MODAL CONDENSATION OF BLADED DISK

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The aim of the paper is modal condensation of a system which represents bladed disk. These kinds of systems are mainly used in steam turbines what is the topic of the article. The model of the bladed disk is based on decomposition into a disk subsystem and a blading subsystem. The finite element method is used for modeling of the both subsystems. All influences of steady-state rotation are respected as centrifugal forces, gyroscopic effects, centrifugal stiffening of blades and dynamic softening. Blades are modeled by using 1D elements and the disk is modeled by using 3D hexahedral elements. Modal condensation is applied to a disk subsystem. Different levels of the condensation are presented and compared on a testing bladed disk.

Keywords: *modal condensation, bladed disk, modal analysis, FEM*

1. Introduction

Many rotating systems with disks are modeled as an one-dimensional rotating shaft with clamped rigid disks [1], [2]. This approach can be usually applied on rotating systems excited in time periodic forces and moments with speed frequency and its multiple. Bladed disks in turbo machines are excited by high frequency hydrodynamical forces in which the high frequency vibration modes of the disks can not be neglected. Many publications are dedicated to the dynamic analysis of the thin rotating disks [3]. In known publications the flexible disk is modeled separately as an isolated subsystem or the flexible disk is linked to rigid body on the interior or external surfaces.

This article is focused on the simulation of the behavior of bladed disks which are main building blocks of steam turbines. The main aim of the research is the investigation of the dynamic behavior of disk blades with nonlinear friction elements, which can reduce undesirable high vibrations of blades. Several approaches of these nonlinearities modeling can be found in [11] and [12]. The treatment, how to mathematically describe nonlinear friction elements, is not the purpose of this paper. But the reduction of degrees of freedom (DoF) of a mathematical model can be useful due to time consumption of numerical simulation of nonlinear systems.

Another phenomenons is requirement on higher efficiency and wide operation range of steam turbines, which leads to higher necessities of accuracy of their finite element (FE) mathematical models. The accuracy of models is directly bounded with higher number of DoF which affects central processing unit (CPU) and memory costs for computation. Usage of condensation techniques is also helpful to decrease this phenomenon. The presented article demonstrates generally accepted modal synthesis method with condensation [4] applied to the mathematical model of disk subsystem. The method is tested on a model of the

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imperfect bladed disk which has been developed for experimental research at Institute of Thermomechanics of the Academy of Sciences of the Czech Republic [5].

2. Mathematical model of a bladed disk

We assume that the bladed disk is centrally clamped on inner radius to rigid shaft rotating by constant angular velocity ω around y-axis. Blades are mounted to the disk by rigid blade feet and shrouds are fixed to free ends of selected blades. The friction elements are embedded between blade shrouds of some blades. Fig. 1 demonstrates a representative disk with a blade. The disk is modeled by 3D hexahedral elements. The blades are modeled using 1D elements. Equations of motion are derived in the rotating coordinate system, which is fixed to a nondeformed bladed disk and rotates with constant angular velocity ω [6]. The equations of motion of these two uncoupled subsystems (disk and blades) can be written in following form [7], [8]

$$\mathbf{M}_{\mathrm{D}}\,\ddot{\mathbf{q}}_{\mathrm{D}}(t) + \omega\,\mathbf{G}_{\mathrm{D}}\,\dot{\mathbf{q}}_{\mathrm{D}}(t) + (\mathbf{K}_{\mathrm{s,D}} - \omega^2\,\mathbf{K}_{\mathrm{d,D}})\,\mathbf{q}_{\mathrm{D}}(t) = \omega^2\,\mathbf{f}_{\mathrm{D}}\ , \tag{1}$$

$$\mathbf{M}_{\mathrm{B}}\,\ddot{\mathbf{q}}_{\mathrm{B}}(t) + \omega\,\mathbf{G}_{\mathrm{B}}\,\dot{\mathbf{q}}_{\mathrm{B}}(t) + (\mathbf{K}_{\mathrm{s,B}} + \mathbf{K}_{\mathrm{C,B}} + \omega^2\,\mathbf{K}_{\omega,\mathrm{B}} - \omega^2\,\mathbf{K}_{\mathrm{d,B}})\,\mathbf{q}_{\mathrm{B}}(t) = \omega^2\,\mathbf{f}_{\mathrm{B}}\ , \tag{2}$$

