# REDUCING EXCESSIVE VIBRATION OF RIGID ROTORS MOUNTED WITH HYDRODYNAMIC BEARINGS BY CONTROLLED EXCITATION OF THE ROTOR SUPPORTS

Petr Ferfecki*, Jaroslav Zapoměl**

*The flexible supports make it possible to decrease the forces transmitted between the working machines and their foundations. At rotating machinery, the hydrodynamic bearings exhibiting compliance and considerable damping together with high load capacity are often used for this purpose. Nevertheless, on certain operating conditions, the hydraulic forces produced in the thin oil film can destabilize the rotor equilibrium position, which consequently leads to development of the rotor self-excited vibration of large amplitude. There are several ways how to increase the critical speed of the rotor rotation at which the self-excited vibration starts. Recently, the attention was focused on increasing damping in the rotor supports or on modification of the bearings geometry. At present time there are available many active control devices working on various physical principles. In this paper the research of the concept based on controlled kinematic excitation of the bearings shells is carried out. The performed computer simulations prove that the investigated technique enables to reduce amplitude of the rotor vibration as a response on their unbalance excitation, to suppress amplitude of the self-excited vibration and to increase the rotor angular speed of rotation, at which the self-excited vibration begins. Suppression of the rotor oscillation is always connected with increasing the force by which the turning rotor acts on its stationary part. The developed computational procedures and results of the performed analyses are intended for finding properties and behaviour of the active control devices for mitigation of excessive vibration of rotating machines and to contribute to their proper design in this way.*

Keywords: *rigid rotor, hydrodynamic bearings, active vibration control, suppression of excessive vibration*

## 1. Introduction

The hydrodynamic bearings exhibit high loading capacity and therefore they are often used as coupling elements placed between the rotor and its stationary part. They work reliably for a large extent of operating speeds, reduce lateral vibration of rotors and because of their flexibility they may decrease the forces transmitted between the rotor and its housing in some speed intervals. But after exceeding a critical value of the rotor angular velocity the hydraulic forces, by which the oil film acts on the shaft journal, start to destabilize the rotor equilibrium position and to induce the self-excited vibration of large amplitude.

The lateral vibrations of rotors can be significantly attenuated if their supports are equipped with efficient damping elements. The essence of the vibration reduction is to dis-

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sipate mechanical energy that gives rise to the oscillations. Among the available methods there are modification of the shape of the bearing gap, increasing the system damping (passive damping devices), or counteracting the external or unbalance excitation directly (active damping devices). The passive damping devices are simple, they do not require expensive and complicated feedback control systems, they work on the principle of dissipation of mechanical energy and therefore they do not destabilize vibration of the rotor. Nevertheless in many cases, the use of passive damping devices or the optimization of the design parameters of the rotor supports is not sufficient and then the active damping elements are needed. The main advantage of the means of active control, versus passive ones, is the versatility in adapting to several load conditions and configurations of the machine.

At present time there are available many active control devices such as piezoelectric bearing pushers, hydraulic actuators, deformable bushes, active bearings with flexible sleeves and others. They work on different physical principles: pneumatic, hydraulic, electric-hydraulic, or electric-mechanic. Wu and Pfeiffer [1] introduced a kind of active journal bearing whose stiffness is controlled by the change of the oil influx into the bearing gap. Krodkiewski and Sun [2] presented a journal bearing having the linen formed by a flexible sleeve whose shape is adapted to the current operating conditions by means of control of the liquid pressure in the pressure chambers under the sleeve. Modelling of a three-pad active bearing (bearing with three flexible sleeves) can be found in [3]. To control the oil pressure in the pressure chambers a PD controller is applied.

A wide class of active damping rotor supports employs piezoelectric elements. Bonneau et al. [4] studied an adaptive bearing consisting of a mobile housing mounted with four piezoelectric actuators. Another concept of application of piezoelectric elements in a journal bearing system was reported by Przybylowicz [5], [6]. The active part is a piezoelectric ring placed between the bearing shell and the bearing housing. Under application of the electric field the piezoelectric ring changes its radial dimension and influences the size of the bearing clearance. The change of the radial dimension of the bearing shell due to radial contraction or expansion of the piezoelectric ring is coupled with the speed of the rotor rotation. Carmignani et al. [7] performed a theoretical and experimental research of another kind of active hydrodynamic bearing. Two piezoelectric actuators, set perpendicularly to each other, act on the bearing housing in the rotor testing device. Since the actuators respond only in expansion, a reaction spring was set against each of them. For their activating a feedback control was applied.

