NONLINEAR VIBRATION OF INITIALLY STRESSED BEAMS WITH ELASTIC END RESTRAINTS ON TWO-PARAMETER FOUNDATION

M. H. Taha*

An analytical solution for nonlinear vibration of an initially stressed beam with elastic end restraints resting on two-parameter foundation is obtained. The mode functions for linear vibration of a beam with elastic end restraints resting on a linear elastic foundation are obtained first and used to solve the nonlinear vibration equation recalling elliptic integrals. The results obtained from the present solution are compared against those obtained from finite element method and found in close agreement. The effects of elastic supports stiffnesses at the beam ends, foundation stiffness, initial axial load and vibration amplitude on the natural frequency are studied.

Keywords: *nonlinear beam vibration, elliptic integrals, two-parameter foundation, mode functions and natural frequencies*

1. Introduction

Many practical engineering applications are modeled as beams resting on elastic foundations and need criteria to be rationally designed. Few analytical solutions limited to special cases for vibrations of such models can be found in the literature due to the complicated mathematical nature of the problem. Numerical methods such as finite element method [1–2], transfer matrix method [3], differential quadrature element method (DQEM) [4–6], perturbation techniques [7–8] are used to obtain the vibration behavior of different types of linear or nonlinear beams resting on linear or nonlinear foundations.

Semi-analytical methods such as series solutions are suggested to obtain analytic expressions for natural frequencies and mode shapes of nonuniform beams resting on elastic foundation [9, 10]. Taha M.H. and Abohadima S. [11] studied the vibration of nonuniform shear beam resting on elastic foundation.

In the present work, the mode functions of linear vibration of axially loaded beam with elastic end supports resting on two- parameter foundation are obtained and employed to solve the nonlinear vibration equation using elliptic integrals. To verify the present solution,

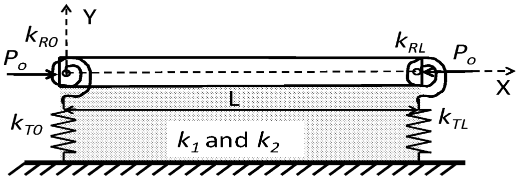


Fig.1: Initially stressed beam with elastic end restraints on two-parameter foundation

* Associate Prof. M. H. Taha, Dept. of Eng. Math. and Physics, Faculty of Engineering, Cairo University, Giza

the obtained results are compared against those obtained from FEM and found in close agreement. The effects of different parameters related to beam and foundation on the natural frequency are studied.

2. Analysis

2.1. Nonlinear vibration equation

The nonlinear vibration equation of an initially stressed beam by an axial force P_0 and resting on two-parameter elastic foundation shown in Fig. 1, is given as:

$$E\,I\,\frac{\partial^4 Y}{\partial X^4}+P_0\,\frac{\partial^2 Y}{\partial X^2}+\mu\,\frac{\partial^2 Y}{\partial t^2}-\frac{E\,A}{2\,L}\left\{\int_0^L\left(\frac{\partial Y}{\partial X}\right)^2\mathrm{d}X\right\}\frac{\partial^2 Y}{\partial X^2}+k_1\,Y(X,t)-k_2\,\frac{\partial^2 Y}{\partial X^2}=0\,, \quad (1)$$

where $E\,I$ is the flexural stiffness of the beam, L is the length of the beam, μ is the mass of the beam per unit length, k_1 and k_2 are the linear and shear foundation stiffnesses per unit length of the beam, E is the modulus of elasticity of the beam material, A is the area of the beam cross section, $Y(X,t)$ is the lateral displacement of the beam, X is the coordinate along the beam and t is time.

Using dimensionless parameters $x = X/L$ and $y = Y/L$, eqn. (1) may be rewritten as:

$$\frac{E\,I}{\mu\,L^4}\,\frac{\partial^4 y}{\partial x^4}+\frac{P_0}{\mu\,L^2}\,\frac{\partial^2 y}{\partial x^2}+\frac{\partial^2 y}{\partial t^2}-\frac{E\,A}{2\,\mu\,L^2}\left\{\int_0^1\left(\frac{\partial y}{\partial x}\right)^2\mathrm{d}x\right\}\frac{\partial^2 y}{\partial x^2}+\frac{k_1}{\mu}\,y(x,t)-\frac{k_2}{\mu\,L^2}\,\frac{\partial^2 y}{\partial x^2}=0\,. \quad (2)$$

The solution of nonlinear partial differential eqn. (2) is obtained by employing the linear mode functions and integrating over the domain of the dimensionless spatial variable x to separate the variation with respect to time. However, the solution of the linear version of eqn. (2) depends on the end conditions.

