EFFECTS OF FIBRE MASS ON THE DYNAMIC RESPONSE OF AN INVERTED PENDULUM DRIVEN BY FIBRES

Pavel Polach*, Michal Hajžman**, Zbyněk Šika***, Jakub Mrštík**, Petr Svatoš***

Fibres, cables, wires and fibres can play an important role in design of many machines. One of the most interesting applications is replacement of chosen rigid elements of a manipulator or a mechanism by cables. The main advantage of this design is the achievement of a lower moving inertia, which leads to a higher mechanism speed, and lower production costs. An inverted pendulum attached and driven by two fibres serves as a typical testing system for the investigation of the fibres properties influence on the system dynamic response. Motion of the pendulum of this nonlinear system is investigated using the alaska simulation tool and an in-house program created in the MATLAB system. Since mass of fibres can influence employed types of fibre models, chosen dynamic quantities (e.g. pendulum angle, vibration amplitude etc.) are investigated in dependence on the excitation frequency for various fibres weights.

Keywords : *inverted pendulum, fibres, vibration*

1. Introduction

One of interesting applications of fibres or cables is the replacement of chosen rigid elements of manipulators or mechanisms by those flexible elements [1]. The main advantage of this design is the achievement of a lower moving inertia, which can lead to a higher machine speed, and lower production costs. Drawbacks can be associated with the fact that cables should be only in tension [2], [3] in the course of a motion. The possible fibre modelling approaches should be tested and their suitability verified in order to create efficient mathematical models of cable-based manipulators mainly intended for the control algorithm design. An inverted pendulum driven by two fibres attached to a frame (see Fig. 1) is a simplified representation of a typical cable manipulator. The motion of the pendulum of this nonlinear system is investigated using the **alaska** simulation tool and using an in-house software created in the MATLAB system. The influence of some parameters of the system of inverted pendulum driven by fibres has already been investigated. The influence of the actuated fibres motion on the pendulum motion in the case of their simultaneous harmonic excitation was investigated in [4] or [5], the influence of the phase shift in the case of non-symmetric harmonic excitation was investigated in [6]. The effect of the fibres preload on the pendulum motion was investigated in [7], the influence of the amplitude of the harmonic kinematic excitation of fibres on the pendulum motion was investigated in [8], validation of

* Dr. Ing. P. Polach, Section of Materials and Mechanical Engineering Research, Výzkumný a zkušební ústav Plzeň s.r.o., Tylova 1581/46, 301 00 Plzeň

** Ing. M. Hajžman, Ph.D., Mgr. J. Mrštík, Department of Computer-Aided Modelling, Section of Materials and Mechanical Engineering Research, Výzkumný a zkušební ústav Plzeň s.r.o., Tylova 1581/46, 301 00 Plzeň

*** prof. Ing. Z. Šika, Ph.D., Ing. P. Svatoš, Department of Mechanics, Biomechanics and Mechatronics, Faculty of Mechanical Engineering, Czech Technical University in Prague, Technická 4, 166 07 Praha 6

permissible load of fibres during their acting under dynamic loading was checked in [9]. The effect of the mass of the fibres (in the case of their simultaneous harmonic excitation) on the pendulum motion is investigated in this article.

2. Possibilities of the cable modelling

The fibre (cable, wire etc.) modelling [10] should be based on considering the cable flexibility and the suitable approaches can be based on the flexible multibody dynamics (see e.g. [11], [12]). The simplest way how to incorporate cables in equations of motion of a mechanism is the force representation of a fibre (e.g. [13]). It is assumed that the mass of cables is low to such an extent comparing to the other moving parts that the inertia of cables is negligible with respect to the other parts. The fibre is represented by the force dependent on the cable deformation and its stiffness and damping properties. This way of the cable modelling is probably the most frequently used model in the cable-driven robot dynamics and control.

A more precise approach is based on the representation of the fibre by a point-mass model (e.g. [14]). The fibre can be considered either flexible or rigid. It has the advantage of a lumped point-mass model. The point masses can be connected by forces or constraints.