In this paper the attention is focused on the research of the strategy minimizing vibration of a symmetric rigid rotor supported by two hydrodynamic bearings. The rotor rotates at a constant angular speed and is excited by its imbalance. Positions of the bearing housings are adjusted in the horizontal and vertical directions by actuators activated by a PD controller. The main objectives are determination of the stability regions of the forced vibration produced by the rotor imbalance and specification of the speed ranges, in which the rotor could be operated without occurring a self-excited vibration. In addition the relation between suppression of the rotor vibration and magnitude of the force transmitted into the rotor casing is studied. Such analysis has not been published yet, so that this represents a new contribution to investigation of behaviour of rotors damped by active damping devices and brings new material for their proper design.

## 2. The investigated rotor system

Rotor of the investigated rotor system (Fig. 1) consists of a central cylindrical part and of two journals. Its coupling with the stationary part is accomplished by two identical hydrodynamic bearings. The rotor is loaded by its weight, it turns at a constant angular velocity and is excited by its unbalance. If the speed of its rotation exceeds a critical value, a subsynchronous self-excited vibration of large amplitude produced by the hydraulic forces in the hydrodynamic bearings (oil whirl, oil whip) takes place. The shells of both hydrodynamic bearings are movable (Fig. 2). Their sliding motion in the direction perpendicular to the rotor axis produces the forces acting on the rotor in the direction opposite to the rotor displacements which results into attenuation of its vibration. The movement of the shells is controlled by a PD feedback controller according to the current position and velocity of the journals relative to the frame of the rotor system.

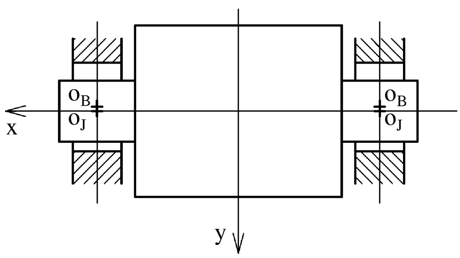


*Fig.1: Scheme of the rigid rotor system supported by fluid-film bearings*

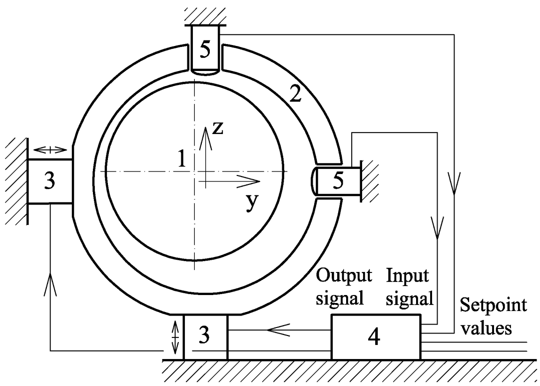


*Fig.2: Scheme of the investigated system (1 – rotor journal, 2 – bearing shell, 3 – actuator, 4 – controller, 5 – sensor)*

The task was to analyse the control strategy especially from the points of view of suppression of the amplitude of the rotor vibration, the critical angular velocity, at which the self-excited vibration starts, and magnitude of the force that is transmitted through the coupling elements between the rotor and its stationary part.

The main technological parameters of the analysed system are: mass of the rotor 0.780 kg, diameter of the rotor journals 29.94 mm, the length of the bearings 15 mm, the bearings clearance 40.5 $\mu$m, and the oil dynamical viscosity 0.004 Pa s. Eccentricity of the rotor centre of gravity is 16.5 $\mu$m.

## 3. The computational model of the investigated rotor system

In the computational model the rotor is considered to be absolutely rigid and symmetric relative to the plane perpendicular to its axis in the middle of the distance between the bearings. The rotor centre of gravity is assumed to be situated in the plane of symmetry slightly shifted from the axis of the rotor rotation. The hydrodynamic bearings are implemented into the computational model by means of force couplings. Determination of the hydraulic forces acting on the rotor journal the classical theory of lubrication based on application of the Reynolds' equation is used. The components of the bearing forces are then a function of differences of displacements and velocities between the rotor and the bearing shells in the $y$ (horizontal) and $z$ (vertical) directions. As the length to diameter ratio of the hydrodynamic bearings is small (0.5), they are implemented into the computational model as short ones. The positions of the bearing shells are controlled with the aim to hold the centres of the rotor journals unmovable at the required location.

