OPTIMIZATION MODEL OF THE VÁH CASCADE OPERATION

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The optimal operation of an electrical power supply subsystem results from respect for the effectiveness criterion for the control of an industrial enterprise. It is essential for the operation of the Váh Cascade hydro power plants – a significant Slovak hydro-energy system. The fundamental principle of this operation is the potential hydropower utilization of water courses in addition to planning the control of the whole electrical system.

Keywords: electrical supply system, optimal operation, hydropower plant, optimization criterion, optimal programming

1. Introduction

The efficient management of the subsystems of an electrical system (ES) must principally be based on respecting the fundamental criterion of efficient ES management. This is also the governing principle of managing the important hydroelectric system within the ENEL Slovenské elektrárne, a.s. (ENEL-SE) national utility company – the Váh Cascade hydro power plant (see Fig. 1). The core management criterion for this subsystem is the cost-effective use of the hydropotential of the ENEL-SE’s water sources, while facilitating all the marginal water management and energy conditions.

The preparation of hydropower plant (HPP) operation seeks to ensure reliable and efficient operation of the hydroelectric system. Planning and preparation of operation within the Dispatching Management System of the Vodná elektrárne Trenčín utility company (VET) is defined as ‘computations and modelling of hydro plant operation based on the

![Scheme of the Váh Cascade](image)

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input of coordinating signals from higher management level in intervals (year, month, day) and respecting the ES hydraulic conditions and requirements to efficiently use the hydro-electric potential. One of the important elements in the HPP’s pre-operating process is the computation and operation at modelling using the hydromodel of the Váh Cascade (‘hydro-model’). The hydro-model represents a programmatic model for a cost-effective use of the HPP Váh Cascade’s hydropotential in covering the Daily Load Chart requirements with respect to the pre-operation of other SE sources (thermo-hydro coordination). The hydro-model’s algorithms are programmed and tuned in the ADA95 program language environment and are currently implemented by means of an external DLL library into the complex IT environment supporting the pre-operating process of the HPPs. The hydro-model’s main task is the processing of hydrological inputs and the modeling of hydraulic states and hydraulic bonds of the hydro power plant cascade on the Váh and Orava rivers (the Váh Cascade). On the output side, a performance plan at the individual HPPs is being generated with due account of their regulatory reserve and in the determined time roster, as proposed under the main efficiency criterion, namely, that the production plan should reflect as closely as possible the required shape of the business chart.

2. The hydro-model’s underlying schemes

Depending on the manner of operation (the manner of the water efficiency) and the waterwork’s (WW) conceptual solution, the individual HPPs of the Váh Cascade can be associated with two underlying schemes:

- regulatory scheme,
- accumulation scheme.

2.1. Regulatory scheme

In operative terms, these HPPs are built on reservoirs with short-term flow regulations (daily or weekly). With respect to the waterwork’s conceptual solution, the regulatory schemes can be divided into weir-surrounding schemes (see Fig. 2) and channel schemes (see Fig. 3).

![Fig.2: Weir-surrounding scheme](image)

![Fig.3: Channel scheme](image)
The weir-surrounding regulatory HPPs of the Váh Cascade include HPP Žilina, HPP Kráľová and HPP Nosice. Among the regulatory channel HPPs of the Váh Cascade are the following HPP groups: Krpeľany – Sučany – Lipovec, Hričov – Mikšová – Považská Bystrica, Ladce – Ilava – Dubnica – Trenčín, Kostolná – Nové Mesto n/Váh – Horná Streda, and the HPP Madunice, which standy alone.

2.2. Accumulation scheme

In terms of operations (water efficiency), these HPPs or pumped-storage HPPs are built at large accumulation reservoirs with long-term flow regulation (annual or more years). The accumulating reservoirs are connected to compensation reservoirs with downstream low-performance HPPs that are designed to generate electricity at given flow rates under these HPPs. The governing HPP of such a group is the HPP or pumped-storage HPP built in a direct hydraulic bond to the accumulation reservoir (see Fig. 4).

The accumulation schemes of the Váh Cascade HPPs include HPP: Orava – Tvrdošín and pumped-storage HPP Liptovská Mara – Bešeňová.

![Fig.4: Accumulation scheme](image)

3. Underlying Computation Scheme

The hydro-model’s basic computation scheme is shown in Fig. 5.

