EVALUATION OF THE SUITABILITY OF THE BLADED DISK DESIGN REGARDING THE DANGER OF THE RESONANT VIBRATION EXCITATION

Pavel Polach*

A suitability of the bladed disk design regarding the possibility of the resonant vibration excitation can be assessed on the basis of several approaches. Most information concerning the evaluation of the bladed disk design suitability is provided in the SAFE diagram but the possibility of exciting the action wheel resonant vibration can also be evaluated from the Campbell diagram. Further criterion is the assessment of the design suitability on the basis of a critical speed of the bladed disk. It especially causes breakdowns of relatively flexible disks. The suitability of the bladed disk with the blades of the ZN340-2 type was evaluated.

Keywords: *bladed disk, resonant vibration, SAFE diagram, continuous binding*

1. Introduction

The suitability of the design of the steam turbine bladed disk from the point of view of the possibility of a resonant vibration excitation can be evaluated on the basis of several approaches. Knowledge of the bladed disk natural vibration characteristics, i.e. its natural frequencies and mode shapes, is the precondition of applying those approaches. Most information for the evaluation of the suitability of the bladed disk design can be found in the SAFE diagram (e.g. [1], [2] and [3]) but the possibility of exciting the bladed disk resonant vibration can also be evaluated from the Campbell diagram. Further criterion is the assessment of the design suitability on the basis of a critical speed of the bladed disk, which is especially the cause of breakdowns of relatively flexible disks [4].

In this case the suitability of design of the bladed disk with 66 moving blades of the ZN340-2 type [5] (Fig. 1) coupled with the continuous binding formed by the integral shrouding and by the tie-boss connection in the middle of the blade is evaluated. At this structural design of the moving blades the creation of connection ('locking') of adjoining blades in the zone of shrouding and in the tie-boss zone occurs at the definite revolutions of the bladed disk. Blades are continuously coupled by the blades untwist due to the centrifugal forces acting. The advantage of the structural design is that the moving blades do not include structural notches, which can be holes for continuous wire or pins [6], [7], rivets in the common integral shrouding etc. when the continuous binding is used.

Natural frequencies and mode shapes of the bladed disk with the blades of the ZN340-2 type were calculated on several virtual models [5], [8] by means of the method using the rotational periodicity of the system (e.g. [9]).

* Dr. Ing. P. Polach, Section of Materials and Mechanical Engineering Research, Výzkumný a zkušební ústav Plzeň s.r.o., Tylova 1581/46, 301 00 Plzeň

2. SAFE diagrams

Theoretical knowledge for generating the SAFE diagrams and their importance for evaluating the suitability of the design of rotating machines bladed disks is taken especially from [1], [2] and [3].

The acting of a time variable force is a precondition for exciting the resonant vibration of any structure. In the case of steam turbines bladed disks the resonant vibration can be caused by a non-uniform steam pressure on the circumference of the bladed disk, by the disk dynamic imbalance, by a bending vibration of a rotor and by any time variable force acting on a moving blade or a rotating action wheel. A non-uniform distribution of the steam pressure on the circumference of the bladed disk is extremely dangerous. It is caused by the steam partial inlet through nozzles (steam pressure acts on the moving blade only in the certain part of the bladed disk circumference and furthermore the pressure between adjacent nozzles decreases nearly to zero) or at a full admission by the decrease in the steam pressure in the places of stator blades (if the number of stator and moving blades is commensurable number, the exciting force acting on the bladed disk is represented by a periodic force, the frequency of which in one rotor revolution equals just the common divisor). From the experience in the steam turbines operation the periodicity of exciting forces acting on the steam turbine action wheel is dependent on the rotor speed frequency and its multiples.