Because of the symmetry, the rotor performs a planar motion and displacements of the rotor centre and of the centres of the rotor journals in both bearings are the same. The rotor has two degrees of freedom from the point of view of the lateral vibration and therefore its position is defined by two parameters, displacements of the rotor centre in the horizontal and vertical directions. Then the equations of motion take the form

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{y}_\mathrm{J} \\ \ddot{z}_\mathrm{J} \end{bmatrix} = 2 \begin{bmatrix} f_\mathrm{By}(y_\mathrm{J}-y_\mathrm{B}, z_\mathrm{J}-z_\mathrm{B}, \dot{y}_\mathrm{J}-\dot{y}_\mathrm{B}, \dot{z}_\mathrm{J}-\dot{z}_\mathrm{B}) \\ f_\mathrm{Bz}(y_\mathrm{J}-y_\mathrm{B}, z_\mathrm{J}-z_\mathrm{B}, \dot{y}_\mathrm{J}-\dot{y}_\mathrm{B}, \dot{z}_\mathrm{J}-\dot{z}_\mathrm{B}) \end{bmatrix} + \\ + \begin{bmatrix} 0 \\ -m\,g \end{bmatrix} + \begin{bmatrix} m\,e_\mathrm{T}\,\omega^2\cos(\omega\,t) \\ m\,e_\mathrm{T}\,\omega^2\sin(\omega\,t) \end{bmatrix} , \tag{1}$$

which can be also expressed in a matrix way

$$\mathbf{M}\,\ddot{\mathbf{q}}_\mathrm{J} = 2\,\mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{J}-\mathbf{q}_\mathrm{B}, \dot{\mathbf{q}}_\mathrm{J}-\dot{\mathbf{q}}_\mathrm{B}) + \mathbf{f}_\mathrm{G} + \mathbf{f}_\mathrm{U}(t) \ . \tag{2}$$

$m$ is the rotor mass, $f_\mathrm{By}$ and $f_\mathrm{Bz}$ are the horizontal and vertical components of the bearing force, $y_\mathrm{J}$, $z_\mathrm{J}$, $y_\mathrm{B}$, $z_\mathrm{B}$ denote displacements of the rotor journals and bearing shells centres in the $y$ and $z$ directions respectively, $e_\mathrm{T}$ is eccentricity of the rotor centre of gravity, $\omega$ is angular velocity of the rotor rotation, $t$ is the time, $g$ is the gravity acceleration and dots $(\dot{\ })$, $(\ddot{\ })$ denote the first and second derivatives with respect to time. In equation (2), $\mathbf{M}$ denotes the mass matrix of the rotor, $\mathbf{f}_\mathrm{B}$, $\mathbf{f}_\mathrm{G}$ and $\mathbf{f}_\mathrm{U}$ are the vectors of bearing forces, constant loading caused by the rotor weight and unbalance forces respectively and $\mathbf{q}_\mathrm{J}$, $\mathbf{q}_\mathrm{B}$ are the displacement vectors of the rotor and the bearing shells centres.

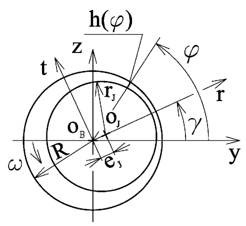


*Fig.3: Scheme of a journal bearing*

Fig. 3 shows the scheme of a cylindrical hydrodynamic bearing. $y$, $z$ and $r$, $t$ are axes of the stationary and rotor-fixed coordinate systems and $O_\mathrm{B}$ and $O_\mathrm{J}$ denote the centers of the bearing shell and of the shaft journal respectively. The thickness of the oil film $h$ at any location of the bearing circumference is defined by the following relation derived e.g. in [8]

$$h(\varphi) = c - e_\mathrm{J}\,\cos(\varphi-\gamma) \ , \qquad c = R - r_\mathrm{J} \tag{3}$$

where $c$ is the radial clearance between the bearing shell and the rotor journal, $e_\mathrm{J}$ is the eccentricity of the rotor journal, $R$ is the radius of the hole in the bearing shell, $r_\mathrm{J}$ is the radius of the shaft journal, $\gamma$ is the position angle of the line of centres and $\varphi$ is the circumferential coordinate (Fig. 3).

A simple geometric analysis gives the relations for the rotor journal eccentricity

$$e_\mathrm{J} = \sqrt{(z_\mathrm{J} - z_\mathrm{B})^2 + (y_\mathrm{J} - y_\mathrm{B})^2} \tag{4}$$

and angle $\gamma$ is obtained by solving a set of goniometric equations

$$\cos(\gamma) = \frac{y_\mathrm{J} - y_\mathrm{B}}{e_\mathrm{J}} , \qquad \sin(\gamma) = \frac{z_\mathrm{J} - z_\mathrm{B}}{e_\mathrm{J}} . \tag{5}$$

The radial and tangential components of the hydraulic bearing forces ($f_\mathrm{Br}$, $f_\mathrm{Bt}$ respectively) are obtained by integration of the pressure distribution around the circumference and along the length of the bearing

$$f_\mathrm{Br} = -R \int_0^L \int_\gamma^{\gamma+2\pi} p_\mathrm{d} \cos\varphi \, \mathrm{d}\varphi \, \mathrm{d}x , \qquad f_\mathrm{Bt} = -R \int_0^L \int_\gamma^{\gamma+2\pi} p_\mathrm{d} \sin\varphi \, \mathrm{d}\varphi \, \mathrm{d}x . \tag{6}$$

$L$ is the length of the bearing, $p_\mathrm{d}$ is the pressure distribution in the layer of lubricant and $x$ is the axial coordinate.

