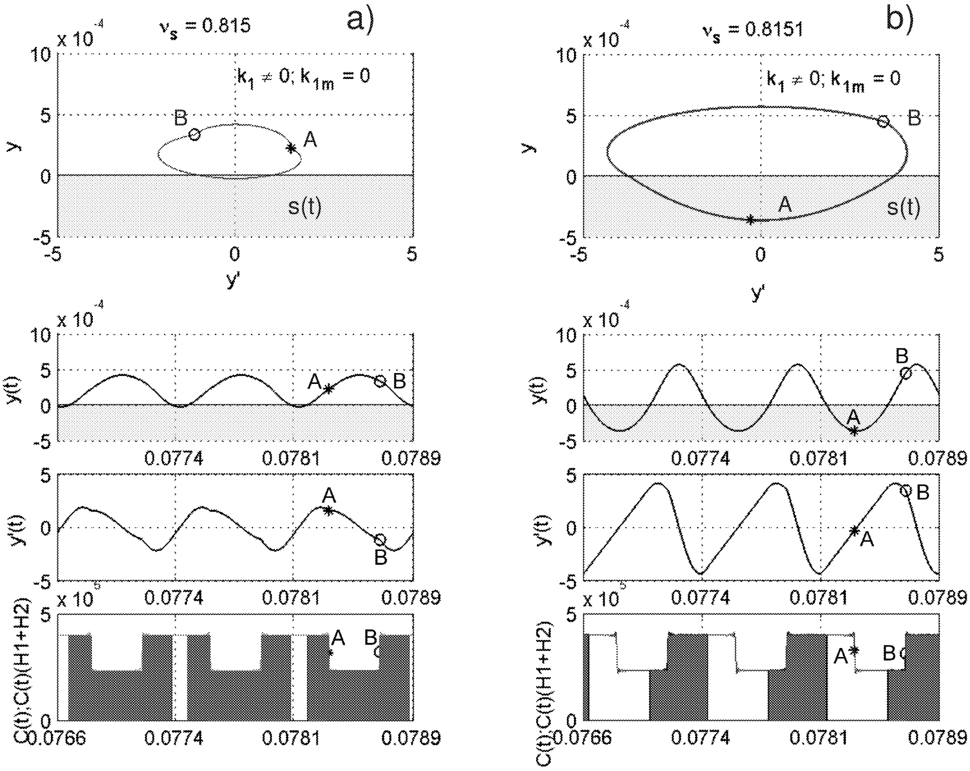


Fig.6: Comparison of influence of modify stiffness functions $C(t)(H1 + H2)$ on the relative motion $y(t)$ in gear mesh by normal mesh and by phase of contact bounce for the tuning: a) $\nu_s = 0.815$; b) $\nu_s = 0.8151$

The explanation of the so large springs in the phase planes in mentioned small interval of frequency tuning $\nu_s = 0.81$ and $\nu_s = 0.82$ is evident from Fig. 6. Here are mentioned for the tuning $\nu_s = 0.815$ and $\nu_s = 0.8151$ the phase planes $\{y'; y\}$ and courses of $y(t)$, $y'(t)$ including the courses of $C(t)$ and of modify stiffness function $C(t)(H1 + H2)$ for the same values of damping k_1 and for $\kappa = 0.5879$. For the tuning $\nu_s = 0.815$ is the value of frequency tuning $\nu_{max} = 0.7391$ and $\nu_{min} = 0.9639$, for $\nu_s = 0.8151$ is $\nu_{max} = 0.7392$ and $\nu_{min} = 0.9640$. Both of the values ν_{min} are near the main resonance.

