PASSIVE AND ACTIVE MEANS FOR SELF-EXCITED VIBRATION SUPPRESSING: TWO-MASS MODEL

Aleš Tondl*

Dedicated to my dear friend Prof. Dr.-Ing. G. Benz on the occasion of his 80th birthday.

A model with two masses is considered where the upper mass is self-excited due to the negative linear damping component and the lower mass represents the damped foundation subsystem. The passive means represents the positive linear damping of foundation subsystem and the active one the linear parametric excitation due to the periodic changing of the foundation mounts stiffness. In the case when only passive means are used the optimal tuning can be reached when stability limits for both vibration modes merge together. The condition for such tuning of the system in question is formulated. But when the stability condition is not met then this tuning of the system is not suitable for using in addition the active means in the form of parametric excitation.

Key words : *self-excited vibration, passive and active suppression means, parametric excitation*

1. Introduction

Self-excited vibration represents an important phenomenon in physical and mechanical systems. There exist different sources of self-excitation, which need different mathematical models describing important properties of the self-excitation. In most cases the self- excited vibration represents a danger for the save run of different systems and devices. Therefore, it is necessary to use means for vibration suppressing or, at least, for reducing the vibration intensity.

There exists a lot of literature dealing with the analysis of self-excited systems and the basic theory can be found in any book on oscillatory systems. The attention is given, first of all, to the explanation of the excitation mechanism and to mathematical models (see e.g. [1], [2]). Less attention is given to different means for vibration suppressing. This is dealt especially in book [3]. The active means using parametric excitation represents quite a new approach (see [4] to [18]).

One important group of these suppressing means is represented by additional subsystems (e.g. a tuned absorber or foundation mass) where the suppressing effect is due to the action of damping (e.g. absorber mass or foundation mass motion). In this contribution the effect of both suppression means – passive and active – is analyzed on a simple two-mass system. The upper mass m_1 mounted on a spring having stiffness k_1 is self-excited and this basic subsystem is attached to a foundation subsystem characterized by mass m_2 and spring having stiffness k_2 (see Fig. 1). The deflections are y_1, y_2. The foundation mass motion is

* A. Tondl, Zborovská 41, Praha 5, Czech Republic

damped. The self-excitation is supposed of van der Pol type. For the case of parametric excitation this is due to the stiffness variation $k_2 = k_{20}\,(1 + \varepsilon\,\gamma\,\cos\omega t)$.

Note: The scheme in Fig. 1 does not mean that deflections have to be in vertical directions. In most real systems of high structures these are lateral horizontal deflections.

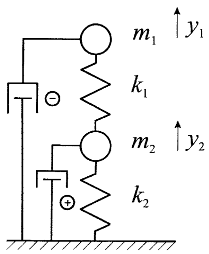

Fig.1: Schema of the system

2. Differential equations of motion

The system in question is governed by the following equations:

$$\begin{aligned} m_1\,\ddot{y}_1 + k_1\,(y_1 - y_2) + \varepsilon\,(-b + d\,y_1^2)\,\dot{y}_1 = 0\ , \\ m_2\,\ddot{y}_2 - k_1\,(y_1 - y_2) + k_2\,y_2 + b_0\,\dot{y}_2 = 0\ . \end{aligned} \tag{1}$$

Using the time transformation $\omega_1\,t = \tau$ ($\omega_1 = \sqrt{k_1/m}$) equations (1) get the form:

$$\begin{aligned} y_1'' + y_1 - y_2 + \varepsilon(-\beta + \delta\,y_1^2)\,y_1' = 0\ , \\ y_2'' - M\,(y_1 - y_2) + q^2\,y_2 + \varepsilon\,(\kappa\,y_2' + \gamma\,y_2\,\cos\eta\,\tau) = 0 \end{aligned} \tag{2}$$

where

$$\beta = \frac{b}{m_1\,\omega_1}\ ,\quad \delta = \frac{d}{m_1\,\omega_1}\ ,\quad \eta = \frac{\omega}{\omega_1}\ ,\quad M = \frac{m_1}{m_2}\ ,\quad q^2 = \frac{\dfrac{k_{20}}{m_2}}{\dfrac{k_1}{m_1}}\ ,\quad \kappa = \frac{b_0}{m_2\,\omega_1}\ .$$

Equations (2) can be transformed into the quasi-normal form using transformation

$$y_1 = x_1 + x_2\ ,\qquad y_2 = a_1\,x_1 + a_2\,x_2 \tag{3}$$

where

$$a_1 = \frac{M}{q^2 + M - \Omega_1^2}\ ,\qquad a_2 = \frac{M}{q^2 + M - \Omega_2^2}\ ,$$

$$(\Omega^2)_{1,2} = \frac{1}{2}\left[1 + q^2 + M \pm \sqrt{(1 + q^2 + M)^2 - 4\,q^2}\right]\ .$$