The pressure distribution in the oil film is determined by solving the Reynolds' equation [8]. In cavitated areas pressure of the medium is assumed to be constant and equal to the pressure in the ambient space. For the case of short and $\pi$-film cavitated bearings (the cavitation occurs in the area where the oil film thickness rises with increasing value of the circumferential coordinate $\varphi$) the horizontal and vertical components of the bearing forces ($f_\mathrm{By}$, $f_\mathrm{Bz}$ respectively) can be obtained in a closed form

$$f_\mathrm{By} = f_\mathrm{Br} \cos(\gamma) - f_\mathrm{Bt} \sin(\gamma) , \qquad f_\mathrm{Bz} = f_\mathrm{Br} \sin(\gamma) + f_\mathrm{Bt} \cos(\gamma) \tag{7}$$

where $f_\mathrm{Br}$ and $f_\mathrm{Bt}$ are the radial and tangential components of the hydraulic force [8]

$$f_\mathrm{Br} = \eta \, R \, L \left(\frac{L}{c}\right)^2 \left[(\omega - 2\,\dot\gamma) \frac{\varepsilon_\mathrm{J}^2}{(1-\varepsilon_\mathrm{J}^2)^2} + \frac{\pi\,(1 + 2\,\varepsilon_\mathrm{J}^2)\,\dot\varepsilon_\mathrm{J}}{2\,(1-\varepsilon_\mathrm{J}^2)^{5/2}}\right] , \tag{8}$$

$$f_\mathrm{Bt} = -\eta \, R \, L \left(\frac{L}{c}\right)^2 \left[(\omega - 2\,\dot\gamma) \frac{\pi\,\varepsilon_\mathrm{J}}{4\,(1-\varepsilon_\mathrm{J}^2)^{3/2}} + \frac{2\,\varepsilon_\mathrm{J}\,\dot\varepsilon_\mathrm{J}}{(1-\varepsilon_\mathrm{J}^2)^2}\right] . \tag{9}$$

$\eta$ is dynamical viscosity of the lubricating oil and $\varepsilon_\mathrm{J}$ is the relative journal eccentricity

$$\varepsilon_\mathrm{J} = \frac{e_\mathrm{J}}{c} . \tag{10}$$

The bearing shells can move independently in the horizontal and vertical directions and their movement is controlled by a PD feedback controller. The goal of the control is to reach zero displacements of the rotor journal centres relative to the rotor stationary part in both bearings. Then it holds

$$\mathbf{q}_\mathrm{B} = -k_\mathrm{p}\,\mathbf{q}_\mathrm{J} - k_\mathrm{d}\,\dot{\mathbf{q}}_\mathrm{J} , \tag{11}$$

$$\dot{\mathbf{q}}_\mathrm{B} = -k_\mathrm{p}\,\dot{\mathbf{q}}_\mathrm{J} - k_\mathrm{d}\,\ddot{\mathbf{q}}_\mathrm{J} , \tag{12}$$

where $k_\mathrm{p}$ and $k_\mathrm{d}$ are the proportional and derivative gains.

If the rotor system is not controlled, the bearing shells do not move and then it holds

$$\mathbf{q}_\mathrm{B} = \mathbf{o} \,, \qquad \dot{\mathbf{q}}_\mathrm{B} = \mathbf{o} \,. \tag{13}$$

$\mathbf{o}$ is a zero vector.

## 4. The computational analyses

If the rotor turns at a constant angular speed, is perfectly balanced and if it is loaded only by its weight, then it takes the equilibrium position. Its determination arrives at solving a set of nonlinear algebraic equations that are obtained from the equations of motion (2) by setting the velocities and accelerations to zero. Then it holds for the uncontrolled rotor

$$2\,\mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{JEQ} - \mathbf{q}_\mathrm{BEQ}, \mathbf{o}) + \mathbf{f}_\mathrm{G} = \mathbf{o} \,. \tag{14}$$

$\mathbf{q}_\mathrm{JEQ}$ and $\mathbf{q}_\mathrm{BEQ}$ denote the vectors of displacements of the rotor journal and bearing shell centres respectively corresponding to the system equilibrium position. Utilizing relations (11), (12), the equilibrium equation (14) valid for the controlled system can be rewritten in the form

$$2\,\mathbf{f}_\mathrm{B}[(1 + k_\mathrm{p})\,\mathbf{q}_\mathrm{JEQ}, \mathbf{o}] + \mathbf{f}_\mathrm{G} = \mathbf{o} \,. \tag{15}$$

To evaluate stability of the rotor equilibrium position, the stiffness and damping parameters of the hydrodynamic bearings must be linearized in its neighbourhood. This requires to expand the nonlinear vector of hydraulic forces into a Taylor series and to take into account only its linear portion.