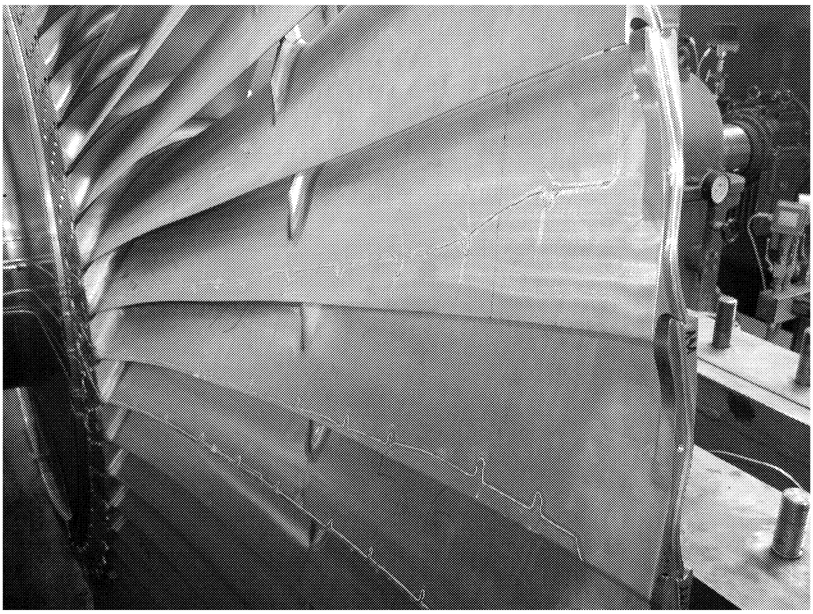

Fig.1: Bladed disk with blades of the ZN340-2 type

At the non-uniform distribution of the steam pressure on the circumference of the bladed disk a dynamic force is acting on each blade of the rotating action wheel. The moving blade dynamic response level is dependent on :

1. the natural frequencies of the bladed disk and their associated mode shapes,
2. the frequency, the shape and the magnitude of the exciting dynamic force, which is dependent on the turbine speed and on the number and location of the stator blades (or on the number of steam flow interruptions),

3. the energy dissipation – i.e. on damping properties of the material, of which the moving blades are made, on frictional forces acting in the places of assembly, on aerodynamic damping from steam etc. (the influence of those factors cannot be evaluated in a qualified way on the basis of the calculations not supported by experimental measurements).

A turbine bladed disk may get into a state of vibration where the energy built up is a maximum. This is exemplified by maxima in its response and minima in its resistance to the exciting force. The condition is called a state of 'resonance'.

In order to excite the resonant vibration of any construction two conditions must be fulfilled (they are derived in [1] from the relation for the positive work, i.e. work done by exciting forces acting on the moving blade) :

1. frequency of the exciting force (so called 'nozzle passing frequency') must equal the natural frequency of the structure,
2. the exciting force profile must be identical with the associated mode shape of vibration (according to [1] the harmonic multiple of the exciting force must equal the number of nodal diameters for the rotating machine action wheel).

If both conditions are not fulfilled at the same time a resonant state does not occur.

Forces acting on the moving blade are dependent on its relative position regarding the stator blades (nozzles) in one complete rotor revolution. On the basis of steady-state conditions of rotation the periodic forces also repeat in successive rotor revolutions. The time dependent periodic forces may be composed of any harmonics. The frequencies of those harmonics are integer multiples of the angular speed of the turbine and at the same time they are dependent on the number of stator blades (or on the number of steam flow interruptions) :

$$\mathbf{F} = \mathbf{F}_0 + \mathbf{F}_1 \sin(\omega_\mathrm{b}\, t + \varphi_1) + \mathbf{F}_2 \sin(2\,\omega_\mathrm{b}\, t + \varphi_2) + \dots \;, \tag{1}$$

where ω_b is the angular frequency of dynamic forces, t is the time, $\mathbf{F}_0$, $\mathbf{F}_1$, ... are constant vectors and φ_1, φ_2, ... are angles of phase difference.

Angular frequency ω_b of dynamic forces acting on the moving blades can be expressed by the relation

$$\omega_\mathrm{b} = \frac{n_\Omega\, M_\mathrm{S}}{60}\ [\mathrm{rad\,s^{-1}}] \;, \tag{2}$$

where M_S is the number of the stator blades (or the number of steam flow interruptions) and n_Ω is the turbine rotor speed (in rpm).