2.2. Boundary conditions

The boundary conditions of elastic restraints control the lateral displacement and rotation at $x = 0$ can be expressed as:

$$k_{\mathrm{T0}}\,y(0,t) = -\frac{E\,I}{L^3}\,\frac{\partial^3 y(0,t)}{\partial x^3}\,, \quad (3a)$$

$$k_{\mathrm{R0}}\,y(0,t) = \frac{E\,I}{L}\,\frac{\partial^2 y(0,t)}{\partial x^2} \quad (3b)$$

and at $x = 1$ are:

$$k_{\mathrm{TL}}\,y(1,t) = \frac{E\,I}{L^3}\,\frac{\partial^3 y(1,t)}{\partial x^3}\,, \quad (3a)$$

$$k_{\mathrm{RL}}\,y(1,t) = -\frac{E\,I}{L}\,\frac{\partial^2 y(1,t)}{\partial x^2}\,, \quad (3b)$$

where k_{T0} and k_{TL} are the stiffnesses of elastic lateral supports at $x = 0, 1$ respectively and k_{R0} and k_{RL} are the stiffnesses of elastic rotational support at $x = 0, 1$ respectively.

2.3. Solution of linear vibration equation

A linear version of eqn. (2) may be assumed as:

$$\frac{E\,I}{\mu\,L^4}\,\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} + \frac{k_1}{\mu}\,y(x,t) = 0\ . \tag{4}$$

Following the separation of variables analogy, the solution of eqn. (4) may be assumed as:

$$y(x,t) = y_0\,\phi(x)\,\psi(t)\ , \tag{5}$$

where y_0 is the dimensionless vibration amplitude (obtained from initial conditions), $\phi(x)$ is the linear mode function and $\psi(t)$ is a function representing the variation of the lateral displacement along the beam with time. Substituting eqn. (5) into eqn. (4), the partial differential eqn. (4) is separated into the following two ordinary differential equations:

$$\frac{\mathrm{d}^4\phi}{\mathrm{d}x^4} - \lambda_\mathrm{f}^4\,\phi(x) = 0\ , \tag{6}$$

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + \omega^2\,\psi(x) = 0\ , \tag{7}$$

where ω is the separation constant (which represents the natural frequency) and λ_f is called the frequency parameter which is given as:

$$\lambda_\mathrm{f}^4 = \frac{\mu\,L^4}{E\,I}\left(\omega^2 - \frac{k_1}{\mu}\right)\ . \tag{8}$$

The general solution of eqn. (6) is given as:

$$\phi(x) = C_1\cos(\lambda_\mathrm{f}\,x) + C_2\sin(\lambda_\mathrm{f}\,x) + C_3\cosh(\lambda_\mathrm{f}\,x) + C_4\sinh(\lambda_\mathrm{f}\,x) \tag{9}$$

and the solution of eqn. (7), assuming at $t = 0$, $\psi = 1$ and $\mathrm{d}\psi/\mathrm{d}t = 0$ is given as:

$$\psi(t) = \cos(\omega\,t)\ . \tag{10}$$

Substitution eqn. (9) into boundary conditions; eqns. (3), yields a system of homogeneous algebraic equations in unknown constants C_i, $i = 1,2,3,4$. However, the condition of nontrivial solution for such a system leads to the frequency equation as:

$$A_{11}\,A_{22} - A_{12}\,A_{21} = 0\ , \tag{11}$$

where:

$$A_{11} = -\xi_1 - \alpha_1\,\xi_2 + \alpha_2\,\xi_4\ , \tag{12a}$$

$$A_{12} = \xi_3 - \alpha_2\,\xi_2 + \alpha_1\,\xi_4\ , \tag{12b}$$

$$A_{21} = \eta_1 + \alpha_1\,\eta_2 + \alpha_2\,\eta_4\ , \tag{12c}$$

$$A_{22} = \eta_3 + \alpha_2\,\eta_2 + \alpha_1\,\eta_4\ , \tag{12d}$$

$$\alpha_1 = \frac{k_{\mathrm{R}0}\,L}{2\,E\,I\,\lambda_\mathrm{f}} - \frac{E\,I\,\lambda_\mathrm{f}^3}{2\,k_{\mathrm{T}0}\,L^3} \qquad \text{and} \qquad \alpha_2 = \frac{k_{\mathrm{R}0}\,L}{2\,E\,I\,\lambda_\mathrm{f}} + \frac{E\,I\,\lambda_\mathrm{f}^3}{2\,k_{\mathrm{T}0}\,L^3}\ , \tag{13}$$

$$\xi_1 = \cos(\lambda_{\mathrm{f}}) + \frac{K_{\mathrm{TL}}}{\lambda_{\mathrm{f}}^3} \sin(\lambda_{\mathrm{f}}) \ , \tag{14a}$$