Taylor expansion of the bearing force referred to the uncontrolled rotor takes the form

$$\mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{J} - \mathbf{q}_\mathrm{B}, \dot{\mathbf{q}}_\mathrm{J} - \dot{\mathbf{q}}_\mathrm{B}) = \mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{JEQ} - \mathbf{o}, \mathbf{o}) + \mathbf{D}_\mathrm{B}\,(\dot{\mathbf{q}}_\mathrm{J} - \mathbf{o}) + \mathbf{D}_\mathrm{K}\,(\mathbf{q}_\mathrm{J} - \mathbf{q}_\mathrm{JEQ}) + \ldots \tag{16}$$

where $\mathbf{D}_\mathrm{B}$ and $\mathbf{D}_\mathrm{K}$ are the Jacobi matrices of partial derivatives with respect to the velocity components and displacements respectively

$$\begin{aligned} \mathbf{D}_\mathrm{B} &= \left[\frac{\partial \mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{J} - \mathbf{q}_\mathrm{B}, \dot{\mathbf{q}}_\mathrm{J} - \dot{\mathbf{q}}_\mathrm{B})}{\partial \dot{\mathbf{q}}_\mathrm{J}}\right]_{\substack{\mathbf{q}_\mathrm{J} = \mathbf{q}_\mathrm{JEQ} \\ \mathbf{q}_\mathrm{B} = \mathbf{o} \\ \dot{\mathbf{q}}_\mathrm{J} = \mathbf{o} \\ \dot{\mathbf{q}}_\mathrm{B} = \mathbf{o}}} \qquad \text{and} \\ \mathbf{D}_\mathrm{K} &= \left[\frac{\partial \mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{J} - \mathbf{q}_\mathrm{B}, \dot{\mathbf{q}}_\mathrm{J} - \dot{\mathbf{q}}_\mathrm{B})}{\partial \mathbf{q}_\mathrm{J}}\right]_{\substack{\mathbf{q}_\mathrm{J} = \mathbf{q}_\mathrm{JEQ} \\ \mathbf{q}_\mathrm{B} = \mathbf{o} \\ \dot{\mathbf{q}}_\mathrm{J} = \mathbf{o} \\ \dot{\mathbf{q}}_\mathrm{B} = \mathbf{o}}} \,. \end{aligned} \tag{17}$$

Taking into account (11) and (12) components of the hydraulic forces acting on the rotor journals of the system controlled by a PD controller are functions of displacements, velocities and accelerations of the rotor journal centre

$$\mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{J} - \mathbf{q}_\mathrm{B}, \dot{\mathbf{q}}_\mathrm{J} - \dot{\mathbf{q}}_\mathrm{B}) = \mathbf{f}_\mathrm{B}[(1 + k_\mathrm{p})\,\mathbf{q}_\mathrm{J} + k_\mathrm{d}\,\dot{\mathbf{q}}_\mathrm{J}, (1 + k_\mathrm{p})\,\dot{\mathbf{q}}_\mathrm{J} + k_\mathrm{d}\,\ddot{\mathbf{q}}_\mathrm{J}] \,. \tag{18}$$

Consequently, expansion of the vector of bearing forces (18) into a Taylor series acquires the form

$$\begin{aligned} \mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{J} - \mathbf{q}_\mathrm{B}, \dot{\mathbf{q}}_\mathrm{J} - \dot{\mathbf{q}}_\mathrm{B}) = \mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{JEQ} + k_\mathrm{p}\,\mathbf{q}_\mathrm{JEQ}, \mathbf{o}) + \mathbf{D}_\mathrm{M}\,(\ddot{\mathbf{q}}_\mathrm{J} - \mathbf{o}) + {} \\ + \mathbf{D}_\mathrm{B}\,(\dot{\mathbf{q}}_\mathrm{J} - \mathbf{o}) + \mathbf{D}_\mathrm{K}\,(\mathbf{q}_\mathrm{J} - \mathbf{q}_\mathrm{JEQ}) + \ldots \,. \end{aligned} \tag{19}$$