The most often used way of the evaluation of the suitability (from the point of view of the possibility of a resonant vibration excitation) of the steam turbine bladed disk design is the Campbell diagram (bladed disk natural frequencies in dependence on the rotor speed are plotted there, e.g. [4]). But the Campbell diagram does not show any pieces of information important for the analysis of the possibility of moving blades' damage. The example (taken from [2] and [3]) of the Campbell diagram with designated 'possible' (states, during which the natural frequency of the bladed disk is identical with the frequency of exciting force – i.e. the fulfilling of only the first condition for the excitation of the resonant vibration) and 'probable' (states, in which the first condition is fulfilled for exciting the resonant vibration and the second condition is fulfilled in a 'probable way' – i.e. the shape of the exciting force from the multiple of the speed frequency can be of an identical shape as the mode shape associ-

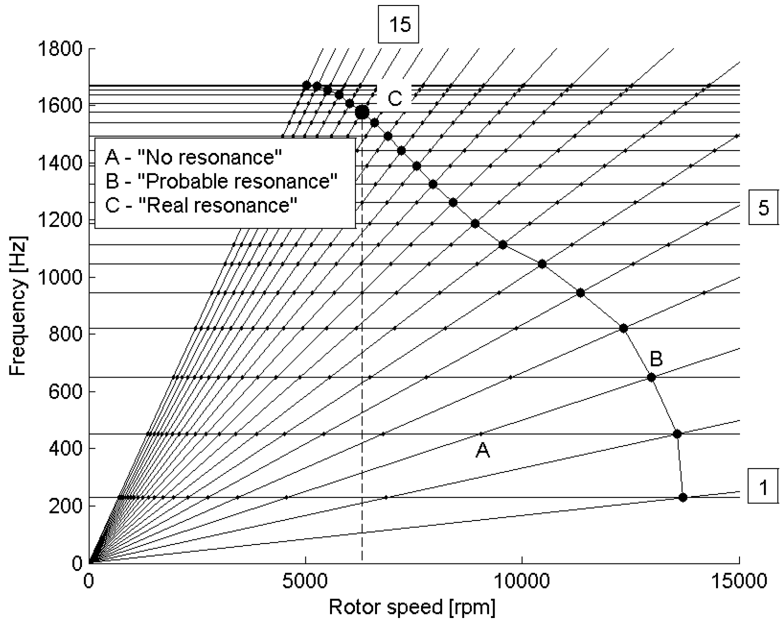

Fig.2: Campbell diagram of the bladed disk with blades connected with the continuous binding (example taken from [2], [3])

ated to the corresponding natural frequency; the curve connecting B points is called an 'impulse line' or a 'speed line') resonances of the steam turbine action wheel (for the steam inlet through 15 nozzles into the bladed disk with the blades connected with the continuous binding) is shown in Fig. 2. Utilising the Campbell diagram it is only possible to determine the states of 'possible' (in Fig. 2 points A 'no resonance'; in [2] indicated as 'probable resonance') resonances and 'probable' (in Fig. 2 points B 'probable resonance'; in [2] specified as 'possible resonance') resonances, not the states of 'real' resonances [2]. Therefore, in compliance with the methodology of the Dresser-Rand Company, a more suitable SAFE (Singh's Advanced Frequency Evaluation) diagram is used for the analysis of the possibility of damaging the moving blades by the resonant vibration excitation (e.g. [1], [2] and [3]; see Fig. 4).

The SAFE diagram is three-dimensional. One projection plane of the SAFE diagram is the Campbell diagram, the second projection plane is the 'SAFE plane', in which the frequencies in dependence on the number of nodal diameters of the bladed disk are plotted (see Fig. 3).

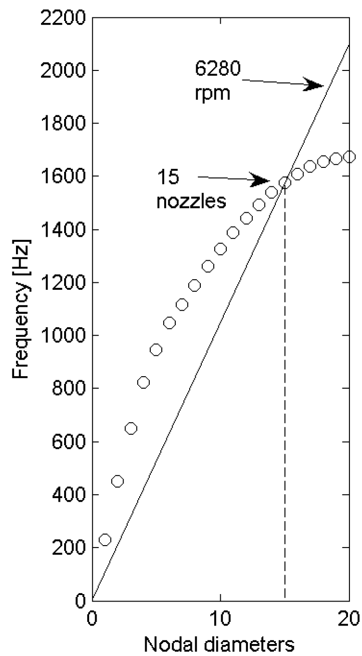

Fig.3: 'Probable' and real resonances of the action wheel plotted in the 'SAFE plane' (example taken from [2], [3])

As stated before, knowledge of the frequencies and shapes of exciting forces for the identification of the real resonances of the bladed disk is necessary. The frequency of the exciting forces is given by the number of stator blades (nozzles) and by the turbine rotor speed, the exciting force shape is given by the number of stator blades (nozzles) [2]. The example (taken from [2]) of the identified real resonance and plotting the 'probable' resonances (i.e. B points) from the Campbell diagram in Fig. 2 into the 'SAFE plane' is given in Fig. 3.