The exciting modifying function are significantly different for the fifth revolution of cog wheels and three last period accordant with the mesh frequency ω_c. Their courses are denoted by dark grey colour. While the time course in Fig. 6a begin with contact bounces of cogs into the tooth backlash $s(t)$ (denoted by light grey colour), in the Fig. 6b run the gear mesh at the potential stiffness level C_{max}. After the follow-up contact of cogs sides with impact run the gear mesh in the Fig. 6a both at the maximal level C_{max} and at the minimal stiffness level C_{min} till the follow-up contact bounce in the tooth backlash $s(t)$. The gear mesh in the Fig. 6b run after the longer contact bounce at the lower potential level C_{min} (i.e. the amplitude of relative motion $y(t)$ grow fast) at the level C_{max} and there occurs the contact bounce of cogs (light grey area). The amplitude of relative motion $y(t)$ in Fig. 6a reaches in contrast to its course in Fig. 6b the lower value in normal gear mesh and as well the lower value in the phase of contact bounces, see the singular area of resonance

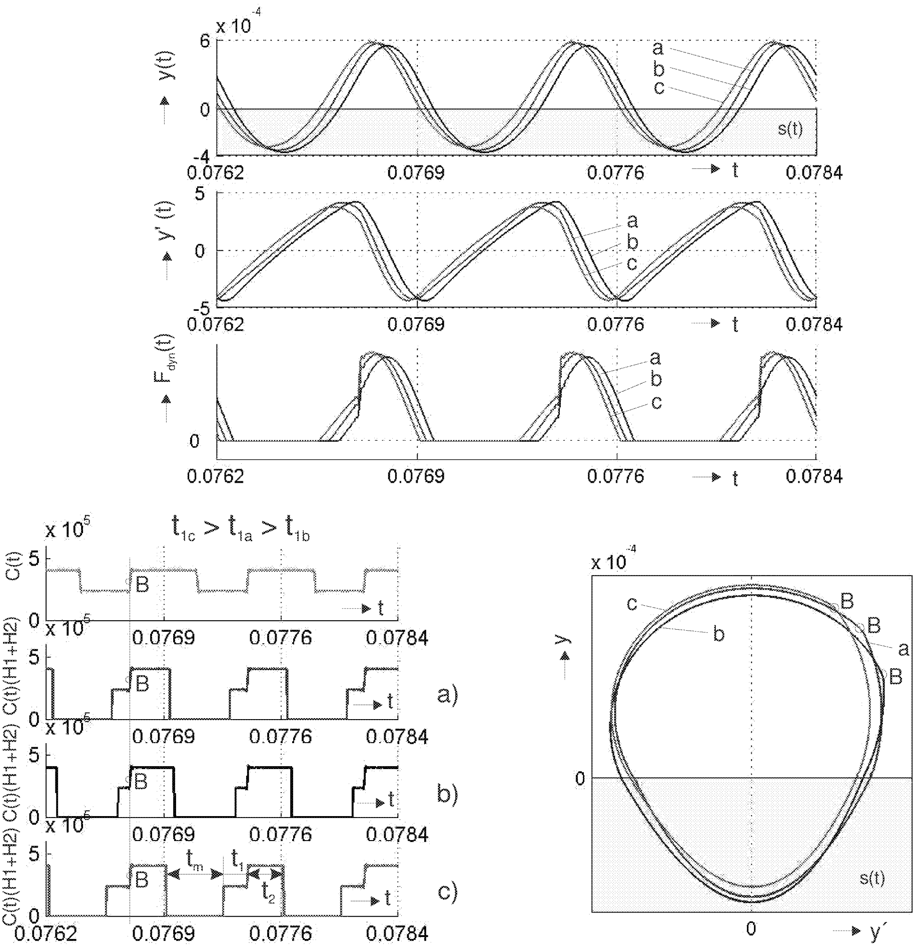


Fig.7: The phase planes $\{y';y\}$ of relative motion $y(t)$ in gear mesh for the combinations of linear material k_1 of cogs in mesh and the viscous mediums $k_{1\mathrm{m}}$ ($k_1 = k_{1\mathrm{m}} = 3.9537\,N\,mm^{-1}\,s$) a) $k_1 \neq 0$, $k_{1\mathrm{m}} = 0$; b) $k_1 = 0$, $k_{1\mathrm{m}} \neq 0$; c) $k_1 \neq 0$, $k_{1\mathrm{m}} \neq 0$

characteristics in vicinity $\nu_\mathrm{s} = 0.8$ in Fig. 3. The phase planes $\{y';y\}$ of solved system are varied considerably. In the resonance characteristic they created the area with singular – non-continuous course.