$\mathbf{D}_\mathrm{M}$, $\mathbf{D}_\mathrm{B}$ and $\mathbf{D}_\mathrm{K}$ are the Jacobi matrices of partial derivatives with respect to accelerations, velocities and displacements

$$\mathbf{D}_\mathrm{M} = \left[\frac{\partial \mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{J}-\mathbf{q}_\mathrm{B},\dot{\mathbf{q}}_\mathrm{J}-\dot{\mathbf{q}}_\mathrm{B})}{\partial \ddot{\mathbf{q}}_\mathrm{J}}\right]_{\substack{\mathbf{q}_\mathrm{J}=\mathbf{q}_\mathrm{JEQ}\\ \mathbf{q}_\mathrm{B}=-k_\mathrm{p}\,\mathbf{q}_\mathrm{JEQ}\\ \dot{\mathbf{q}}_\mathrm{J}=\mathbf{o}\\ \dot{\mathbf{q}}_\mathrm{B}=\mathbf{o}}} ,$$

$$\mathbf{D}_\mathrm{B} = \left[\frac{\partial \mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{J}-\mathbf{q}_\mathrm{B},\dot{\mathbf{q}}_\mathrm{J}-\dot{\mathbf{q}}_\mathrm{B})}{\partial \dot{\mathbf{q}}_\mathrm{J}}\right]_{\substack{\mathbf{q}_\mathrm{J}=\mathbf{q}_\mathrm{JEQ}\\ \mathbf{q}_\mathrm{B}=-k_\mathrm{p}\,\mathbf{q}_\mathrm{JEQ}\\ \dot{\mathbf{q}}_\mathrm{J}=\mathbf{o}\\ \dot{\mathbf{q}}_\mathrm{B}=\mathbf{o}}} \quad \text{and} \tag{20}$$

$$\mathbf{D}_\mathrm{K} = \left[\frac{\partial \mathbf{f}_\mathrm{B}(\mathbf{q}_\mathrm{J}-\mathbf{q}_\mathrm{B},\dot{\mathbf{q}}_\mathrm{J}-\dot{\mathbf{q}}_\mathrm{B})}{\partial \mathbf{q}_\mathrm{J}}\right]_{\substack{\mathbf{q}_\mathrm{J}=\mathbf{q}_\mathrm{JEQ}\\ \mathbf{q}_\mathrm{B}=-k_\mathrm{p}\,\mathbf{q}_\mathrm{JEQ}\\ \dot{\mathbf{q}}_\mathrm{J}=\mathbf{o}\\ \dot{\mathbf{q}}_\mathrm{B}=\mathbf{o}}} .$$

The substitution of (16) or (19) into (2) gives the linearized equation of motion. Calculation of the system eigenvalues arrives then at solving the characteristic equation

$$\det\left(\lambda\begin{bmatrix}\mathbf{O} & \mathbf{M}+2\,\mathbf{D}_\mathrm{M}\\ \mathbf{M}+2\,\mathbf{D}_\mathrm{M} & 2\,\mathbf{D}_\mathrm{B}\end{bmatrix}+\begin{bmatrix}-\mathbf{M}-2\,\mathbf{D}_\mathrm{M} & \mathbf{O}\\ \mathbf{O} & 2\,\mathbf{D}_\mathrm{K}\end{bmatrix}\right) \tag{21}$$

where $\lambda$ is the eigenvalue and $\mathbf{O}$ is a zero matrix. In the case of uncontrolled system, the Jacobi matrix of partial derivatives $\mathbf{D}_\mathrm{M}$ in (21) is zero. The position of equilibrium of the rotor is stable if the real parts of all the system eigenvalues are negative.

From the physical point of view the negatives of the Jacobi matrices of partial derivatives $\mathbf{D}_\mathrm{B}$, $\mathbf{D}_\mathrm{K}$ and $\mathbf{D}_\mathrm{M}$ represent the linearized damping, stiffness, and mass matrices of the fluid film bearings. For the uncontrolled rotors their elements depend only on the position of the rotor journals relative to the bearing shells and on the rotor rotational speed. If the rotor system is equipped with a feedback controller, then elements of the Jacobi matrices depend also on the gain constants.

## 5. Results of the computational simulations

The coordinates of the equilibrium position of the rotor are obtained by solving a set of nonlinear algebraic equations (14) or (15). For this purpose a Newton-Raphson method was applied. Even if the rotor is loaded only by its weight, the rotor centre is displaced both

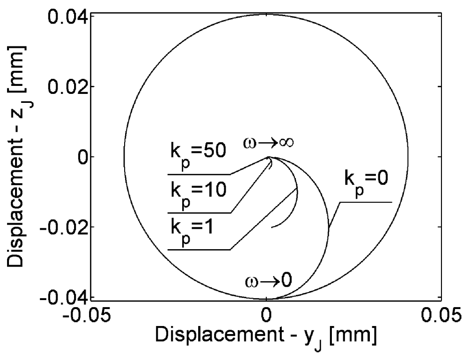


*Fig.4: The equilibrium position of the uncontrolled rotor and of the rotor controlled by the P controller*

in the vertical and horizontal directions (Fig. 4). The equilibrium position of the journals of the uncontrolled rotor approaches with increasing rotational speed the centres of the bearing shells but the reference position (the centre of bearing shell) would be achieved only if the speed were infinitely high. The same holds for the rotors controlled by a proportional controller.