In order to represent bending behaviour of cables their discretization using the finite segment method [11] or so called rigid finite elements [15] is possible. Standard multibody codes (SIMPACK, MSC.ADAMS, **alaska** etc.) can be used for this purpose. Other more complex approaches can utilize nonlinear three-dimensional finite elements [16] or can employ the absolute nodal coordinate formulation (ANCF) elements [11], [12], [17].

Investigation of the approaches to the modeling of the system of inverted pendulum driven by fibres was investigated in [18] and [19]. Implementation of the model based on the finite rigid elements into the **alaska** simulation tool proved to be unsuitable [18]. The ANCF elements cannot be implemented in the **alaska** simulation tool, verification on this approach was carried out utilizing the MATLAB system [19].

3. Inverted pendulum

As an example of the investigation of fibres behaviour an inverted pendulum, which is attached and driven by two fibres (see Fig. 1) and is affected by a gravitation force, was chosen.

When the pendulum is displaced from the equilibrium position, i.e. from the ‘upper’ position, it is returned back to the equilibrium position by the tightened fibre. As it has already been mentioned, this system was selected with respect to the fact that it is a simplification of possible cable-based manipulators. In addition it was supposed that the nonlinear system of the inverted pendulum attached to a frame by two fibres would show an unstable behaviour under specific excitation conditions (e.g. [20]).

In order to investigate the effect of the mass of the fibres on the pendulum motion the point-mass model of fibres in the inverted pendulum models [4] is used.

For better description of the solved problem a simple massless model is presented. The massless model is shown in Fig. 1 (the used model of the fibre based on the point-mass model with lumped point masses corresponding to the mass of the fibre is geometrically

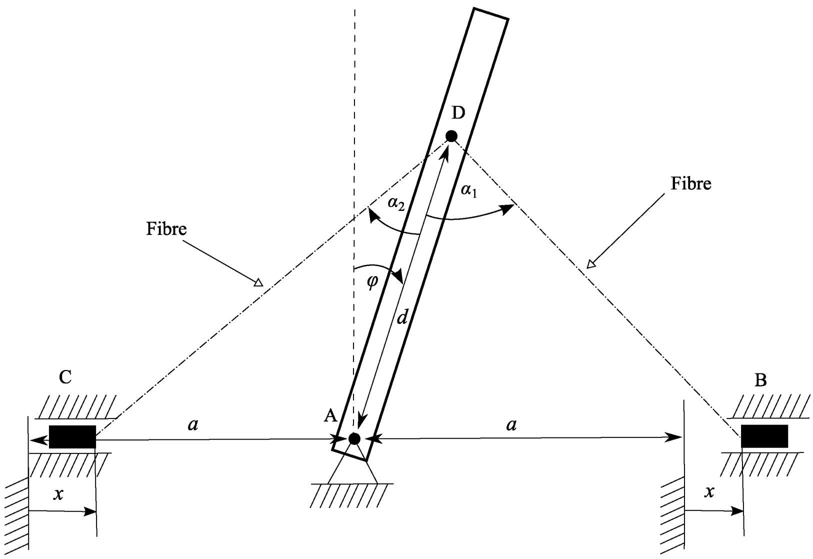


Fig.1: Inverted pendulum actuated by the fibres

identical) [4]. The models of the system of the inverted pendulum are considered to be two-dimensional.

The system kinematics can be described by angle φ of the pendulum with respect to its vertical position (one degree of freedom), angular acceleration $\ddot{\varphi}$ and prescribed kinematic excitation $x(t)$. The equation of motion is of the form

$$\ddot{\varphi} = \frac{1}{I_\mathrm{A}} \left(F_{\mathrm{v}1}\, d\, \sin\alpha_1 + F_{\mathrm{v}2}\, d\, \sin\alpha_2 + m\, g\, \frac{l}{2}\, \sin\varphi \right) , \tag{1}$$