where index D corresponds to a disk subsystem and index B corresponds to a subsystem of blades. $\mathbf{M}_{\mathrm{D}}$ and $\mathbf{M}_{\mathrm{B}}$ are mass matrices, $\omega\,\mathbf{G}_{\mathrm{D}}$ and $\omega\,\mathbf{G}_{\mathrm{B}}$ express gyroscopic effects, $\mathbf{K}_{\mathrm{s,D}}$ and $\mathbf{K}_{\mathrm{s,B}}$ represent static stiffness matrices, $\mathbf{K}_{\mathrm{C,B}}$ is contact stiffness matrix between shrouds and friction elements, $\omega^2\,\mathbf{K}_{\omega,\mathrm{B}}$ represents blade centrifugal stiffening, $\omega^2\,\mathbf{K}_{\mathrm{d,D}}$ and $\omega^2\,\mathbf{K}_{\mathrm{d,B}}$ are matrices of dynamic spin softening of 1D and 3D continuum in centrifugal field. All described matrices are symmetrical except skew-symmetrical matrices of gyroscopic effects. Centrifugal load vectors $\omega^2\,\mathbf{f}_{\mathrm{D}}$ and $\omega^2\,\mathbf{f}_{\mathrm{B}}$ are constant in time.

The equations of motion for deformable disks are written in a configuration space defined by vector $\mathbf{q}_{\mathrm{D}} = [\cdots\ u_j\ v_j\ w_j\ \ \cdots]^{\mathrm{T}} \in \mathbf{R}^{n_{\mathrm{D}}}$ of nodal displacements with respect

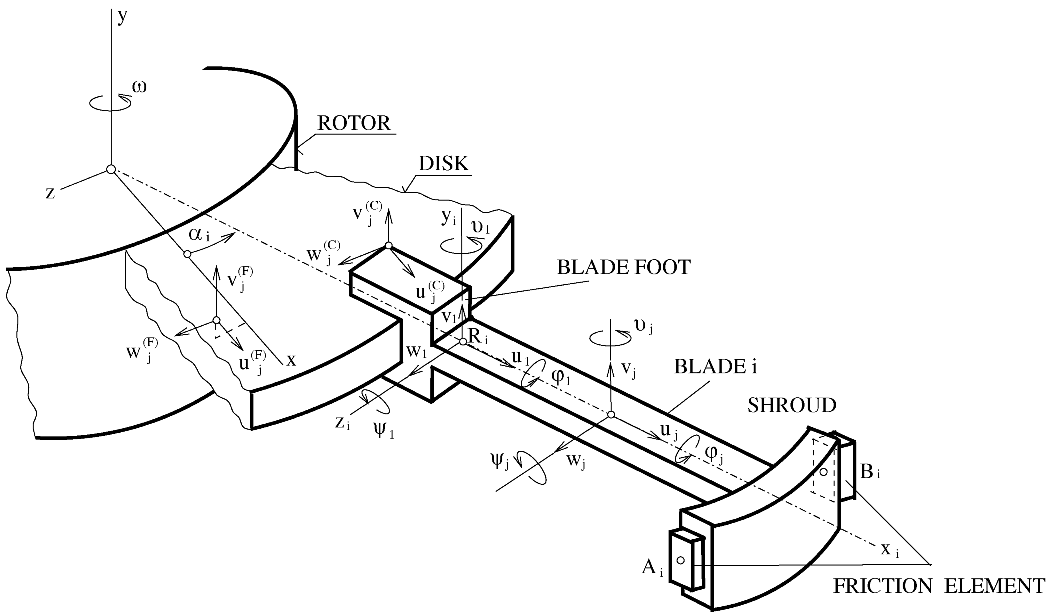

Fig.1: The blade i with a shroud and a friction elements mounted to a disk

to the rotating xyz-coordinate system. The motion equations of the blades with shrouds and friction elements (see Fig. 1) are written in configuration space defined by vector $\mathbf{q}_\mathrm{B} = [\cdots\ u_j\ v_j\ w_j\ \varphi_j\ \vartheta_j\ \psi_j\ \cdots]^\mathrm{T} \in \mathbf{R}^{n_\mathrm{B}}$ of nodal displacements with respect to the same rotating xyz-coordinate system.