The influence of damping, i.e. combination of linear material k_1 of cogs in the gear mesh and the viscous mediums $k_{1\mathrm{m}}$, on the relative motion $y(t)$, its velocity $y'(t)$ and on the dynamic force $F_\mathrm{dyn}(t)$ in gear mesh are shown from the Fig. 7. On the basis of Fig. 5 are plotted here among others the course of exciting modify stiffness functions $C(t)(H1 + H2)$ for tuning value $\nu_\mathrm{s} = 0.82$ and for the combinations of damping k_1, $k_{1\mathrm{m}}$. From the courses of exciting modify stiffness functions is evident the influence of damping combinations on the time distances of contact bounce of cogs in gear mesh at the potential levels C_max or C_min. These take effect in the courses of $y(t)$, $y'(t)$ and $F_\mathrm{dyn}(t)$ partly by the phase – time shift, partly by the amplitude size of relative motion $y(t)$ or dynamic force $F_\mathrm{dyn}(t)$ in gear mesh. From the figure is evident that during the time, when the gear mesh wages at stiffness level $C_{\min}$, validates for given tuning and mentioned combinations of damping the relation $t_{1c} > t_{1a} > t_{1b}$. The influence of damping is then also evident from the courses of phase planes of relative motion.

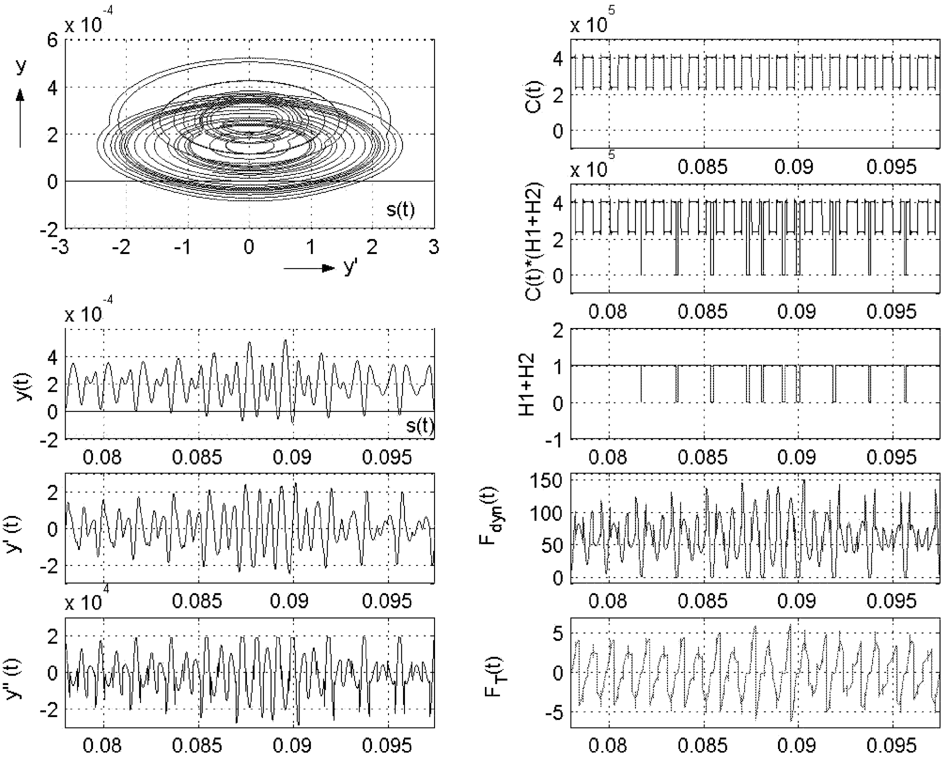


Fig.8: The phase planes $\{y'; y\}$ and time courses of relative motion $y(t)$ in gear mesh, resulting stiffness function $C(t)$, its modification $C(t)(H1 + H2)$ with respect to phase of gear mesh, dynamic $F_{\rm dyn}(t)$ and friction $F_{\rm T}(t)$ forces for tuning value $\nu_{\rm s} = 0.66$ and damping $k_1 = k_{1\rm m} = 0$ during the fifth revolution of the analysed gearing system with six degrees of freedom