where I_A is the moment of inertia with respect to the axis in point A (see Fig. 1), α_1 and α_2 are angles between the pendulum and the fibres, m is the mass of the pendulum, g is the gravity acceleration and l is the length of the pendulum. The forces acting on the pendulum from the fibre are

$$\begin{aligned} F_{\mathrm{v}1} &= \left[k_\mathrm{v}\, (l_{\mathrm{v}1} - l_{\mathrm{v}0}) + b_\mathrm{v}\, \frac{\mathrm{d}l_{\mathrm{v}1}}{\mathrm{d}t} \right] H(l_{\mathrm{v}1} - l_{\mathrm{v}0}) , \\ F_{\mathrm{v}2} &= \left[k_\mathrm{v}\, (l_{\mathrm{v}2} - l_{\mathrm{v}0}) + b_\mathrm{v}\, \frac{\mathrm{d}l_{\mathrm{v}2}}{\mathrm{d}t} \right] H(l_{\mathrm{v}2} - l_{\mathrm{v}0}) , \end{aligned} \tag{2}$$

where k_v is the fibre stiffness, b_v is the fibre damping coefficient, $l_{\mathrm{v}0}$ is the original length of the fibres and $H(\cdot)$ is the Heaviside function. It is supposed, that forces act in the fibres only when the fibres are in tension.

Actual lengths $l_{\mathrm{v}1}$ and $l_{\mathrm{v}2}$ of the fibres should be calculated in each time

$$\begin{aligned} l_{\mathrm{v}1} &= \sqrt{(d\, \cos\varphi)^2 + (a + x(t) - d\, \sin\varphi)^2} , \\ l_{\mathrm{v}2} &= \sqrt{(d\, \cos\varphi)^2 + (a - x(t) + d\, \sin\varphi)^2} , \end{aligned} \tag{3}$$

In the fibre model based on the point masses each fibre is discretized using 10 point masses (e.g. [4]). Each point mass is unconstrained (i.e. number of degrees of freedom is 3) in two-dimensional model of the system of the inverted pendulum. The adjacent point masses are connected using spring-damper elements. Only axial (spring and damping) forces are considered in these spring-damper elements. The stiffness and the damping coefficients between the masses are determined in order to keep the global properties of the massless fibre model. The validation of the point-mass model is given in [21].

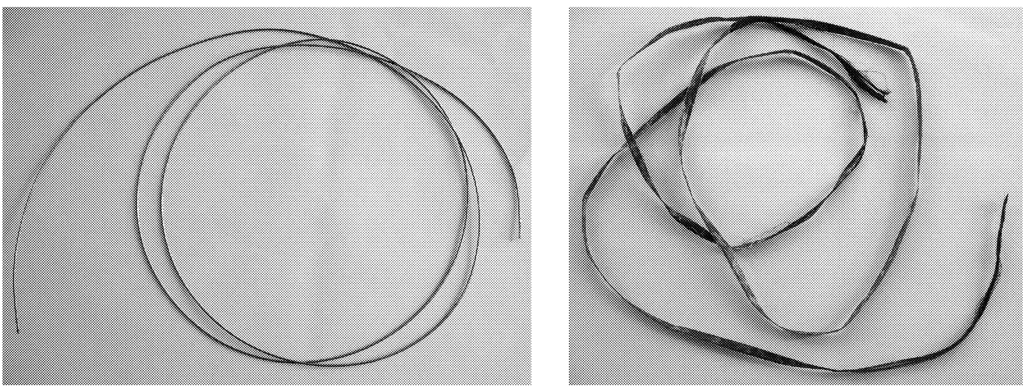
The kinematic excitation is given by function

$$x(t) = x_0 \sin(2\pi f t) , \tag{4}$$

where x_0 is the chosen amplitude of motion and f is the excitation frequency. The influence of the excitation frequency on the pendulum motion is investigated. Excitation in points designated B and C (see Fig. 1) is considered to be symmetrical (without any mutual phase shift) and of the same amplitude x_0.