$$\xi_2 = \sin(\lambda_{\mathrm{f}}) - \frac{K_{\mathrm{TL}}}{\lambda_{\mathrm{f}}^3} \cos(\lambda_{\mathrm{f}}) \ , \tag{14b}$$

$$\xi_3 = \cosh(\lambda_{\mathrm{f}}) - \frac{K_{\mathrm{TL}}}{\lambda_{\mathrm{f}}^3} \sinh(\lambda_{\mathrm{f}}) \ , \tag{14c}$$

$$\xi_4 = \sinh(\lambda_{\mathrm{f}}) - \frac{K_{\mathrm{TL}}}{\lambda_{\mathrm{f}}^3} \cosh(\lambda_{\mathrm{f}}) \ , \tag{14d}$$

$$\eta_1 = \frac{K_{\mathrm{RL}}}{\lambda_{\mathrm{f}}} \cos(\lambda_{\mathrm{f}}) - \sin(\lambda_{\mathrm{f}}) \ , \tag{15a}$$

$$\eta_2 = \frac{K_{\mathrm{RL}}}{\lambda_{\mathrm{f}}} \sin(\lambda_{\mathrm{f}}) + \cos(\lambda_{\mathrm{f}}) \ , \tag{15b}$$

$$\eta_3 = \frac{K_{\mathrm{RL}}}{\lambda_{\mathrm{f}}} \cosh(\lambda_{\mathrm{f}}) + \sinh(\lambda_{\mathrm{f}}) \ , \tag{15c}$$

$$\eta_4 = \frac{K_{\mathrm{RL}}}{\lambda_{\mathrm{f}}} \sinh(\lambda_{\mathrm{f}}) + \cosh(\lambda_{\mathrm{f}}) \ , \tag{15d}$$

$$K_{\mathrm{T0}} = \frac{k_{\mathrm{T0}}\, L^3}{E\, I} \qquad \text{and} \qquad K_{\mathrm{TL}} = \frac{k_{\mathrm{TL}}\, L^3}{E\, I} \ , \tag{16}$$

$$K_{\mathrm{R0}} = \frac{k_{\mathrm{R0}}\, L}{E\, I} \qquad \text{and} \qquad K_{\mathrm{RL}} = \frac{k_{\mathrm{RL}}\, L}{E\, I} \ . \tag{17}$$

Solving the frequency equation (11) using any proper iterative technique one obtains the frequency parameters $\lambda_{\mathrm{f}m}$, $m = 1, 2, \ldots$, hence the natural frequency ω_m can be calculated by means of eqn. (8).

The normalized mode function is obtained assuming $C_1 = 1$ then, the values of the other three constants can be obtained in terms of α_1 and α_2. The m-mode function is obtained as:

$$\begin{aligned} \phi_m(x) = \sin(\lambda_{\mathrm{f}m}\, x) + (\alpha_2\, \alpha_0 - \alpha_1) \cos(\lambda_{\mathrm{f}m}\, x) - \\ - \alpha_0 \sinh(\lambda_{\mathrm{f}m}\, x) + (\alpha_2 - \alpha_1\, \alpha_0) \cosh(\lambda_{\mathrm{f}m}\, x) \ , \end{aligned} \tag{18}$$

where $\alpha_0 = A_{11}/A_{12}$.

Using eqn. (18), then the general solution for eqn. (4) may be expressed as:

$$\begin{aligned} y(x,t) = \sum_{m=1}^{\infty} \Big\{ \sin(\lambda_{\mathrm{f}m}\, x) + (\alpha_2\, \alpha_0 - \alpha_1) \cos(\lambda_{\mathrm{f}m}\, x) - \alpha_0 \sinh(\lambda_{\mathrm{f}m}\, x) + \\ + (\alpha_2 - \alpha_1\, \alpha_0) \cosh(\lambda_{\mathrm{f}m}\, x) \Big\} [C_m \sin(\omega_m\, t) + D_m \cos(\omega_m\, t) \ . \end{aligned} \tag{19}$$

where constants C_m and D_m can be obtained from initial conditions and orthogonality properties of mode functions.