The vector of disk generalized coordinates can be partitioned with respect to the couplings between disk and blades in the form

$$\mathbf{q}_\mathrm{D} = \begin{bmatrix} \mathbf{q}_\mathrm{D}^{(\mathrm{F})} \\ \mathbf{q}_\mathrm{D}^{(\mathrm{C})} \end{bmatrix} , \quad \mathbf{q}_\mathrm{D}^{(\mathrm{F})} \in \mathbf{R}^{n_\mathrm{D}^{(\mathrm{F})}} , \quad \mathbf{q}_\mathrm{D}^{(\mathrm{C})} \in \mathbf{R}^{n_\mathrm{D}^{(\mathrm{C})}} , \tag{3}$$

where the displacements of the disk nodes, which are coupled to blade foots, can be expressed by the displacements of the first blade nodes. This relation for displacements of coupled disk nodes j on the foot of the blade i and the first node of blade is

$$\begin{bmatrix} u_j^{(\mathrm{C})} \\ v_j^{(\mathrm{C})} \\ w_j^{(\mathrm{C})} \end{bmatrix} = \begin{bmatrix} \cos\alpha_i & 0 & \sin\alpha_i \\ 0 & 1 & 0 \\ -\sin\alpha_i & 0 & \cos\alpha_i \end{bmatrix} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & z_j & -y_j \\ 0 & 1 & 0 & -z_j & 0 & x_j \\ 0 & 0 & 1 & y_j & -x_j & 0 \end{array}\right] \begin{bmatrix} u_1 \\ v_1 \\ w_1 \\ \varphi_1 \\ \vartheta_1 \\ \psi_1 \end{bmatrix}_{\mathrm{B}_i} \tag{4}$$

or shortly

$$\mathbf{q}_j^{(\mathrm{C})} = \mathbf{T}_{j,i}\, \mathbf{q}_i , \tag{5}$$

where α_i is the angle between the rotating disk axis x and rotating blade axis x_j and x_j, y_j, z_j are coordinates of the coupled disk nodes j in the coordinate system $x_i y_i z_i$ of the blade i with the origin in the first blade node R_i.

The displacements of the free (uncoupled) disk nodes are localized in vector $\mathbf{q}_\mathrm{D}^{(\mathrm{F})} \in \mathbf{R}^{n_\mathrm{D}^{(\mathrm{F})}}$. The total transformation between displacements of all coupled nodes of the disk with blades can be expressed in the matrix form

$$\left[\begin{array}{c} \vdots \\ \hline \mathbf{q}_j^{(\mathrm{C})} \\ \hline \vdots \end{array}\right] = \left[\begin{array}{c|c|c} & \vdots & \\ \hline \cdots & \mathbf{T}_{j,i} & \cdots \\ \hline & \vdots & \end{array}\right] \left[\begin{array}{c} \vdots \\ \hline \mathbf{q}_i \\ \hline \vdots \end{array}\right] \Longrightarrow \mathbf{q}_D^{(\mathrm{C})} = \mathbf{T}_{\mathrm{D,B}}\, \mathbf{q}_\mathrm{B} , \tag{6}$$

where the rectangular transformation matrix is $\mathbf{T}_{\mathrm{D,B}} \in \mathbf{R}^{n_\mathrm{D}^{(\mathrm{C})}, n_\mathrm{B}}$ and $n_\mathrm{D}^{(\mathrm{C})} = n_\mathrm{D} - n_\mathrm{D}^{(\mathrm{F})}$ is DoF number corresponding to coupled nodes of the disk.