Note: It can be proved that (see [19]) the following relations are valid:

$$a_1 > 0\ ,\qquad a_2 < 0\ ,\qquad a_1\,a_2 = -M\ . \tag{4}$$

In this way equations (2) get the form:

$$\begin{aligned} x_1'' + \Omega_1^2\,x_1 + \frac{\varepsilon}{a_1 - a_2}\,(-a_2\,F_1 + F_2) = 0\ , \\ x_2'' + \Omega_2\,x_2 + \frac{\varepsilon}{a_1 - a_2}\,(a_1\,F_1 - F_2) = 0 \end{aligned} \tag{5}$$

where

$$F_1 = \left[-\beta + \delta\,(x_1 + x_2)^2\right](x_1' + x_2') \ ,$$
$$F_2 = q^2\,(a_1\,x_1 + a_2\,x_2)\,\cos\eta\,\tau + \kappa\,(a_1\,x_1' + a_2\,x_2') \ .$$

3. Passive suppression means

For this case it is considered that $\gamma = 0$. Seeking the single-frequency vibration the harmonic balance method can be used to determine the approximate solution. The solution with the first mode can be sought in the form:

$$x_1 = X_1\,\cos\Omega\,\tau \ , \qquad x_2 = 0 \ , \qquad (X_1 > 0) \ , \tag{6}$$

the solution with the second mode in the form:

$$x_1 = 0 \ , \qquad x_2 = X_2\,\cos\Omega\,t \ , \qquad (X_2 > 0) \ . \tag{7}$$

Inserting these into equations (5) and comparing the coefficients at $\cos\Omega\,t$ (considering above mentioned assumption on the form of functions F_1, F_2) the following results are obtained for the first and second mode vibrations:

$$\Omega = \Omega_1 \ , \qquad \Omega = \Omega_2 \ . \tag{8}$$

When comparing the coefficients at $\sin\Omega\tau$ the following relations are obtained for the first mode:

$$\frac{\Omega}{\pi}\int\limits_0^{2\pi/\Omega_1}\left[-a_2\,F_1\,(-\Omega_1\,X_1\,\sin\Omega_1\tau, X_1\,\cos\Omega_1\tau) + \right.$$
$$\left. + F_2\,(-a_1\,X_1\,\sin\Omega_1\tau, a_1\,X_1\,\cos\Omega_1\tau)\right]\,\sin\Omega_1\tau\,\mathrm{d}\tau = 0 \ . \tag{9}$$

Similarly for the second mode we obtain:

$$\frac{\Omega}{\pi}\int\limits_0^{2\pi/\Omega_2}\left[a_1\,F_1\,(-\Omega_2\,X_2\,\sin\Omega_2\tau, X_2\,\cos\Omega_2\tau) - \right.$$
$$\left. - F_2\,(-a_2\,X_2\,\sin\Omega_2\tau, a_2\,X_2\,\cos\Omega_2\tau)\right]\,\sin\Omega_2\tau\,\mathrm{d}\tau = 0 \ . \tag{10}$$

For the first mode vibration we obtain equation

$$a_2\left[-\Omega_1\,X_1\left(-\beta + \frac{1}{4}\,\delta\,X_1^2\right)\right] - a_1\,\kappa\,X_1 = 0 \ . \tag{11}$$

Then the amplitude is given by equation:

$$X_1^2 = \frac{4}{\delta}\left(\beta + \frac{a_1}{a_2}\,\kappa\right) \ . \tag{12}$$

Considering that $a_1/a_2 < 0$ for the real value of X_1 reads:

$$\beta > -\frac{a_1}{a_2}\,\kappa \ . \tag{13}$$

Then relation

$$\kappa > -\frac{a_2}{a_1}\,\beta \tag{14}$$

is the condition for the full suppressing of the vibration with the first mode. Similarly for X_2:

$$X_2^2 = \frac{4}{\delta}\left(\beta + \frac{a_2}{a_1}\,\kappa\right) . \tag{15}$$

The condition for the full suppression of the second mode vibration reads:

$$\kappa > -\frac{a_1}{a_2}\,\beta . \tag{16}$$

It is necessary mention that in the case of self-excitation due to the flow coefficient β depends on the flow velocity, i.e. increasing with increasing with flow velocity. How the ratio $-a_1/a_2$ depends on tuning coefficient q for different mass ratio M shows the following table.