2.4. Solution of nonlinear vibration equation

The linear m-mode function which satisfies the end conditions is substituted in the nonlinear vibration equation (eqn. (20)) to obtain the solution of nonlinear case. The nonlinear vibration is assumed as:

$$y(x,t) = y_0\, \phi_m(\lambda_{\mathrm{f}m}\, x)\, \psi(t) \ , \tag{20}$$

where y_0 is the dimensionless vibration amplitude. Substitution of eqn. (20) into eqn. (2) and integrating over the x-domain leads to:

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}t^2} + \gamma_{1m}\,\psi(t) + \gamma_{2m}\,\psi^3(t) = 0 \;, \tag{21}$$

where:

$$\gamma_{1m} = \omega_m^2 + \frac{P_0}{\mu\,L^2}\,\frac{\int_0^1 \phi''_m\,\mathrm{d}x}{\int_0^1 \phi_m\,\mathrm{d}x} - \frac{k_2}{\mu\,L^2}\,\frac{\int_0^1 \phi''_m\,\mathrm{d}x}{\int_0^1 \phi_m\,\mathrm{d}x} \;, \tag{22a}$$

$$\gamma_{2m} = -\frac{E\,A\,y_0^2}{2\,\mu\,L^2}\left\{\int_0^1 (\phi'_m)^2\,\mathrm{d}x\right\}\frac{\int_0^1 \phi''_m\,\mathrm{d}x}{\int_0^1 \phi_m\,\mathrm{d}x} \;, \tag{22b}$$

where ω_m is the natural frequency of the linear m-mode. Integration eqn. (21) once with respect to time, with the initial conditions at $t = 0$, $\psi = 1$ and $\mathrm{d}\psi/\mathrm{d}t = 0$, one obtains:

$$\left(\frac{\mathrm{d}\psi}{\mathrm{d}t}\right)^2 = -\gamma_{1m}\,\psi^2 - \frac{1}{2}\,\gamma_{2m}\,\psi^4(t) + c \;, \tag{23}$$

where c is the integration constant. Obtaining c and regrouping coefficients of terms in the right hand side of eqn. (23), one obtains:

$$\left(\frac{\mathrm{d}\psi}{\mathrm{d}t}\right)^2 = \varrho_m^2\,(1-\psi^2)\,(k_m^2\,\psi^2 - k_m^2 + 1) \;, \tag{24}$$

where:

$$\varrho_m^2 = \gamma_{1m} + \gamma_{2m} \qquad \text{and} \qquad k_m^2 = \frac{\gamma_{2m}}{2\,\varrho_m^2} \;.$$

Substituting $\psi(t) = \cos(\varphi)$, where $\varphi = \varphi(t)$ into equation (23b), one gets:

$$t\,\varrho_m = \int_0^{\varphi} \frac{\mathrm{d}\varphi}{\sqrt{1 - k_m^2\,\sin^2(\varphi)}} \;. \tag{25}$$

The integration in eqn. (25) is the elliptic integral of the first kind, the inversion of which yields the Jacobi elliptic function $\mathrm{cn}[\varrho_m\,t, k_m]$.

Then, the variation in the lateral displacement of the beam at any location with time can be expressed as:

$$\psi_m(t) = \mathrm{cn}[\varrho_m\,t, k_m] \;. \tag{26}$$

The period of the Jacobi elliptic function is defined by the complete elliptic integral:

$$T_m = \frac{4}{\varrho_m}\int_0^{\pi/2} \frac{\mathrm{d}\varphi}{\sqrt{1 - k_m^2\,\sin^2(\varphi)}} \;. \tag{27}$$

Then, the natural frequency for m-mode in nonlinear vibration is:

$$\Omega_m = \frac{2\pi}{T_m} \ . \tag{28}$$

3. Numerical results

To verify the obtained solutions, values of the frequency parameter calculated using the present solution are graphed against those obtained from FEM [1] for different values of load ratio γ in Fig. 2. It is obvious that the obtained results are in close agreement with FEM results. The load ratio γ is defined as:

$$\gamma = \frac{P_0}{P_\mathrm{c}} \quad \text{and} \quad (0 \leq \gamma \leq 1) \ , \tag{29}$$

where P_c is the critical (buckling) load.

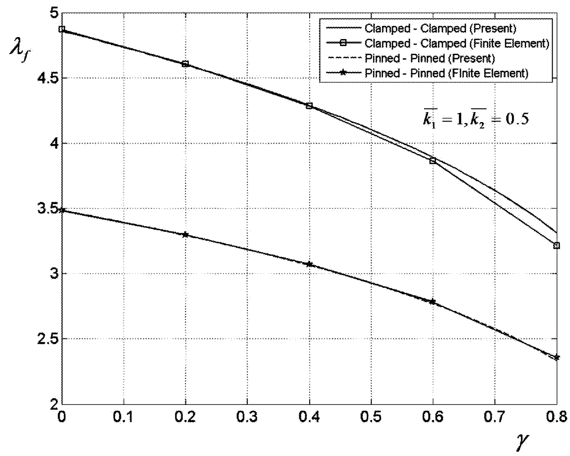


Fig.2: Present results against FEM results

The derived expressions are used to investigate the influence of different parameters on the natural frequency of the beam-foundation system shown in Fig. 1. However, the natural frequency of such a system increases as the overall stiffness of the system increases. The overall stiffness of the beam-foundation system is the resultant of the flexural stiffness of the beam, the stiffness of the foundations and the stiffness of elastic supports at ends. In deformed configuration, the lateral component of the axial load P_0 in case of compression is in the opposite direction of the overall system stiffness lateral restoring force, while in the same direction in the case of axial tension load. In other words, the axial load decreases the overall stiffness of the beam-foundation system in case of compression and increases it in case of tension. In the light of these facts, the behavior of the beam-foundation system can be qualitatively expected.