3. Condensed mathematical model of the system

The nodal coordinates of the disk FE models can be used directly as elastic coordinates, although the number of degrees of freedom required to adequately represent the deformation may be very large. Hence, the number of free node elastic coordinates $\mathbf{q}_\mathrm{D}^{(\mathrm{F})}$ of the disk is desirable reduced by the use of modal condensation [9]. Similar approach was used for condensation of disks mounted to a shaft [10]. For that purpose each matrix and vector in the disk mathematical model (1) can be rearranged according to decomposition (3)

$$\mathbf{X}_D = \begin{bmatrix} \mathbf{X}_\mathrm{D}^{\mathrm{FF}} & \mathbf{X}_\mathrm{D}^{\mathrm{FC}} \\ \mathbf{X}_\mathrm{D}^{\mathrm{CF}} & \mathbf{X}_\mathrm{D}^{\mathrm{CC}} \end{bmatrix} , \qquad \mathbf{X} = \mathbf{M}, \mathbf{G}, \mathbf{K}_\mathrm{s}, \mathbf{K}_\mathrm{d} , \qquad \mathbf{f}_\mathrm{D} = \begin{bmatrix} \mathbf{f}_\mathrm{D}^{(\mathrm{F})} \\ \mathbf{f}_\mathrm{D}^{(\mathrm{C})} \end{bmatrix} . \tag{7}$$

Let modal properties of the conservative model of the non-rotating disk (for $\omega = 0$) with blade foots be characterized by spectral and modal matrices satisfying the orthogonality conditions

$$\mathbf{V}_{\mathrm{D}}^{\mathrm{T}}\,\mathbf{M}_{\mathrm{D}}\,\mathbf{V}_{\mathrm{D}} = \mathbf{E}\ , \qquad \mathbf{V}_{\mathrm{D}}^{\mathrm{T}}\,\mathbf{K}_{\mathrm{D}}\,\mathbf{V}_{\mathrm{D}} = \mathbf{\Lambda}_{\mathrm{D}}\ , \tag{8}$$

where $\mathbf{E}$ is unit matrix. Modal matrices of disks can be rearranged into the block form

$$\mathbf{V}_{\mathrm{D}} = \begin{bmatrix} {}^{m}\mathbf{V}_{\mathrm{D}}^{\mathrm{F}} & {}^{s}\mathbf{V}_{\mathrm{D}}^{\mathrm{F}} \\ {}^{m}\mathbf{V}_{\mathrm{D}}^{\mathrm{C}} & {}^{s}\mathbf{V}_{\mathrm{D}}^{\mathrm{C}} \end{bmatrix}\ , \tag{9}$$

corresponding to decomposition (3) and eigenvectors separation into frequency lower eigenvectors (so called master – superscript m) and frequency higher eigenvectors (so called slave – superscript s). The vectors $\mathbf{q}_{\mathrm{D}}^{(\mathrm{F})}$, corresponding to free disk nodes, can be approximately transformed in the form

$$\mathbf{q}_{\mathrm{D}}^{(\mathrm{F})} = {}^{m}\mathbf{V}_{\mathrm{D}}^{\mathrm{F}}\,\mathbf{x}_{\mathrm{D}}\ , \tag{10}$$

where ${}^{m}\mathbf{V}_{\mathrm{D}}^{\mathrm{F}} \in \mathbf{R}^{n_{\mathrm{D}}^{(\mathrm{F})},m_{\mathrm{D}}}$ is the modal sub-matrix of the disk corresponding to free disk generalized coordinates and frequency lower eigenmodes. Higher frequency modes usually contribute less to the disk deformation and their influence can be neglected.