	$M = 0.1$	$M = 0.2$	$M = 0.5$	$M = 1$	$M = 2$	$M = 5$
$q = 0.4$	7.322	3.782	1.611	0.852	0.450	0.190
$q = 0.6$	4.697	2.584	1.218	0.699	0.395	0.178
$q = 0.8$	2.226	1.429	0.820	0.533	0.332	0.163
$q = 1$	0.730	0.642	0.500	0.382	0.268	0.146
$q = 1.2$	0.213	0.264	0.287	0.262	0.210	0.128
$q = 1.4$	0.076	0.116	0.164	0.177	0.161	0.111
$q = 1.6$	0.034	0.057	0.096	0.118	0.122	0.095

Tab.1: The dependence of $-a_1/a_2$ on q for different values of mass ratio M

For $-a_1/a_2 < 1$ the stability limit β for the first vibration mode is lesser than for the second mode. For $-a_1/a_2 > 1$ it is just a reverse case. The second case can be expected for smaller mass ratio M. Some more information (diagrams and tables) on a_1, a_2, Ω_1, Ω_2 and other functions see [19], Appendix I.

When the absolute values of a_1/a_2 and a_2/a_1 are different then also differ the conditions for the existence of vibration with a certain mode. For the full suppression of self-excited vibration both modes could be suppressed, i.e. the conditions for the not existence of vibration for both modes must be met. The stability boundaries for both vibration modes are the same in the case when

$$\frac{a_1}{a_2} = \frac{a_2}{a_1} . \tag{17}$$

Using the relations for a_1, a_2 we obtain

$$\frac{a_1}{a_2} = \frac{q^2 + M - 1 + \left[(1 + q^2 + M)^2 - 4\,q^2\right]}{q^2 + M - 1 - \left[(1 + q^2 + M)^2 - 4\,q^2\right]} . \tag{18}$$

When

$$q^2 + M = 1 \tag{19}$$

the following equations are valid:

$$\frac{a_1}{a_2} = \frac{a_2}{a_1} = -1 ,$$
$$\Omega_1 = \sqrt{1 - \sqrt{1 - q^2}} , \tag{20}$$
$$\Omega_2 = \sqrt{1 + \sqrt{1 - q^2}} .$$

In the case when relation (19) is met then for the above-mentioned alternatives only a single condition is necessary both vibration modes to be stable:

$$\beta < \kappa \ . \tag{21}$$

Of course condition (21) cannot be realized for any system because the mass ratio M had to be small.

4. Active means

The system in question is governed by differential equations having the nonlinear terms only for expressing positive damping and the equilibrium position is given by the trivial solution ($y_s = 0;\ s = 1, 2$). When investigating the stability of the equilibrium position given by the trivial solution of the governing differential equations the linear equations can be considered. These read for our system:

$$x_s'' + \Omega_s^2\, x_s + \frac{\varepsilon}{a_1 - a_2} \sum_{k=1}^{2} (\Theta_{sk}\, x_k' + Q_{sk}\, x_k\, \sin\eta\tau) = 0 \ . \tag{22}$$

For $\Theta_{kk} > 0$ the k-th vibration mode is stable, for $\Theta_{ss} < 0$ the s-th vibration mode unstable. The conditions that trivial solution is stable at $\eta = \eta_0 = \Omega_2 - \Omega_1$ read:

$$\Theta_{11} + \Theta_{22} > 0 \ , \tag{23}$$

$$\frac{Q_{12}\, Q_{21}}{4\, \Omega_1\, \Omega_2} + \Theta_{11}\, \Theta_{22} > 0 \ , \tag{24}$$

$$\Theta_{11} = a_2\, \beta + a_1\, \kappa \ , \quad \Theta_{22} = -a_1\, \beta - a_2\, \kappa \ , \quad Q_{12} = a_2\, q^2 \ , \quad Q_{21} = -a_1\, q^2 \ . \tag{25}$$

Condition (23) reads that $(a_1 - a_2)(\kappa - \beta)$ should be positive value, which is identical with condition (21) considering that $(a_1 - a_2)$ is a positive value. From it follows that the tuning of the system so that relation (17) is met is not convenient for the use of the additional active suppression means using parametric excitation by variation of the foundation spring stiffness. The necessary condition for the successful use of parametric excitation is: One vibration mode must be stable for passive means.

5. Conclusion

In the case when only passive means are used the optimal tuning is for the case when stability limits for both vibration modes merge together and so only one stability condition is sufficient. However when this stability condition is not met then this tuning is not suitable for using in addition the active means in the form of parametric excitation because the necessary condition (23) would not be met. Two stability conditions must be met to stabilize the equilibrium position at parametric excitation frequency equal to the difference of the natural frequencies of the abbreviated system. The enough stability of one vibration mode represents the necessary condition.

In this contribution the parametric excitation by spring stiffness variation is considered. Similar effect would be possible to achieve by parametric excitation using periodic mass variation, e.g. using a mechanism changing the corresponding reduced mass but this type of parametric excitation would need a special analysis for the considered system.

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