The m-frequency parameter $\lambda_{\mathrm{f}m}$ for nonlinear case is defined as:

$$\lambda_{\mathrm{f}m}^4 = \frac{\mu\, L^4\, \Omega_m^2}{E\, I} \ , \tag{30}$$

where Ω_m is the natural frequency of nonlinear vibration of the system calculated using eqn. (28) and $m = 1, 2, \ldots$ is the mode number. The fundamental frequency parameter

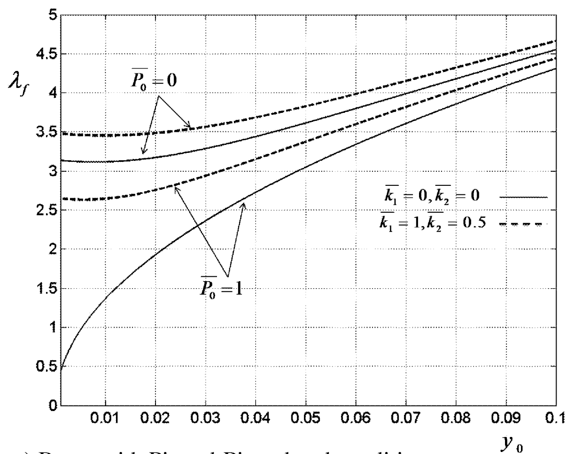
($m = 1$) is defined as λ_f. The dimensionless load parameter and foundation stiffness parameters are introduced as:

$$\bar{P}_0 = \frac{P_0\,L^2}{\pi^2\,E\,I}\ , \tag{31}$$

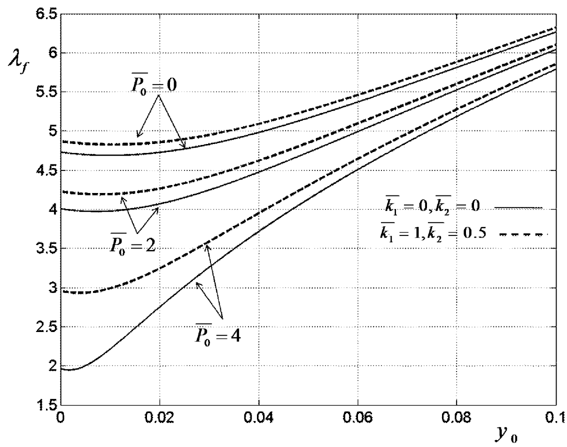
$$\bar{k}_1 = \frac{k_1\,L^4}{E\,I} \qquad \text{and} \qquad \bar{k}_2 = \frac{k_2\,L^2}{E\,I}\ . \tag{32}$$

The usage of dimensionless parameters defined in eqn. (30), eqn. (31) and eqn. (32) with respect to beam length and flexural stiffness separates the effects of geometric properties of the beam on the frequency parameter.

The effect of lateral vibration amplitude on the fundamental frequency parameter λ_f (simply called as the frequency parameter hereinafter) of the beam-foundation system for different values of load parameter $\bar{P}_0$ and foundation parameters ($\bar{k}_1$ and $\bar{k}_2$) is shown in Fig. 3. Fig. 3a represents the case of a beam with Pinned-Pinned (P-P) end conditions and Fig. 3b represents the case of a beam with Clamped-Clamped (C-C) end conditions.



a) Beam with Pinned-Pinned end conditions



b) Beam with Clamped-Clamped end conditions

Fig.3: Influence of vibration amplitude y_0 and load parameter $\bar{P}_0$ on frequency parameter λ_f

Actually, the lateral vibration of the beam causes stretching in the beam length which produce axial tension load. As the amplitude of the lateral vibration increases, the tension axial load resulting from stretching increases, leading to increase in the frequency parameter.