The motion equations of the fictive system assembled from uncoupled subsystems – the disk and the blades – in the configuration space

$$\mathbf{q} = \left[(\mathbf{q}_{\mathrm{D}}^{(\mathrm{F})})^{\mathrm{T}}\ (\mathbf{q}_{\mathrm{D}}^{(\mathrm{C})})^{\mathrm{T}}\ \mathbf{q}_{\mathrm{B}}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{11}$$

can be formally rewritten as

$$\mathbf{M}\,\ddot{\mathbf{q}}(t) + \omega\,\mathbf{G}\,\dot{\mathbf{q}}(t) + (\mathbf{K}_{\mathrm{s}} + \omega^2\,\mathbf{K}_{\omega} - \omega^2\,\mathbf{K}_{\mathrm{d}})\,\mathbf{q}(t) = \omega^2\,\mathbf{f}\ , \tag{12}$$

where, according to mathematical models (1) and (2), matrices have the block-diagonal form $\mathbf{X} = \mathrm{diag}(\mathbf{X}_{\mathrm{D}}, \mathbf{X}_{\mathrm{B}})$, $\mathbf{X} = \mathbf{M}, \mathbf{G}, \mathbf{K}_{\mathrm{d}}$, and $\mathbf{K}_{\mathrm{s}} = \mathrm{diag}(\mathbf{K}_{\mathrm{s,D}}, \mathbf{K}_{\mathrm{s,B}} + \mathbf{K}_{\mathrm{C,B}})$, $\mathbf{K}_{\omega} = \mathrm{diag}(\mathbf{0}, \mathbf{K}_{\omega,\mathrm{B}})$, and $\mathbf{f} = [\mathbf{f}_{\mathrm{D}}, \mathbf{f}_{\mathrm{B}}]^{\mathrm{T}}$. The vector of generalized coordinates $\mathbf{q}$ in consequence of the couplings (6) and modal transformations (10) can be transformed into new vector $\tilde{\mathbf{q}} = [\mathbf{x}_{\mathrm{D}}^{\mathrm{T}}, \mathbf{q}_{\mathrm{B}}^{\mathrm{T}}]^{\mathrm{T}}$ of the dimension $m = m_{\mathrm{D}} + n_{\mathrm{B}}$. The transformation is given by

$$\begin{bmatrix} \mathbf{q}_{\mathrm{D}}^{(\mathrm{F})} \\ \mathbf{q}_{\mathrm{D}}^{(\mathrm{C})} \\ \mathbf{q}_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} {}^{m}\mathbf{V}_{\mathrm{D}}^{\mathrm{F}} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_{\mathrm{D,B}} \\ \mathbf{0} & \mathbf{E}^{n_{\mathrm{B}}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\mathrm{D}} \\ \mathbf{q}_{\mathrm{B}} \end{bmatrix}\ . \tag{13}$$

The condensed mathematical model of the bladed disk in the configuration space $\tilde{\mathbf{q}}$ takes the form

$$\tilde{\mathbf{M}}\,\ddot{\tilde{\mathbf{q}}}(t) + \omega\,\tilde{\mathbf{G}}\,\dot{\tilde{\mathbf{q}}}(t) + (\tilde{\mathbf{K}}_{\mathrm{s}} + \omega^2\tilde{\mathbf{K}}_{\omega} - \omega^2\,\tilde{\mathbf{K}}_{\mathrm{d}})\,\tilde{\mathbf{q}}(t) = \omega^2\,\tilde{\mathbf{f}}\ , \tag{14}$$

where condensed mass, gyroscopic, static stiffness, centrifugal stiffening and dynamic spin softening matrices are given by $\tilde{\mathbf{X}} = \mathbf{T}^{\mathrm{T}}\,\mathbf{X}\,\mathbf{T}$, $\mathbf{X} = \mathbf{M}, \mathbf{G}, \mathbf{K}_{\mathrm{s}}, \mathbf{K}_{\omega}, \mathbf{K}_{\mathrm{d}}$, and $\tilde{\mathbf{f}} = \mathbf{T}^{\mathrm{T}}\,\mathbf{f}$.