The evaluation of the possibility of resonant vibration excitation is rather more complicated in the case, in which the moving blades are connected with a shroud into packets. Mode shapes of the packeted bladed disk can be generally expressed using the Fourier decomposition of the displacement of each blade

$$ {}_{\beta}\mathbf{U} = \sum_{L} \mathbf{A}_L \, \sin\left(L \, {}_{\beta}\omega + \varphi_L\right) \tag{3}$$

where β is the number of nodal diameters of the action wheel, ${}_{\beta}\omega$ is the angular natural frequency of the action wheel at β nodal diameters (e.g. [9]), $\mathbf{A}_L$ is the constant vector, φ_L is the angle of a phase difference, $L = i\,n \pm \beta$ and $i = 0, 1, 2, \ldots$ (n is the number of blades packets in the action wheel – in the case of the continuous binding $n = 0$ must be considered) and at the same time L must comply with the condition $0 \leq L \leq M/2$ (in the case of an odd number of moving blades $0 \leq L \leq (M-1)/2$; M is the total number of moving blades).

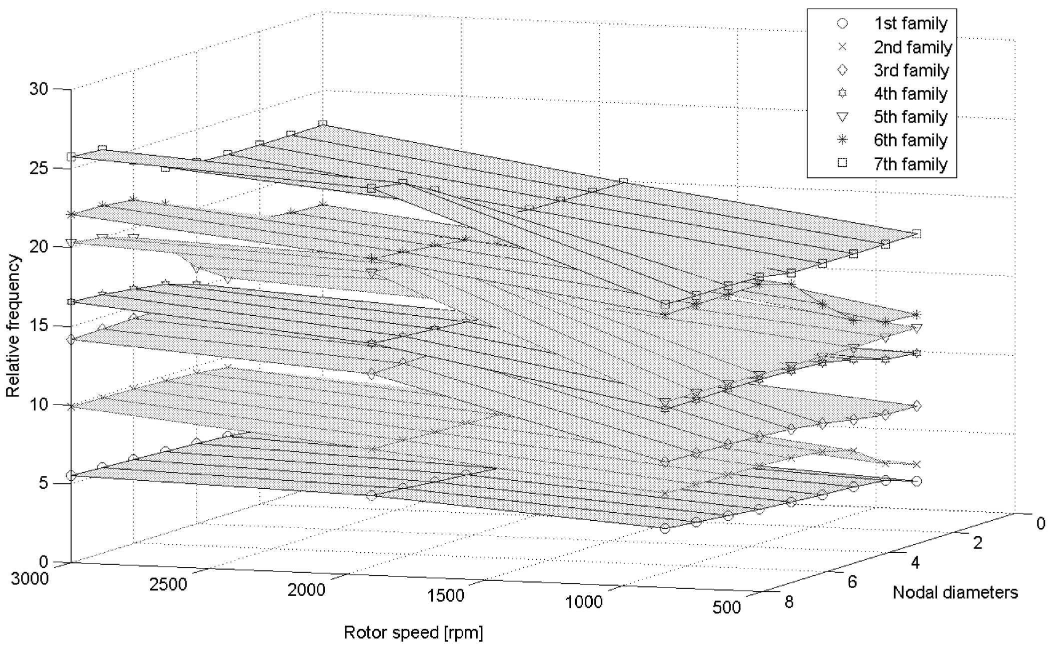

Fig.4: The SAFE diagram of the bladed disk with blades of the ZN340-2 type (results from the computational model with the 'rigid' binding [5])