![Fig.5: Basic computation scheme of the solution](image)
The hydro-model’s main inputs are as follows:
- required business chart or evaluation,
- required values of supporting services (SS) (i.e., primary regulation of power output (PPR+/−), secondary regulation of power output (PSR+/−) and tertiary regulation of power output (PTR+/−)),
- hydrological forecasts from the Slovak Institute of Hydrometeorology Institute (SHMÚ),
- the WW’s construction parameters and limits,
- required manipulation at water structures,
- optimization criterion.

The hydro-model’s main outputs are as follows:
- plan of power output and generation at individual HPPs of the Váh Cascade,
- supporting services (SS) offered.

As a fundamental prerequisite, the search for the most efficient solution of any system must be based on derivation of the optimization criterion that, as concerns the HPP sub-system, must comply with the basic criteria of efficient management of the entire ES. The overriding optimization criterion of the pre-operation of the HPPs of the Váh Cascade is to achieve the closest business chart/proposed performance ratio as required. The optimization criterion is then defined by the so-called criterial (objective) function. The objective function can be written as:

\[\text{min } Z = \sum_{i=1}^{n} (N_i - P_i)^2,\]

where \(i\) – index of time period in the planning problem, \(i = 1, 2, \ldots, n\); \(N_i\) – required power output in period \(i\); \(P_i\) – total power output produced from the HPPs of the Váh Cascade in period \(i\).

Optimization will thus be understood as minimizing the objective function \(Z(P)\), which is represented by the dependent variable of parameters, whose optimum values \(P^* = (P_1^*, P_2^*, \ldots, P_n^*)\) are to be sought. The tasks of this formulation can be resolved by using theoretical methods of optimum system management. One suitable way to solve the given optimization task which is usable for drafting the pre-operation of the HPP system is the use of a HPP system simulation model and the optimization method in the simulation model. The choice of a suitable optimization method should take due account of the fact that the accuracy of the computations in the management efficiency solution will be higher than the accuracy of the possible acquisition of the input data.

In view of the above and in an effort to achieve the utmost efficiency of the HPP pre-operating process (in particular, regarding the hydro-model’s ‘velocity’), the objective function had necessarily been transformed to linear function.

\[\text{max } Z = \sum_{k=1}^{m} \sum_{i=1}^{n} \left( N_i - \sum_{j=1}^{k-1} jP_i \right)^k Q_i \]

subject to:

\[kQ_{\text{MIN},i} \leq kQ_i \leq kQ_{\text{MAX},i},\]

\[jP_{\text{MIN},i} \leq jP_i \leq jP_{\text{MAX},i},\]
\[ kV_{\text{MIN}_i} \leq kV_i \leq kV_{\text{MAX}_i}, \]
\[ kV_i = kV_{i-1} + kI_i - kQ_i t, \]

where \( t \) – time period, \( k \) – index of HPP, \( k = 1, 2, \ldots, m \) (set of downstream), \( m \) – number of HPP, \( kQ_i \) – discharge for HPP \( k \) in period \( i \), \( kQ_{\text{MIN}_i} \) – minimum discharge for HPP \( k \) in period \( i \), \( kQ_{\text{MAX}_i} \) – maximum discharge for HPP \( k \) in period \( i \), \( kP_i \) – power output produced by HPP \( k \) in period \( i \), \( kP_{\text{MIN}_i} \) – minimum power output limit of HPP \( k \) in period \( i \), \( kP_{\text{MAX}_i} \) – maximum power output limit of HPP \( k \) in period \( i \), \( kV_{\text{i}} \) – storage volume of the reservoir \( k \) at the end of period \( i \), \( kV_{\text{MIN}_i} \) – minimum storage of the reservoir \( k \) in period \( i \), \( kV_{\text{MAX}_i} \) – maximum storage of the reservoir \( k \) in period \( i \), \( kP_{\text{i}} \) – net inflow volume to the reservoir \( k \) in period \( i \), including seepage and evaporation losses, and diversions.

As all coefficients of the task (coefficients of all the functions) are real numbers, the problem of the HPP’s efficiency of operations could have been transformed to a linear programming task that can briefly be stated as the following matrix:

\[
\text{for } k = 1 \text{ to } m
\]
\[
\max Z = c^TQ \quad \text{in compliance with } A \mathbf{Q} \leq \mathbf{b}, \quad \mathbf{Q} \geq 0
\]
\[
\text{next } k
\]

where \( A \) – matrix of structural coefficients, \( c \) – vector of evaluations, \( Q \) – vector of solutions (optimum flow through HPPs profile), \( b \) – vector of restrictions.