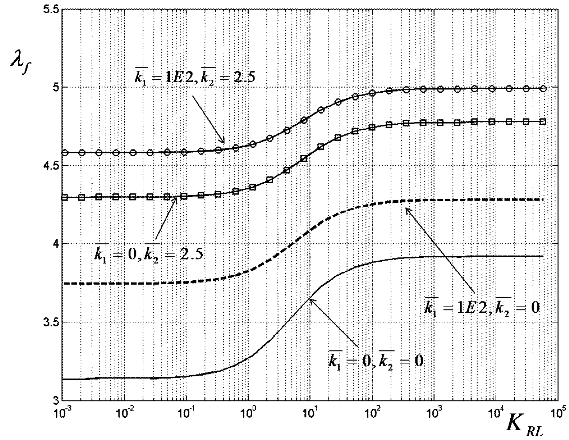


Fig.4: Influence of support rotational stiffness K_{RL} and foundation parameters ($\bar{k}_1$ and $\bar{k}_2$) on frequency parameter λ_{f} ($K_{\mathrm{T0}} = K_{\mathrm{TL}} = K_{\mathrm{R0}} = 1E5$, $\bar{P}_0 = 1$ and $y_0 = 0.001$)

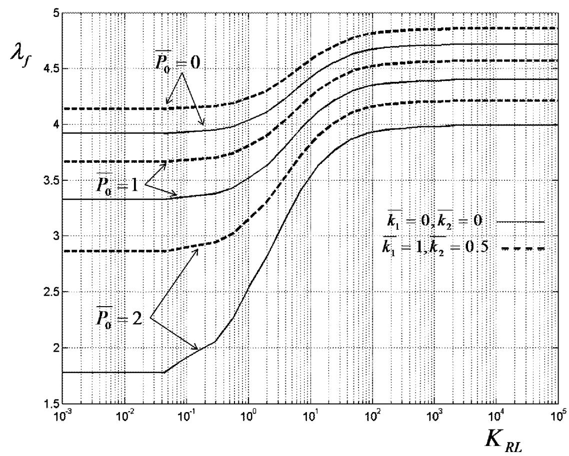


Fig.5: Influence of support rotational stiffness K_{RL} and load parameter $\bar{P}_0$ on frequency parameter λ_{f} ($K_{\mathrm{T0}} = K_{\mathrm{TL}} = K_{\mathrm{R0}} = 1E5$ and $y_0 = 0.001$)

Figure 4 shows the influence of elastic rotational stiffness parameter K_{RL} at one end on frequency parameter λ_{f} for different values of foundation parameters ($\bar{k}_1$ and $\bar{k}_2$) for beam loaded by axial compression load $P_0 = E\,I\,(\pi^2/L^2)$. It is clear that the frequency parameter increases as foundation parameters increase and as the rotational stiffness increases. The effect of foundation stiffness parameters is more significant for small values of rotational stiffness than for large values of rotational stiffness.

In Fig. 5, the effect of rotational stiffness parameter at one end on the frequency parameter is depicted for different values of load parameter and foundation stiffnesses parameters. The frequency parameter increases as rotational stiffness increases, the foundation parameters increase and as the load parameter decreases. The effects of both the foundation parameters and the load parameter are more noticeable for flexible rotational restrains.

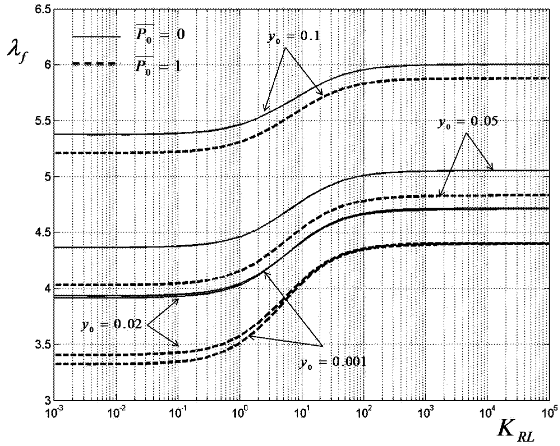


Fig.6: Influence of support rotational stiffness K_{RL} and vibration amplitude y_0 on frequency parameter λ_{f}. ($K_{\mathrm{T0}} = K_{\mathrm{TL}} = K_{\mathrm{R0}} = 1E5$ and $\bar{k}_1 = \bar{k}_2 = 0$)

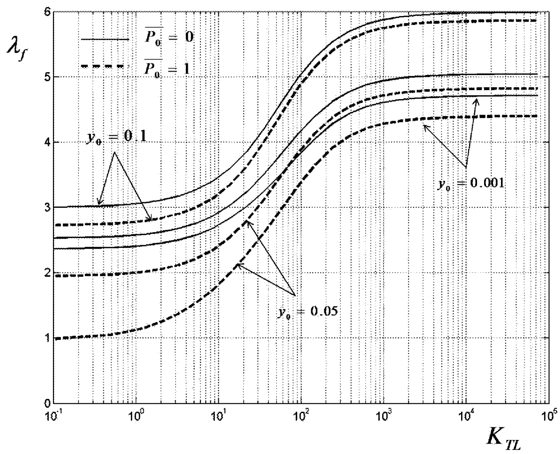


Fig.7: Influence of support lateral stiffness K_{TL} and vibration amplitude y_0 on frequency parameter λ_{f} ($K_{\mathrm{T0}} = K_{\mathrm{TL}} = K_{\mathrm{R0}} = 1E10$ and $\bar{k}_1 = \bar{k}_2 = 0$)