An interference diagram is a graphic aid for the interpretation of relation (3), which provides information on possible terms of the Fourier series. In a horizontal axis there is plotted harmonic content and in a vertical axis a number of nodal diameters of the action wheel. The diagrams are constructed by drawing ±45 degree lines emanating from each integer multiple of the number of blades packets in the action wheel. Fig. 5 shows the example (inspired by [2]), in which the action wheel with 32 free blades or 32 blades coupled with continuous binding (Fig. 5a) or the action wheel with 32 blades in 8 packets – each of 4 blades – (Fig. 5b) are considered. Interference diagrams provide the following important pieces of information :

1. When the 45 degree lines cross each other and have the same horizontal and vertical designation, the mode with the proper number of nodal diameters will split, i.e. will have some mode designation but have two different natural frequencies.
2. In the interference diagrams the harmonic index of any mode shape is indicated. In the case of moving blades with a continuous binding each mode shape contains only one harmonic content while in the case of the moving blades packets one mode shape has more harmonic contents (the example for 8 packets, the 15 nodal diameter mode shape respond to the 15th, 9th, 7th and 1st harmonics of exciting force but with different magnitude – see Fig. 5b).

a)

Nodal diameter \ Harmonic content	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	x																
1		x															
2			x														
3				x													
4					x												
5						x											
6							x										
7								x									
8									x								
9										x							
10											x						
11												x					
12													x				
13														x			
14															x		
15																x	
16																	x

b)

Nodal diameter \ Harmonic content	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	x								X								x
1		x						x		x						x	
2			x				x				x				x		
3				x		x						x		x			
4					**X**								X				
5				x		x						x		x			
6			x				x				x				x		
7		x						x		x						x	
8	X								**X**								X
9		x						x		x						x	
10			x				x				x				x		
11				x		x						x		x			
12					X								**X**				
13				x		x						x		x			
14			x				x				x				x		
15		x						x		x						x	
16	x								X								x

Fig.5: Example of interference diagrams : a) bladed disk with 32 free blades or 32 blades connected with continuous binding, b) bladed disk with 32 blades in 8 packets, each with 4 blades

Natural frequencies and mode shapes of the action wheel are influenced by the number of nodal diameters, which manifest in vibration of moving blades in out-of-phase. The series of bladed disk natural frequencies which have identical mode shape from the point of view of one moving blade and differ in the number of nodal diameters from the point of view of the bladed disk is called 'family' (the series of associated mode shapes is called 'family of modes') – see Fig. 4.

The maximum number of mode shapes (number of nodal diameters) plotted on an interference diagram for M moving blades in the disk is $M/2$ (or $(M-1)/2$, when M is odd). In [2] there is examined the problem if the higher order of harmonic(s) than $M/2$ of an exciting force can excite modes of lower harmonic(s) content. On the basis of the problem analysis it is concluded in [2] that the mode shape of the action wheel with β nodal diameters can be excited by a force (see relation (1)) of the shape of $\cos(\beta\,\omega)$, $\cos[(M-\beta)\,\omega]$, $\cos[(M+\beta)\,\omega]$, $\cos[(2\,M-\beta)\,\omega]$, $\cos[(2\,M+\beta)\,\omega]$ etc. It means that the mode shape with β nodal diameters can be excited by a force having $\cos(L\,\omega)$ shape where $L = i\,M \pm \beta$; $i = 0, 1, 2, \ldots, M$ (note : M is the total number of moving blades).

On the basis of [1], [2] and [3] dealing with the theoretical bases, with the creation and the importance of the SAFE diagrams (in compliance with the methodology used in the company of Dresser-Rand) it can be confirmed that the SAFE diagrams are beneficial for the evaluation of the suitability of designs of action wheels of rotating machines from the point of view of identification of the 'probable' and the real resonant states. But the approach to the evaluation of resonant states ('probable' or real?) in the cases, in which the exciting force shape corresponds to the multiple of the rotor speed frequency (according to [1] real, according to [3] 'probable', in [2] both possibilities are mentioned) is not fully evident from [1], [2] and [3]. In [1], [2] and [3] the information capacity of the Campbell diagrams is (probably on purpose) underestimated too much.