4. Application

The presented method is tested on a simple test example of an imperfect bladed disk [5]. The imperfect bladed disk consists of a disk, which is fixed on inner radius to rigid shaft,

eig. mod.	full model	condensed models (condensation level c)					
		$c = 20$		$c = 80$		$c = 320$	
i	f_i [Hz]	f_i [Hz]	ε [%]	f_i [Hz]	ε [%]	f_i [Hz]	ε [%]
1	0.775	0.775	0.000	0.775	0.000	0.775	0.000
2	0.775	0.775	0.000	0.775	0.000	0.775	0.000
3	0.775	0.775	0.000	0.775	0.000	0.775	0.000
4	0.775	0.775	0.000	0.775	0.000	0.775	0.000
5	0.775	0.775	0.000	0.775	0.000	0.775	0.000
6	0.775	0.775	0.000	0.775	0.000	0.775	0.000
7	0.775	0.775	0.000	0.775	0.000	0.775	0.000
8	0.775	0.775	0.000	0.775	0.000	0.775	0.000
9	89.233	89.343	0.123	89.239	0.007	89.233	0.000
10	93.729	93.823	0.100	93.738	0.010	93.730	0.001
11	110.407	110.444	0.034	110.410	0.003	110.407	0.000
12	120.384	120.440	0.047	120.389	0.004	120.384	0.000
13	123.601	124.084	0.391	123.610	0.007	123.602	0.001
14	137.329	137.616	0.209	137.354	0.018	137.334	0.004
15	139.865	140.099	0.167	139.884	0.014	139.869	0.003
16	140.669	140.707	0.027	140.691	0.016	140.675	0.004
17	140.848	140.931	0.059	140.874	0.018	140.854	0.004
18	141.073	141.089	0.011	141.087	0.010	141.080	0.005
19	141.172	141.184	0.009	141.182	0.007	141.176	0.003
20	141.181	141.191	0.007	141.190	0.006	141.185	0.003
21	141.261	141.341	0.057	141.278	0.012	141.267	0.004
22	141.667	141.907	0.169	141.681	0.010	141.671	0.003
23	148.171	148.402	0.156	148.181	0.007	148.173	0.001
24	148.908	149.098	0.128	148.921	0.009	148.910	0.001

Tab.1: Comparison of rotor eigenfrequencies with different condensation level of disks

and two kinds of blades. Blades of the first type are mounted to disk by a rigid joint. A rigid shroud is placed at the end of the second type of blades. Friction elements are placed between shrouds. Each type of blades is separated into two sets (2×25 blades of the first type, 2×5 blades of the second type), which create bladed disk with two perpendicular axis of symmetry. There are 2×4 friction elements in the both sets of the second type of blades.

Total number of DoF of the fixed disk is $n_\mathrm{D} = 7200$ from that $n_\mathrm{D}^{(\mathrm{F})} = 2880$ and $n_\mathrm{D}^{(\mathrm{C})} = 4320$. The 60 blades and 8 friction elements are described by using $n_\mathrm{B} = 2208$ DoF. Imperfect bladed disk is represented by $n = n_\mathrm{D}^{(\mathrm{F})} + n_\mathrm{B} = 5088$ DoF.

In the table 1 the three condensation levels of non-rotating bladed disk are compared for the 24 lowest eigenfrequencies. In the first column the eigenfrequencies of the non-condensed (referential) model with $n = 5088$ DoF number is placed. The other columns correspond to condensed models with $c = 20$, 80, and 320 DoF number corresponding to the lowest eigenmodes of isolated disk. The total number of DoF of condensed models is 2228, 2288 and 2528 for corresponding level of condensation.