In Fig. 6, the effect of elastic rotational stiffness parameter at one end K_{RL} on the frequency parameter is shown for different values of load parameter and vibration amplitude. However, the other three elastic end restraints are assumed very rigid. It is obvious that the frequency parameter increases as the lateral vibration amplitude increases and as the rotational stiffness increases. Figure 7 is similar to Fig. 6, but for lateral elastic restraint parameter K_{TL}. It is concluded that the increase in the restraint stiffness increases the overall stiffness of the system and leads to increase in the frequency parameter. Also, Fig. 8 is another version of Fig. 4, but the rotational stiffness parameter K_{RL} in Fig. 4 is replaced by the lateral displacement stiffness parameter K_{TL} and $\bar{P}_0 = 0$ in Fig. 8.

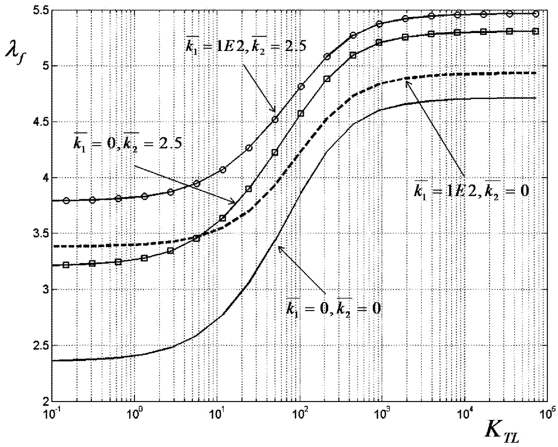


Fig.8: Influence of support lateral stiffness K_{TL} and foundation parameters ($\bar{k}_1$ and $\bar{k}_2$) on frequency parameter λ_{f} ($K_{\mathrm{T0}} = K_{\mathrm{TL}} = K_{\mathrm{R0}} = 1E10$, $\bar{P}_0 = 0$ and $y_0 = 0.001$)

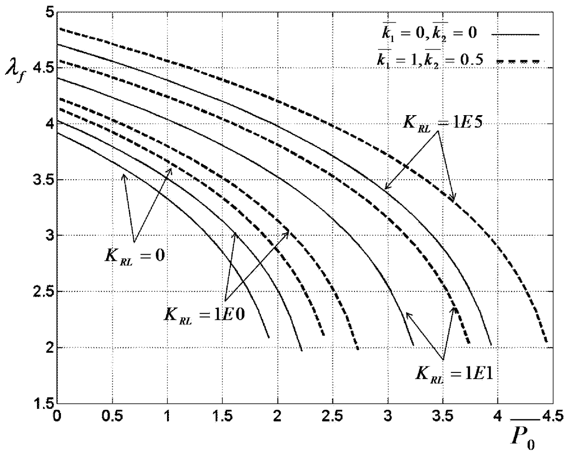


Fig.9: Influence of load parameter $\bar{P}_0$ and support rotation stiffness K_{RL} on frequency parameter λ_{f} ($K_{\mathrm{T0}} = K_{\mathrm{TL}} = K_{\mathrm{R0}} = 1E5$ and $y_0 = 0.001$)

The influences of load parameter on the frequency parameter are presented in Figures 9–11 for different values of elastic restraints stiffness at one end and foundation stiffness parameters. It is obvious in all cases that as the load parameter increases, the frequency parameter decreases. However, as the value of the compression axial load approaches a critical load P_{cr} for certain configuration, the beam-foundation system reaches aperiodic case and approaches equilibrium position asymptotically. The graphs predict both the frequency parameter and the critical load for wide range of beam-foundation system characteristics.

4. Conclusions

Analytical solution for the nonlinear vibration of an initially stressed beam with elastic end restraints resting on a two-parameter elastic foundation is obtained.