The transient response of the rotor was determined by a direct integration of the equations of motion utilizing the Runge-Kutta method of the 4th order with adaptive size of the integration step (Dormand-Prince method). The rotor is excited by its imbalance and its vibration is controlled by a feedback controller with the proportional and derivative gains of 5 and 1 s respectively. At the moment of time of 10 s the controller is switched off and the uncontrolled vibration starts. The orbits, the time histories of the rotor journal displacements in the z-direction, their frequency spectra and the course of the hydraulic force components for two angular speeds ($314\,\mathrm{rad\,s^{-1}}, 733\,\mathrm{rad\,s^{-1}}$) are drawn in Figs. 5–14.

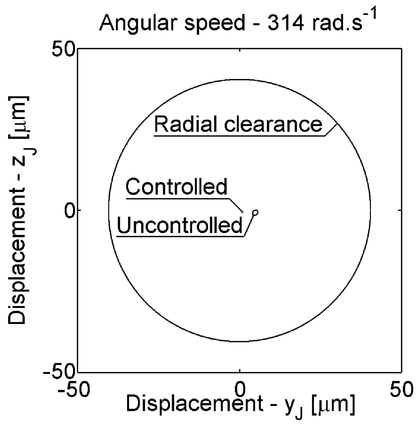


*Fig.5: Orbits of the uncontrolled rotor and the rotor controlled by a PD controller ($k_\mathrm{p} = 5$, $k_\mathrm{d} = 1\,s$, $\omega = 314\,rad\,s^{-1}$, $e_\mathrm{T} = 16.5\,\mu m$)*

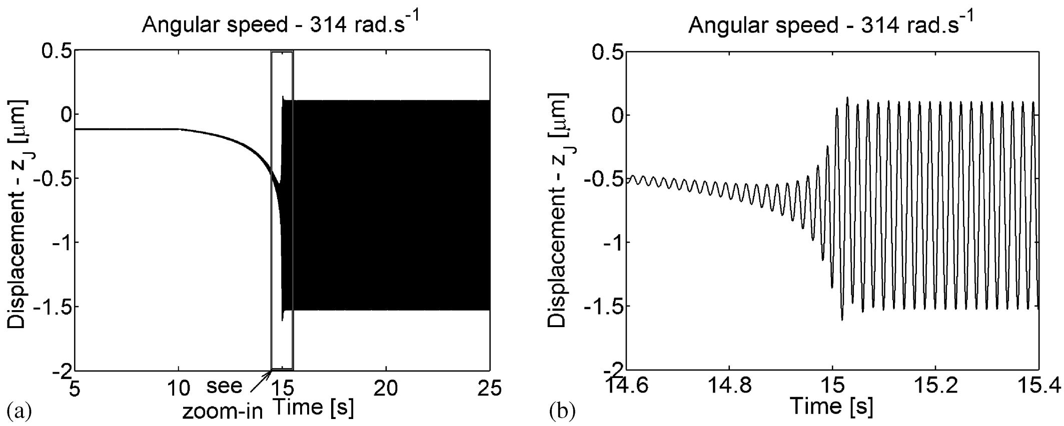


*Fig.6: Time history of the centre of the rotor journal displacement in the z-direction (a) and its detail at time 15 s (b) ($k_\mathrm{p} = 5$, $k_\mathrm{d} = 1\,s$, $\omega = 314\,rad\,s^{-1}$, $e_\mathrm{T} = 16.5\,\mu m$)*

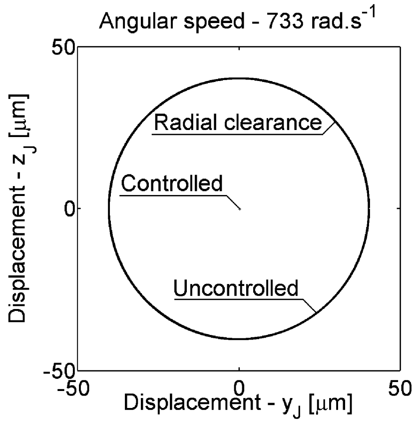


*Fig.7: Orbits of the uncontrolled rotor and the rotor controlled by a PD controller ($k_\mathrm{p} = 5$, $k_\mathrm{d} = 1\,s$, $\omega = 733\,rad\,s^{-1}$, $e_\mathrm{T} = 16.5\,\mu m$)*