The first eight eigenfrequencies correspond to eigenmodes where only friction elements oscillate in the direction of axis of rotation. A deformation of the disk is involved in the rest of eigenmodes. Each eigenfrequency of condensed models is characterized by relative error ε. Relative errors decrease with decreasing condensation level (DoF number c increases). However, the highest condensation level $c = 20$ is sufficient because the relative

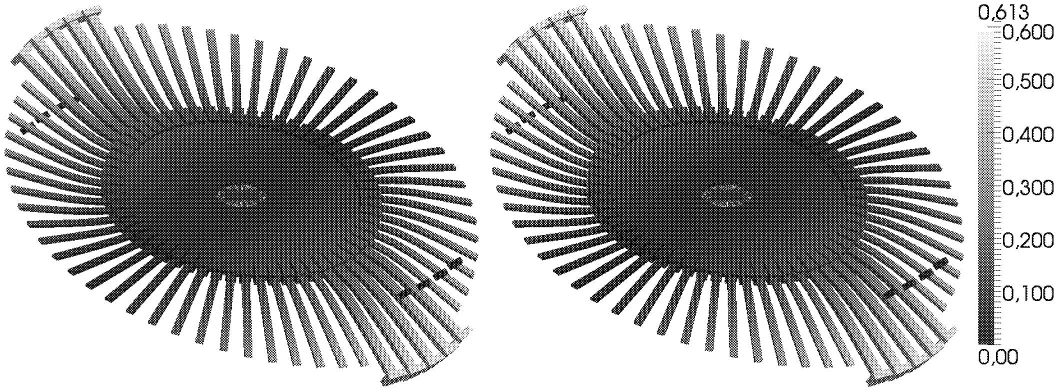

Fig.2: Comparison of the chosen 9th eigenmode of the referential and condensed model of the rotor

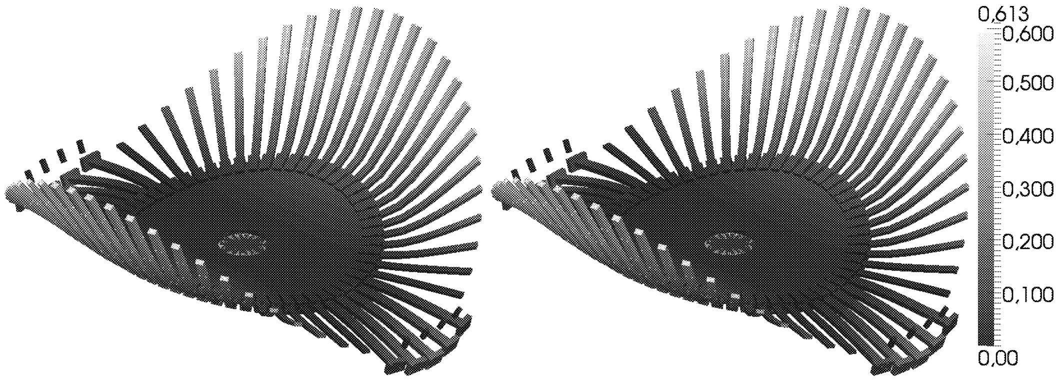

Fig.3: Comparison of the chosen 12th eigenmode of the referential and condensed model of the rotor

errors are below 0.4 %. Two eigenmodes are compared in Figs 2 and 3. The referential eigenmode is on the left side and the corresponding eigenmode of the condensed model with $c = 20$ is on the right side. There is minimal difference between corresponding mode shapes.

The dependency of eigenfrequencies on rotor speed is shown in the diagram (see Fig. 4) for 9th to 17th eigenfrequencies and for rotor speed up to 3000 rpm. Two levels of condensation are compared to the non-condensed bladed disk. The blade centrifugal stiffening effect represented by matrix $\omega^2\,\mathbf{K}_\omega$ influences the eigenfrequencies mostly. The eigenfrequencies are less affected by gyroscopic and dynamic spin softening effects.