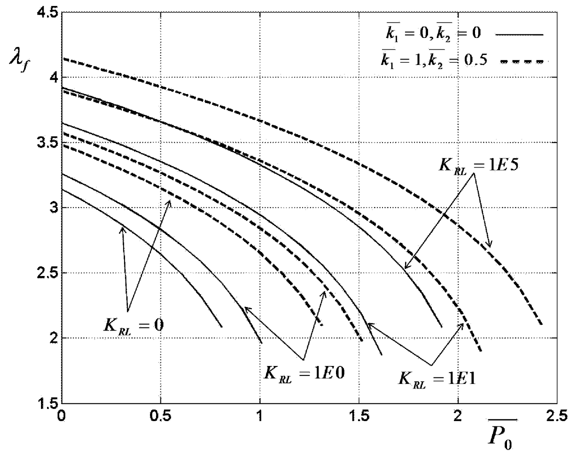


Fig.10: Influence of load parameter $\bar{P}_0$ and support rotation K_{RL} on frequency parameter λ_{f} ($K_{\mathrm{T0}} = K_{\mathrm{TL}} = 1E5$, $K_{\mathrm{R0}} = 0$ and $y_0 = 0.001$)

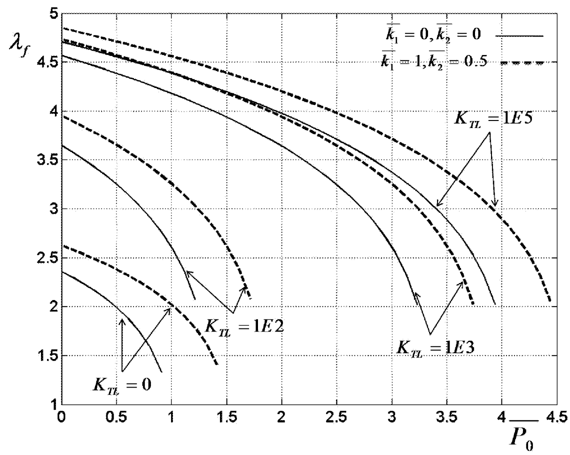


Fig.11: Influence of load parameter $\bar{P}_0$ and support lateral stiffness K_{TL} on frequency parameter λ_{f} ($K_{\mathrm{T0}} = K_{\mathrm{TL}} = K_{\mathrm{R0}} = 1E5$ and $y_0 = 0.001$)

The influences of both lateral translational and rotational elastic restraints at ends are studied in the present analyses. It is found that the natural frequency of the beam-foundation system increases as the overall stiffness of the system increases. The overall stiffness of the beam-foundation system increases with the increase of the beam flexural stiffness, the end restraints stiffness and the foundation stiffness. In case of compression axial load, as the axial load increases, the natural frequency of the system decreases. Furthermore, as the axial load approaches a critical (buckling) value, the system reaches aperiodic conditions and approaches the equilibrium deformed configuration asymptotically. Also, the natural frequency increases as the vibration amplitude increases due to stretching resulting in the beam length, which induces axial tension load. All effects are more significant in flexible configurations than for stiff configurations of the beam-foundation system.

References

[1] Naidu N.R., Rao G.V.: Vibrations of initially stressed uniform beams on a two-parameter elastic foundation, Computers & Structures, 57(5), 941–943, 1995
[2] Thambiratnam D., Zhuge Y.: Free vibration analysis of beams on elastic foundations, Comp. & Str., 60 (6): 971–980, 1996
[3] Ayvaz Y., Ozgan K.: Application of modified Vlasov model to free vibration analysis of beams resting on elastic foundations, Sound Vib.; 255(1): 111–127, 2002
[4] Chen C.N.: Vibration of prismatic beam on an elastic foundation by the differential quadrature element method, Comp. and str.; 77: 1–9, 2000
[5] Chen C.N.: DQEM Vibration analysis of non-prismatic shear deformable beams resting on elastic foundations, Sound Vib.; 255 (5): 989–999, 2002
[6] Chen W.Q., Lu C.F., Bain Z.G.: A mixed method for bending and free vibration of beams resting on Pasternak elastic foundations, Appl. Math. Modeling; 28: 877–890, 2004
[7] Nayfeh A.H., Nayfeh S.A.: On nonlinear modes of continuous systems, Vib. Acous.; ASME; 116: 129–136, 1994
[8] Maccari A.: The asymptotic perturbation method for nonlinear continuous systems, Nonlinear dyn. ; 19: 1–18, 1999
[9] Ruta P.: The vibration of a non-prismatic beam on an inertial elastic half-plane, Sound Vib; 275: 533–556, 2004
[10] Wang J.T.S, Lin C.C.: Dynamic analysis of generally supported beams using Fourier series, Sound and Vib., Vol. 196(3), 285–293, 1996
[11] Taha M.H., Abohadima S.: Mathematical model for vibrations of nonuniform flexural beams, Eng. Mech., 15 (1): 3–11, 2008

Received in editor's office: April 17, 2012
Approved for publishing: August 9, 2012