In the hydro-model, the solution of the defined task of the linear programming is arranged by using the Simplex method. The parameters for computing the values of the structural coefficients of the \( A \) matrix and the \( b \) vector of the restrictions (the vector of the right sides) are arrived at by computations made in the hydro-model’s additional modules.

### 3.1. Hydrological module

The hydrological module provides data as to the flow management in the individual elements of the system. Its task (output) is to provide real data on available water flows for the individual HPPs. The hydrological module is based on the following:

– the system’s elements/flow management impact rate,
– determination of the priority bonds among the system’s reservoirs.

Based on the setting of the possible bonds (interactions) and the system’s elements/flow management impact rate, the hydromodel can be divided as follows:

– sections: profile of the reservoir’s intake – HPP intake profile,
– sections: outlet profile from the group-ultimate HPP – intake profile of the next HPP group,
– sections: weir (profile of an idle outlet from the HPP group) – the orifice of the ultimate HPP’s outlet channel to the old riverbed.

The hydrological module serves to provide data on flows in the sections between the individual HPPs or HPP groups that, although hydraulically uncoupled, have a substantial impact on the time sequence of the regulated flows, and, as a result, impact the engagement of the individual HPPs in time. The ‘inefficient’ sections include: Bešeňová – Krpeľany, Tvrdošín – Krpeľany, Lipovec – Žilina, Žilina – Hričov and Madunice – Kráľová.
Additionally, the hydrological module also serves to provide data on flows in the sections located between a weir (the profile of an idle outlet from the HPP group) and the orifice of the group-ultimate HPP’s outlet channel to the old riverbed.

3.2. Hydraulic module

The purpose of the hydraulic module is to provide data on heads for the individual HPPs. It permits to determination the following characteristics:
- water surface level in the reservoir,
- hydraulic losses in the HPP intakes and outlets,
- hydraulic bonds of the individual HPPs.

The proposed structure of the hydraulic module stems from the configuration of the HPPs. In terms of solving the hydraulic bonds and losses, the module is divided as follows:
- reservoirs,
- channels divided into sections: reservoir – HPP, HPP – HPP, HPP – reservoir of the next HPP group (the downstream water surface level is impacted by the operation of the next HPP group), HPP – inefficiently used flow section,
- penstocks (section reservoir – HPP).

The job of the reservoir’s hydraulic module is to set, on the basis of the supply volume, the reservoir’s immediate water surface level or the reservoir’s immediate water supply volume as based on the water surface level. The job of the hydraulic module of the channels is to set the hydraulic losses resulting from the water flow in these structures. The job of the hydraulic module of the pressure inlet channels (pipes) is to set the hydraulic losses resulting from the pressured water flow in these structures.

3.3. Transformation module

The purpose of the transformation module is to transform, based on the results of both the hydrological and hydraulic modules, the flow rates (hydrological modules) and heads (hydraulic module) to electrical power. Additionally, the transformation module also has the job of determining the values of the flow rates equivalent to the required supporting service values. The transformation of the required regulatory outputs to the equivalent flow rates is apparent from the scheme in Fig. 6.

Fig. 6: Transformation of the required regulatory outputs to the equivalent flow rates
3.4. Module of optimization and correction

The purpose of the optimization module is to propose, based on the existing conditions and marginal criteria of water management and energy management, as well as other criteria, the output plan for the individual HPPs of the Váh Cascade, while using the main optimization criterion. The HPP Output Plan with headroom for a regulatory reserve in the determined time roster is proposed for the stated optimization interval.

The corrective part of the efficiency module serves to make corrections of the values of the structural coefficients of the $A$ matrix and the $b$ vector of restrictions for and after each computation of the values of the solution vector $Q = (Q_1, Q_2, \ldots, Q_n)$. Subsequently, more accurate values of the $Q$ solution vector will be recomputed using the corrected values.

4. Conclusion

The described hydro model had been put into test operation in 2006 and since 2007 it is in the real operation as a part of the ‘Complex information system of the operation planning of the ENEL – SE energy sources’. It replaced a hydro model used in the dispatch centre of hydropower plants for about 20 years. Besides the new visual aspects it offers many new functions, mostly in the field of planning and providing the support services. Its modular structure enables an independent upgrade of particular modules, what assures a trouble free reaction on new knowledge in the field of operation of a HPP system. The output of the hydro model in the control process of the ENEL-SE energy sources is the plan of the total power generation of the HPP subsystem, including its power reserve for providing the support services with the respect to the criterion of the optimal load distribution in the system between particular energy sources.

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References


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