5. Conclusion

The paper deals with a modeling of rotating bladed disk vibrations with flexible disks that are ideally fixed to outer shaft surface. The blades are modeled as a one dimensional continuum on the basis of the Bernoulli-Euler theory. The disk is modeled as a three dimensional continuum discretized using isoparametric hexahedral solid finite elements. The presented new analytical numerical approach is based on the modal synthesis method and DoF number reduction corresponding to elastic displacements of the free disk nodes. The displacements of the coupled disk nodes with the blade feet are eliminated by means of the

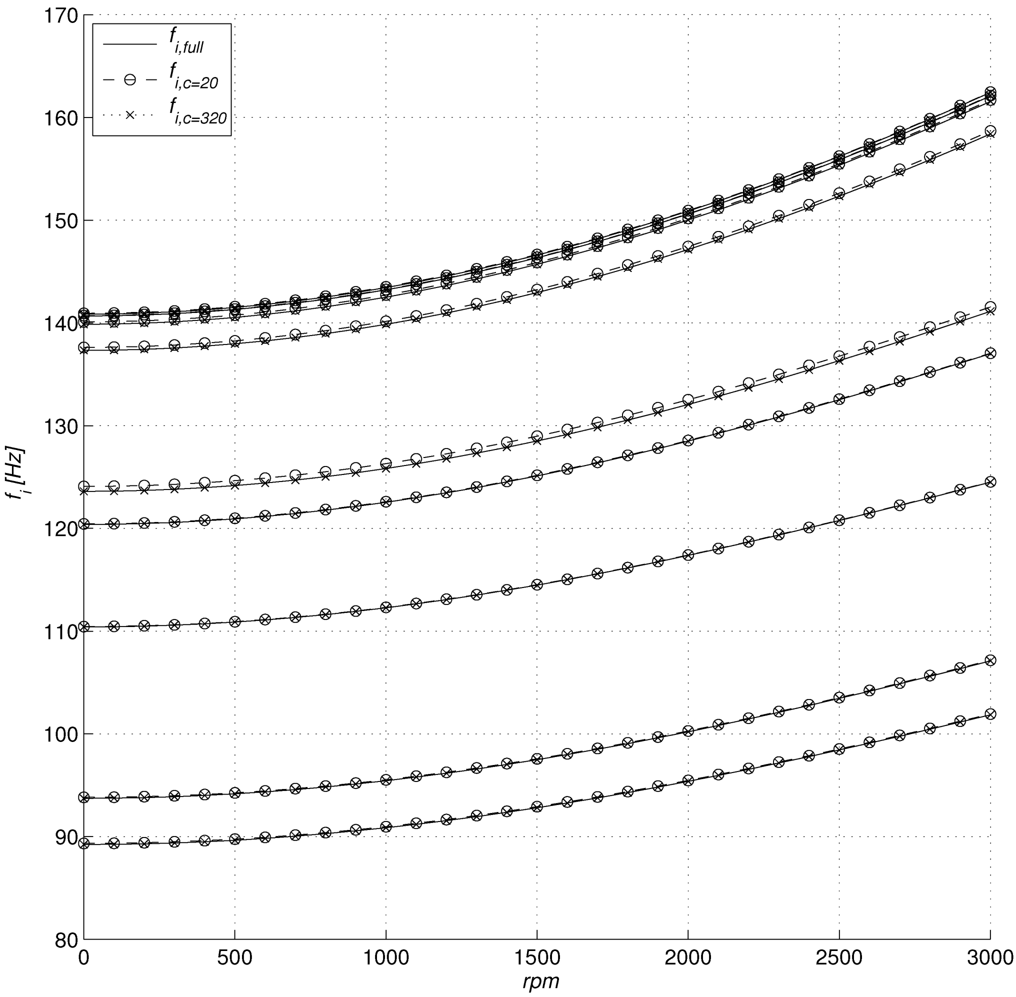

Fig.4: Eigenfrequencies of the imperfect bladed disk in dependence on rotor speed

first blade nodes displacements at blade roots. The method allows to introduce continuous displayed centrifugal and gyroscopic effects. The condensed model of the system can be used effectively for other types of simulations (e.g. steady state harmonic response, dynamical response to excitations of the blades) and estimations of damping and stiffness parameters of friction elements. This approach gives possibilities to use model of a disk with high DoF while preserving lower requirements on CPU and memory. From an assesment of the modal assurance of condensed models follows that the developed software in MATLAB code based on the presented methodology is an effective tool for modeling bladed disk vibrations.

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