The most important model parameters (see Fig. 1) are: $l = 1$ m, $a = 1.2$ m, $d = 0.75$ m, $I_\mathrm{A} = 3.288\,\mathrm{kg\,m^2}$, $m = 9.864$ kg, $k_\mathrm{v} = 8.264\times10^3$ N/m (stiffness), $b_\mathrm{v} = 5\times10^{-4}\,k_\mathrm{v}$ N s/m (damping coefficient). Additional parameters of the point-mass models were originally (in [4], [6] or [7]) fibre cross-section area $A_\mathrm{v} = \pi\,0.000001\,\mathrm{m^2}$ and fibre density $\rho_\mathrm{v} = 4000\,\mathrm{kg\,m^3}$ (these parameters represent a wattled steel wire – see Fig. 2a). In this case mass of one fibre is 17.783 grams. Further a 'light' fibre made of thin carbon fibres in Department of Mechanics, Biomechanics and Mechatronics, Faculty of Mechanical Engineering, Czech Technical University in Prague (the mass of one carbon fibre is 3.846 grams) was considered – see Fig. 2b. Further virtual fibres of the mass ten times higher than the mass of the wattled steel wire (i.e. of the mass 177.83 grams; 'heavier' fibre) and of the mass of 'the heaviest' fibre (i.e. of the mass 1269 grams) were chosen.

Natural frequency of the linearized system of the inverted pendulum in equilibrium position when considering the massless fibre model is 5.04 Hz.



a) b)

Fig.2: Real fibres: a) wattled steel wire, b) 'light' fibre made of thin carbon fibres

4. Simulations results

The kinematic excitation amplitude (defined by equation 4) $x_0 = 0.02$ m was chosen (as in [4] and [6]). An example of the time history of kinematic excitation is given in Fig. 3.

Presented results are obtained using the **alaska** simulation tool. Generated nonlinear equations of motion are solved by means of numerical time integration. The simulations results presented in this article were obtained utilizing the Livermore Solver for Ordinary Differential Equation (LSODE) for stiff systems, the maximum relative error that **alaska** allows at each integration step was chosen 0.0001 and the maximum absolute error that **alaska** allows at each integration step was chosen 0.0001, too. Time step of this integration routine is variable.

Time histories and extreme values of pendulum angle φ (the maximum value of pendulum angle at quasi-static loading is $\varphi = 1.52°$; the minimum value of the pendulum angle at quasi-static loading is logically $\varphi = -1.52°$), of the force in the fibres and of the positions of the point masses are the monitored quantities.

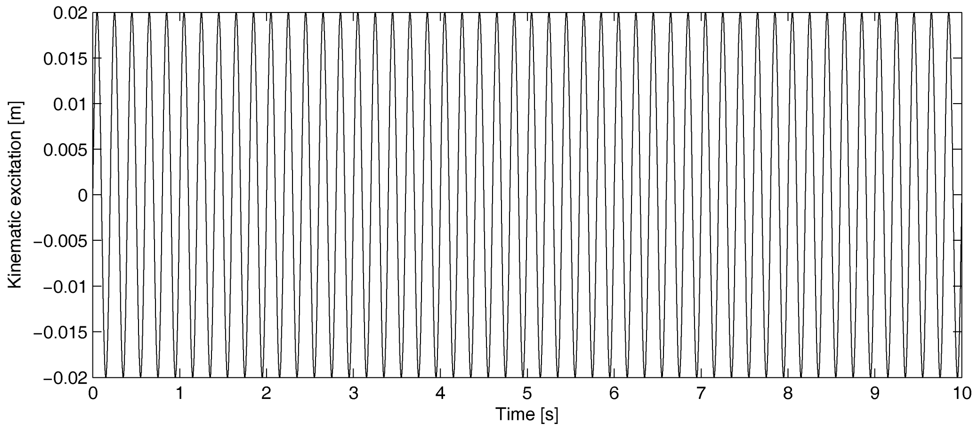


Fig.3: An example of the time history of kinematic excitation ($f = 5\,Hz$)