In Fig. 8 is given the example of analysis of dynamic behaviour of solved system for the damping $k_1 = k_{1\rm m} = 0$ and the tuning $\nu_{\rm s} = 0.66$ in the fifth revolution of gear wheels. The solved system has unsteady relative motion $y(t)$ in gear mesh. That is evident both from phase plane of relative motion $\{y'; y\}$ and from partial time courses of relative motion $y(t)$ in gear mesh, its velocity $y'(t)$ and acceleration $y''(t)$. Provided that the amplitudes of relative motion in gear mesh are greater than the static-elastic deformations, arise to contact bounces of cog profiles and to issue of impact effects. The resulting stiffness function $C(t)$ in gear mesh modifies by theirs influence in the form $C(t)(H1 + H2)$, see time course in Fig. 8, where are according to Fig. 1, relevant Heaviside's functions denoted by $C(t)(H1 + H2)$. The parametric excited function in gear mesh has then the character, which correspond to modifying course $C(t)(H1 + H2)$.

In the Fig. 8 are given more the time courses of dynamic force $F_{\rm dyn}(t)$ and friction force $F_{\rm T}(t)$ in gear mesh. The impact effects in gear mesh caused by dynamic forces influences the tribological conditions in mesh because the transitions between the phases of gear mesh and tooth backlash can induce the disturbance of supporting oil film layer which are connected with the creation of condition for issue of medium-dry and dry friction in the gear mesh, with the consequence of issued self-excited vibration and noise.

4. Concluding remarks

The mentioned exemplifications of damping influence bring out some partial results of analysis of the **internal dynamics** of strongly nonlinear parametric systems which are excited purely parametrical i.e. by only mentioned modify resulting stiffness function $C(t)(H1+H2)$ in the gear mesh.

In conclusion can yet note, that the problems of analysis of parametric vibration in the systems with kinematic coupling – gear pairs is functional dependence of many parameters above all

- the *size of tooth backlash* $s(t)$, which is time variable in consequence of elastic supporting of wheels and possible run-out of pitch circles meshing wheels,
- the *value of the coefficient of mesh duration* ε which determines at which stiffness level (potential level) $C_{\max}$ or $C_{\min}$ and in what time interval run the solution of relative motion in gear mesh,
- the *value of the amplitude modulation* $\kappa = C_{\min}\, C_{\max}^{-1}$,
- the *resonance tuning* of stiffness level at which the solution of relative motion just run, i.e. the relation between the exciting mesh frequency ω_{c} and eingenfrequencies $\Omega_{\max,\min} = C_{\max,\min}\, m_{\mathrm{red}}^{-1}$,
- the *form of modify resulting stiffness function* $C_{\mathrm{M}} \equiv H\, C(t,\varepsilon,\kappa)$ in gear mesh in the framework of vibration of system with impact effects in mesh, where H is Heaviside function,
- the *phase shift* ψ of relative motion $y(t)$ towards stiffness function $C(t,\varepsilon,\kappa)$ alternatively towards its modify form $C_{\mathrm{M}} \equiv H\, C(t,\varepsilon,\kappa)$ caused by linear and nonlinear damping effects both the material in gear mesh and temperature dependent viscosity of environment in the area of tooth backlash.

Detailed analysis of influence of parametric vibrations and influence of system parameters on the quality and quantity of amplitude of relative motion $y(t)$ will be the theme of next research, which is yet not finished. The closing results and principles will be published in next works.

Acknowledgement

This research work was supported by the pilot programme PP 06-361 in the framework of research purpose AVOZ 20760514 in Institute of Thermomechanics of Academy of Sciences of the CR.

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Received in editor's office: March 16, 2006
Approved for publishing: April 28, 2006

Note: The paper is an extended version of the contribution presented at the national colloquium with international participation *Dynamics of Machines 2006, IT AS CR, Prague, 2006.*