In the action wheel the ZN340-2 moving blades are coupled with the continuous binding formed by a shrouding and a tie-boss (or under the rotor lower speed the blades are free or they are coupled only by a shrouding [5]), intrapacket mode shapes then do not exist. That is why the harmonic contents do not develop in the SAFE diagrams (in this case harmonic contents are of the same meaning as nodal diameters of the action wheel; see Fig. 5a). Rotor operating speed is 3000 rpm (i.e. 50 Hz). Steam inlet on the action wheel is performed by a guide wheel with 48 stator blades (i.e. exciting forces frequencies are multiples of 48), it means that the first multiple of the frequency of exciting forces 2400 Hz ('nozzle passing frequency') excited by the steam inlet is considerably above the upper limit of the monitored natural frequencies (350 Hz – see Chapter 3) at operating speed. Thus the SAFE diagrams do not have more important information capability for the bladed disk with blades of the ZN340-2 type than the classical Campbell diagrams. But the parallel presentation of results in the Campbell diagram and the 'SAFE plane' is more transparent for the evaluation of the possibility of the excitation of the action wheel resonant vibration.

3. Evaluation on the basis of the operating speed

In practice the suitability of the design of the steam turbine action wheel from the point of view of the possibility of the resonant vibration excitation is sometimes evaluated only on the basis of comparing the natural frequencies of the action wheel and the multiples of the rotor speed frequency at the operating speed. It is verified if the natural frequencies of the action wheel are identical with lower multiples (up to the septuple) of the speed frequency (in the case of the bladed disk with the blades of the ZN340-2 type multiples by 50, it means the frequencies up to 350 Hz are checked) at the operating speed. The rule that at the operating speed the natural frequencies should be at least by 5 % lower or higher than the multiples of the speed frequency is applied.

In the case of free blades and a relatively rigid disk the nodal diameters do not influence the natural frequencies and mode shapes at all. It means that the SAFE diagram creation is useless and the Campbell diagram is sufficient for the evaluation of the suitability of the action wheel design (from the point of view of the possibility of a resonant vibration excitation). In the case of the bladed disk with the blades connected with the continuous binding all the 'probable' resonant frequencies, i.e. states, in which exciting forces from multiples of speed frequency can be of the same shape as the mode shapes associated to the corresponding natural frequencies not fulfilling the rule of 'five percent', are considered dangerous.

4. Critical speed of the bladed disk

Experience in the steam turbines operation shows that only bending mode shapes without nodal circles (e.g. [4], [5] and [11]) are the source of failures of the disk itself. At the bladed disk vibration at standstill the nodal diameters are motionless regarding the disk. In a simplified way, the relation for the j-th bending mode shape of the bladed disk can be written in the form (this simplification does not have any influence on the deduced conclusions at all, it concerns only the function determining the mode shape amplitude along radius $X(r)$, which does not take into account the possibility of moving blade vibrations also in tangential direction)

$$z = X(r)\,\sin(\beta\,\varphi)\,\cos(\omega_j\,t)\ , \tag{4}$$

where φ is the angle of a phase difference and ω_j is j-th angular natural frequency.

Relation (4) can be modified, using the theorem on the sum and difference of sinus of two angles, into the form (for $\beta \neq 0$ – from experience the mode shape without nodal diameters is not dangerous for the disk damage)

$$z = \frac{1}{2}\,X(r)\,\sin\left[\beta\left(\varphi + \frac{\omega_j}{\beta}\,t\right)\right] + \frac{1}{2}\,X(r)\,\sin\left[\beta\left(\varphi - \frac{\omega_j}{\beta}\,t\right)\right]\ . \tag{5}$$

The first term in the relation (5) is the expression for forward propagating wave and the second one is the expression for backward propagating wave (see e.g. [10]). Those waves will circulate at the angular frequency ω_j/β around the rotation axis of the bladed disk. The amplitude of waves equals the half of the mode shape amplitude $X(r)$.