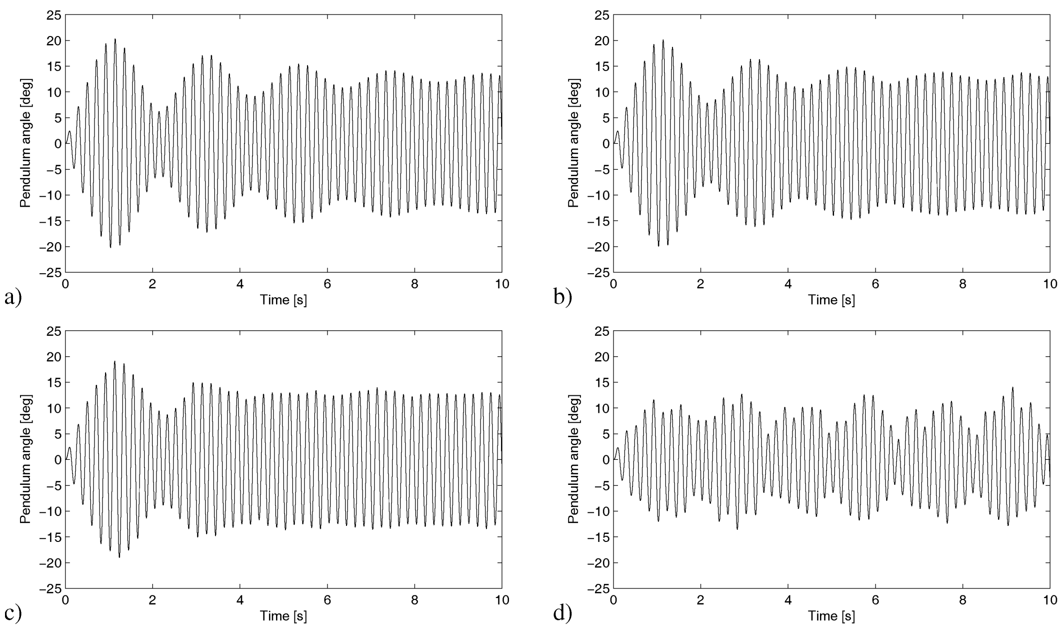


Fig.4: Time history of pendulum angle φ, $f = 5\,Hz$, a) carbon fibres, b) wattled steel wires, c) 'heavier' fibres, d) 'the heaviest' fibres