At rotating the bladed disk a centrifugal force is acting radially from the centre towards the circumference. These force increases stiffness of the bladed disk and thus the values of its natural frequencies are increased as well. The j-th natural frequency of the bladed disk at n_Ω rpm can be expressed by the relation

$$f_{\mathrm{d}_j} = \sqrt{\left(\frac{\omega_j}{2\,\pi}\right)^2 + B\,n_\Omega^2}\ , \tag{6}$$

where B is the coefficient dependent on geometric dimensions of a disk and moving blades and on the number of nodal diameters β. Now the considered waves will circulate around the bladed disk at angle speed $2\pi\,f_{\mathrm{d}_j}/\beta$ (regarding the bladed disk), i.e. the number of the wave revolutions is f_{d_j}/β per second. Regarding the non-rotating coordinate system the number of revolutions (per second) of the forward propagating wave is

$$n_{+j} = \frac{f_{\mathrm{d}_j}}{\beta} + n_\Omega \tag{7}$$

and of the backward propagating wave

$$n_{-j} = \frac{f_{\mathrm{d}_j}}{\beta} - n_\Omega\ . \tag{8}$$

The considered forward and backward propagating waves are formed by a sinusoid with β periods. Vibration frequency regarding the non-rotating coordinate system is a β-multiple of the number of the waves revolutions (per second), i.e. the frequency of the forward propagating wave is

$$f_{+j} = \beta\, n_{+j} = f_{\mathrm{d}_j} + \beta\, n_\Omega \tag{9}$$

and the backward propagating wave is

$$f_{-j} = \beta\, n_{-j} = f_{\mathrm{d}_j} - \beta\, n_\Omega\ . \tag{10}$$

In practice it was verified that the resonant vibration of the disk can be excited at acting the force which is of the same frequency as the forward or the backward propagating wave. The most dangerous is the case in which the backward propagating wave is motionless in a space

$$f_{-j} = 0\ , \tag{11}$$

because in order to excite and keep the resonant vibration of this type it is sufficient to act on the bladed disk with the force of a zero frequency, it means with the constant force.

This case is characterized with a so called disk critical speed. The relation for their calculation can be obtained by substituting the expression for the motionless backward propagating wave (11) into the relation (10), which expresses the frequency of the backward propagating wave

$$n_\Omega = \frac{f_{\mathrm{d}_j}}{\beta}\ . \tag{12}$$

It is known from practice that for the turbine operation it is important that the operating speed should not be the bladed disk critical speed for two up to six nodal diameters at bending vibration without nodal circles (at two nodal diameters the difference between the operating and the critical speed should be at least 15 %, at three or four nodal diameters at least 10 % and at five or six ones at least 5 % [4]). At vibrating with a larger number of nodal diameters the intensity of exciting forces is not sufficient for keeping the amplitude causing the disk failure and the critical speed for vibrating with one nodal diameter does not exist at steam turbines bladed disks (at steam turbines bladed disks the coefficient B is always higher than 1 and after substituting relation (12) for the calculation of a critical speed for $\beta = 1$ into (6) a real solution does not exist).

5. Conclusion

On the basis of the evaluation of one (the most conservative) of the calculation models' results [5] it was possible to conclude that at operating speed the bladed disk with blades of the ZN340-2 type with the continuous binding formed by a shrouding and a tie-boss is, from the dynamic point of view, on the boundary of the endangering by the exciting of the resonant vibration in the mode shape with five nodal diameters, which is associated to the second natural frequency. But from the point of view of an unknown intensity of exciting forces it was not possible to determine in a qualified way if in the case of the natural mode shape with five nodal diameters, which corresponds to the second natural frequency, the 'probable' ('probable resonance' or 'possible resonance') or the real ('real resonance') resonant state was concerned.

The bladed disk critical speed with the blades of the ZN340-2 type was only investigated due to research reasons. As it has been stated the critical speed of the bladed disk is mostly the cause of failures of relatively flexible disks, which does not threaten at the relatively stiff bladed disk with the blades of the ZN340-2 type. Moreover, from the calculated critical speed (given in [5]) it is evident that the operating speed is not the critical speed of the bladed disk.

When using the computational models in solving individual problems of technical practice it is always necessary to realize that the calculation results only describe behaviour of the created virtual models, not of real structures. From the point of view of the verification of the suitability of the approach towards the computational models creation and from the point of view of acquiring new pieces of knowledge for further development of a computational modelling in the field of steam turbines bladed disks with the continuous binding it is necessary to verify the computational models on the basis of results acquired by means of experimental measurements' evaluation. The measurements were performed in 2008 [12].