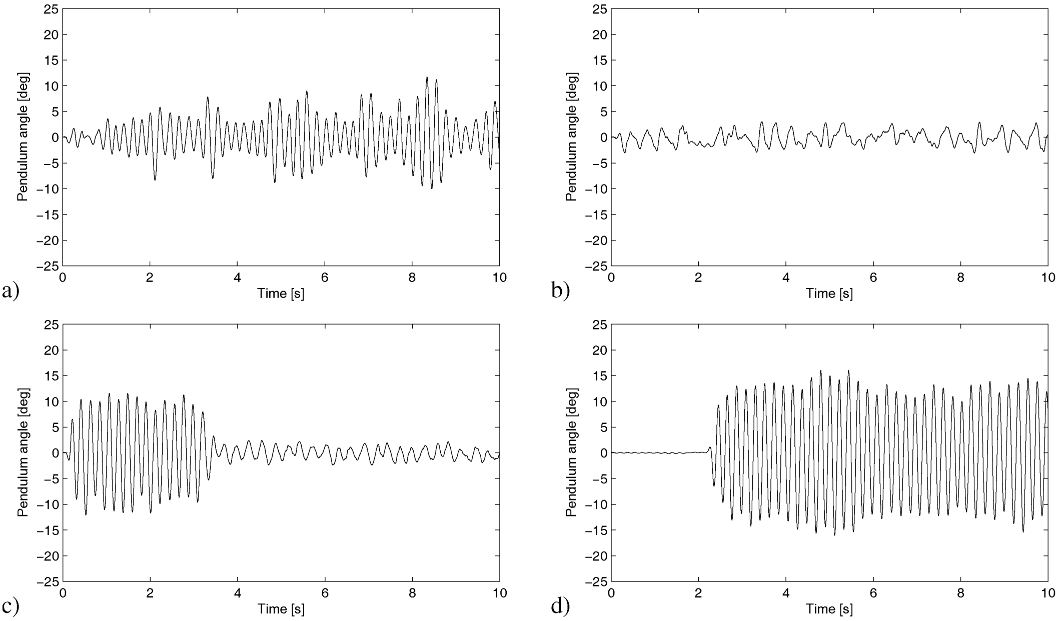


Fig.5: Time history of pendulum angle φ, 'heavier' fibres, a) $f = 170$ Hz, b) $f = 180$ Hz, c) $f = 190$ Hz, d) $f = 200$ Hz

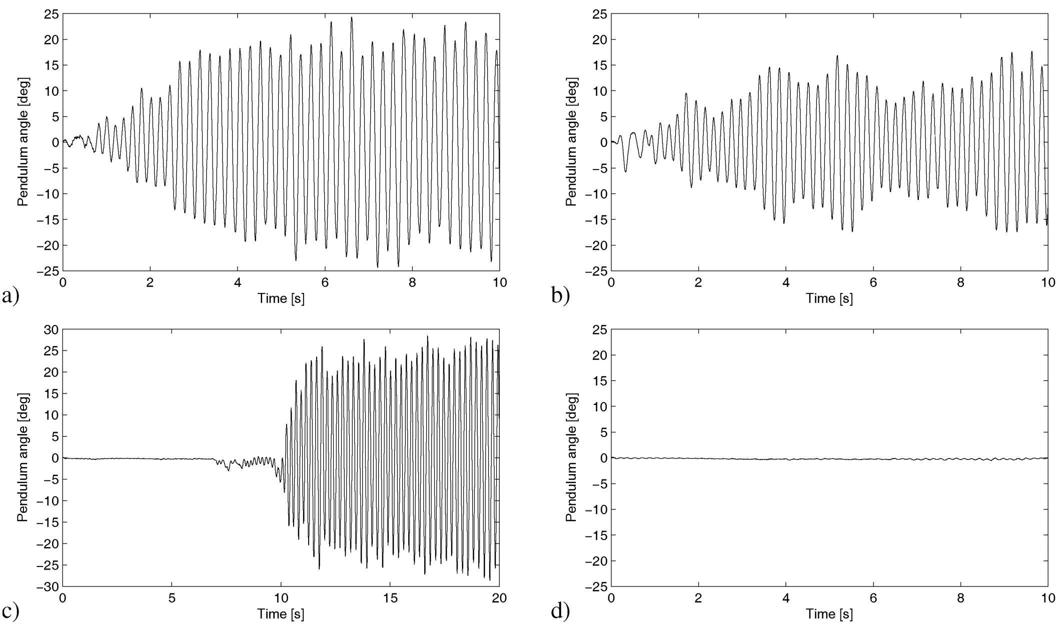


Fig.6: Time history of pendulum angle φ, 'the heaviest' fibres, a) $f = 40$ Hz, b) $f = 80$ Hz, c) $f = 130$ Hz, d) $f = 140$ Hz

Excitation frequency f was considered in the range from 0.1 Hz to 200 Hz. Selected results of the numerical simulations are presented in Figs 4 to 10. To make comparison results obtained using the massless model (partly taken from [4], [5]) are plotted in graphs in Figs 7 and 8. Results in Figs 8 and 10 are given in the frequency range from 0.1 Hz to 10 Hz because the upper limit of excitation frequencies 200 Hz is too high for the practical use in manipulators.

Simulation time is 10 seconds (in some 'special' cases 20 seconds – see Fig. 6c). It was verified that after this period the character of the system response to the kinematic excitation did not change (e.g. [4]).

From the results obtained (see Figs 4 to 10) it is evident that the pendulum motion is influenced (besides by the excitation frequency) by the mass of the moving fibres.

The results which were obtained considering the carbon fibres are (almost) the same as the results which were obtained using the massless models [4], [5]. This is observed up to the excitation frequency of 10 Hz (see Fig. 8). The global extreme values of pendulum angle φ (with exception of using 'the heaviest' fibres, where only the local extreme values are concerned) appear at excitation frequency 5 Hz irrespective of the mass of the fibres (see

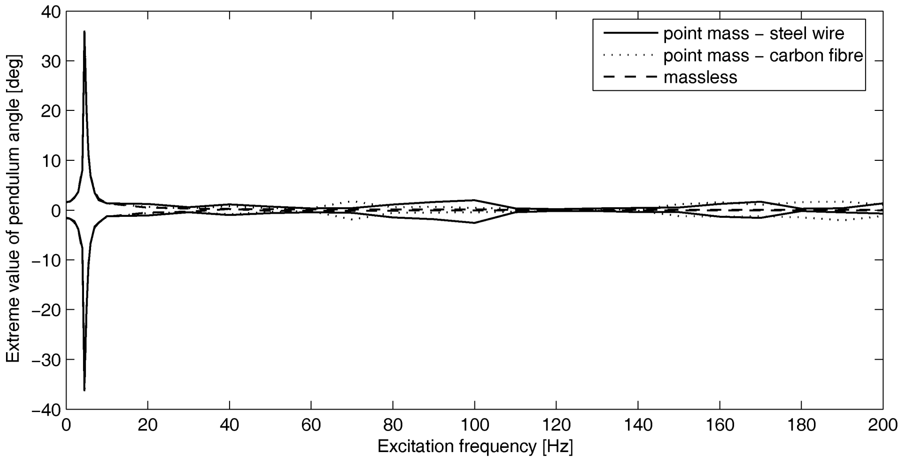


Fig.7: Time history of pendulum angle φ in dependence on excitation frequencies (carbon fibres, steel wires and massless model)

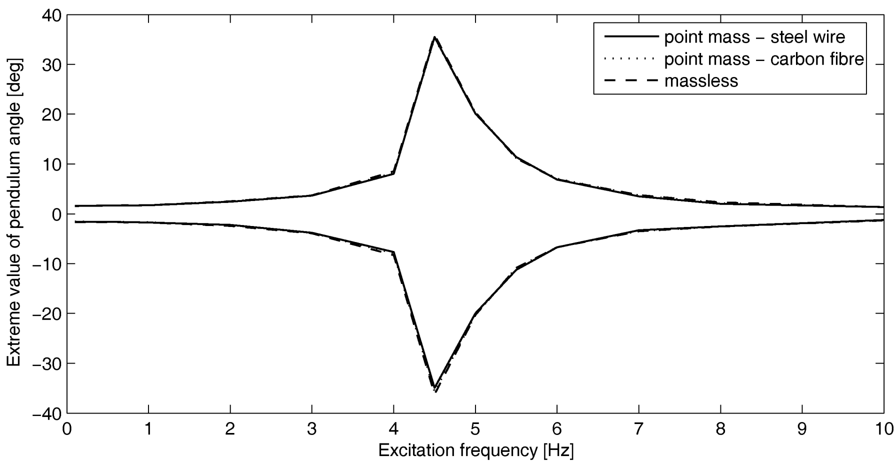


Fig.8: Time history of pendulum angle φ in dependence on excitation frequencies (carbon fibres, steel wires and massless model)