Acknowledgement

The article has originated in the framework of the solution of the Research Plan of the Ministry of Education, Youth and Sports of the Czech Republic MSM4771868401.

References

[1] Singh M.P., Vargo J.J., Schiffer D.M., Dello J.D.: SAFE Diagram – A Design Reliability Tool for Turbine Blading, In: Seventeenth Turbomachinery Symposium, Turbomachinery Laboratory, Texas A&M University, College Station 1988, p. 93

[2] Singh M.: SAFE Diagram – A Dresser-Rand Evaluation Tool for Packeted Bladed Disc Assembly, Technology Report Dresser-Rand Company, TP025, 1984, Wellsville, N.Y.

[3] Dello J.: Frequency Evaluation of a Steam Turbine Bladed Disk, Turbomachinery International, January/February (1987) 19

[4] Levin A.B.: Rabočije lopatki i diski parovych turbin (Moving Blades and Disks of Steam Turbines), Gosenergoizdat Moskva 1953, Moskva (in Russian)

[5] Polach P., Jankovec J., Kindelmann P.: Výpočet dynamických charakteristik oběžného kola parní turbíny s oběžnými lopatkami s průběžnou vazbou bandáží a tie-bossem (Calculation of Dynamic Characteristics of the Steam Turbine Bladed Disk with the Blades with the Continuous Binding in the Form of Shrouding and Tie-boss), Research Report ŠKODA VÝZKUM s.r.o., VYZ 0904/06, 2006, Plzeň (in Czech)

[6] Polach P.: Calculation of Natural Vibration of a Steam Turbine Bladed Disk Using Rotational Periodicity, In: Third International Conference on Mechanical Engineering and Mechanics, eds. R.C. Batra, L.F. Qian, Y. He, X.N. Li, K.D. Zhou, H. Dresig, Y. Morita, E. Cheung, Science Press and Science Press USA Inc., Beijing 2009, Vol. 1, p. 761

[7] Polach P.: Calculation of Natural Vibration of a Steam Turbine Bladed Disk Using Rotational Periodicity and Comparison with Experimental Measurement, In: 7th EUROMECH Solid Mechanics Conference, eds. J.A.C. Ambrósio, M.P.T. Silva, Portuguese Association for Theoretical, Applied and Computational Mechanics, Lisbon 2009, CD-ROM

[8] Polach P.: Using Rotational Periodicity Method for Calculation of Natural Vibration of the Turbine Blade of Progressive Design, In: The Tenth International Conference on Computational Structures Technology CST2010, eds. B.H.V. Topping, J.M. Adam, F.J. Pallarés, R. Bru, M.L. Romero, Civil-Comp Press, Valencia 2010, CD-ROM

[9] Polach P.: Calculation of Natural Vibration of Linear Undamped Rotationally Periodic Structures, Engineering Mechanics 15 (2008) 81

[10] Mester S.S., Benaroya H.: Periodic and Near-Periodic Structures, J. Shock and Vibration 2 (1995) 69

[11] Polach P.: Analysis of Dynamic Properties of Steam Turbine's Acting Wheel with Blades Connected by a Continuous Binding, In: VIII. International Conference on the Theory of Machines and Mechanisms, eds. J. Mrázek, J. Kaniok, M. Bílek, TU of Liberec, Liberec 2000, p. 631

[12] Bordo L., Bevegni M.: Steam turbine 48" last stage blade dynamic test, document Ansaldo Energia S.p.A., 255W547, 2008, Genova

[13] Polach P.: Problematika posuzování vhodnosti konstrukčního řešení oběžného kola turbíny z hlediska možnosti vybuzení rezonančního kmitání (Problems of Evaluating the Suitability of the Bladed Disk Design Regarding the Possibility of the Resonant Vibration Excitation), In: National Colloquium with International Participation Dynamics of Machines 2011, ed. L. Pešek, Institute of Thermomechanics AS CR, v.v.i., Prague 2011, p. 69 (in Czech)

Received in editor's office: March 29, 2011
Approved for publishing: May 30, 2011

Note: This paper is an extended version of the contribution [13] presented at the national colloquium with international participation *Dynamics of Machines 2011* in Prague.