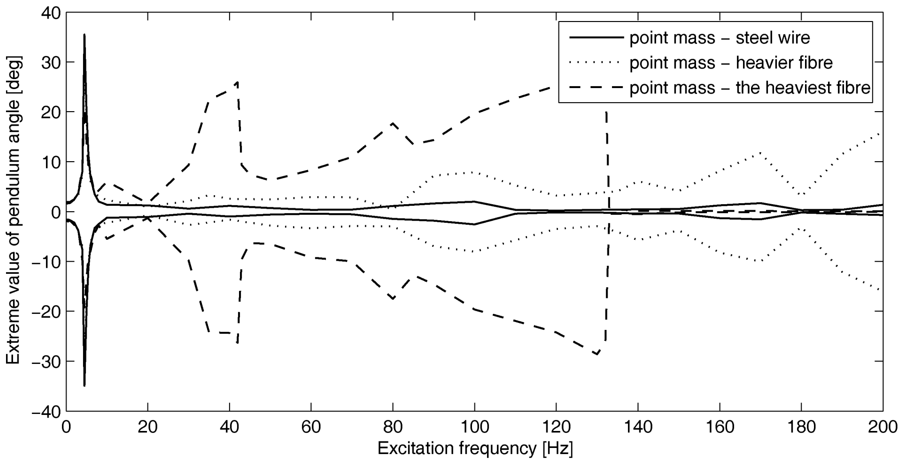


Fig.9: Time history of pendulum angle φ in dependence on excitation frequencies (steel wires, 'heavier' and 'the heaviest' fibres)

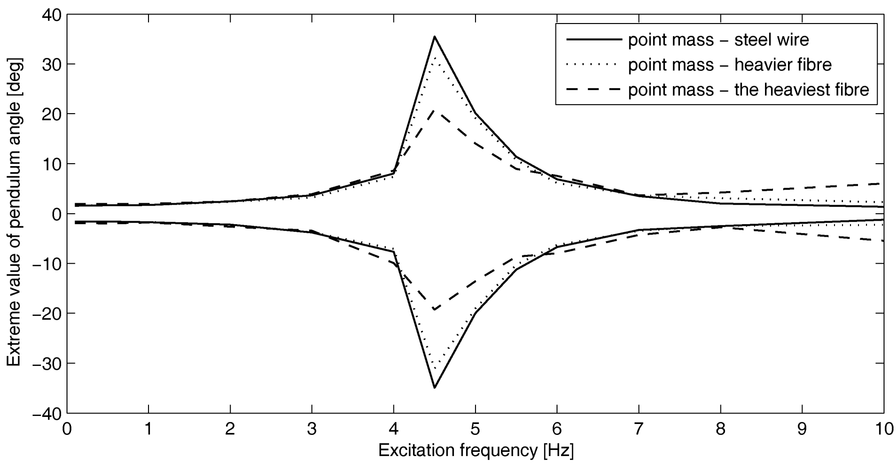


Fig.10: Time history of pendulum angle φ in dependence on excitation frequencies (steel wires, 'heavier' and 'the heaviest' fibres)

Fig. 4 and Figs 7 to 10) in the monitored interval of the excitation frequencies (i.e. up to 200 Hz). The lower the mass of the fibres the higher the extreme values of pendulum angle φ (see Fig. 4) at this frequency. When using 'heavier' fibres pendulum angle φ increases at higher excitation frequencies (above 170 Hz) (see Fig. 5). When using 'the heaviest' fibres pendulum angle φ achieves considerable local extreme values at excitation frequency 40 Hz (see Fig. 6a) and high extreme values of pendulum angle φ appear at the excitation frequency up to 130 Hz (see Fig. 6b and 6c). At higher frequencies the pendulum with 'the heaviest' fibres does not vibrate (see Fig. 6d).

Generally, at higher excitation frequencies f, the results obtained at the simulations with the point-mass models indicate that the character of the system behaviour changes at certain excitation frequencies. The fibres vibration is reflected in the pendulum vibration character. Already mentioned remarkable time histories of pendulum angle obtained at various parameters of system of inverted pendulum model are given in Figs 4 to 6.

5. Conclusion

The approach to the fibre modelling based on the lumped point-mass representations was utilized for the investigation of the effect of the mass of the fibres on the motion of the inverted pendulum driven by two fibres attached to a frame. The influence of the actuated fibres motion on the pendulum motion in the case of their simultaneous harmonic excitation was investigated. It was demonstrated that the mass of the fibres influences the pendulum vibration significantly but not to such a great extent as it had been originally supposed (unstable behaviour of the pendulum).

Experimental verification of the fibre dynamics within the manipulator systems and research aimed at measuring the material properties of selected fibres are considered important steps in further research.

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