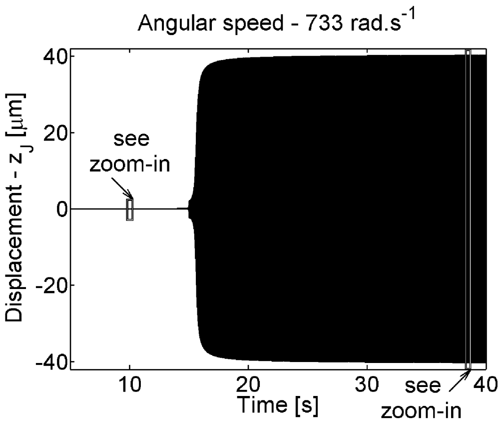


*Fig.8: Time history of the centre of the rotor journal displacement in the z-direction ($k_\mathrm{p} = 5$, $k_\mathrm{d} = 1\,s$, $\omega = 733\,rad\,s^{-1}$, $e_\mathrm{T} = 16.5\,\mu m$)*

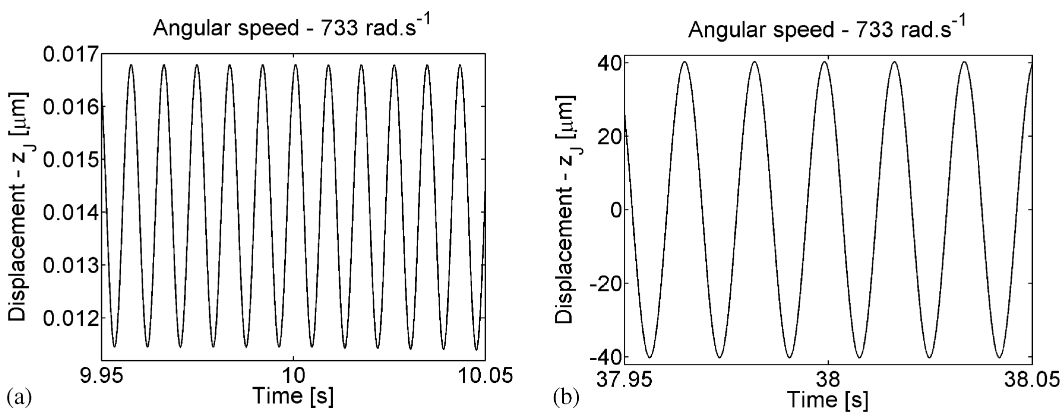


*Fig.9: Detail of the time history of the centre of the rotor journal displacement in the z-direction at time 10 s (a) and 38 s (b) ($k_\mathrm{p} = 5$, $k_\mathrm{d} = 1\,s$, $\omega = 733\,rad\,s^{-1}$, $e_\mathrm{T} = 16.5\,\mu m$)*

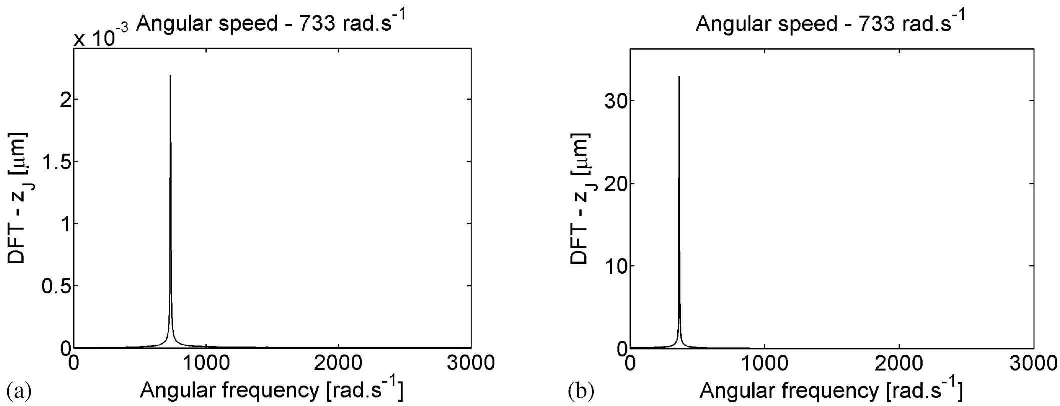


*Fig.10: The frequency spectrum of the displacement in the z-direction of the journal centre of the rotor with a PD controller switched on (a) and off (b) ($k_\mathrm{p} = 5$, $k_\mathrm{d} = 1\,\mathrm{s}$, $\omega = 733\,\mathrm{rad\,s^{-1}}$, $e_\mathrm{T} = 16.5\,\mu\mathrm{m}$)*

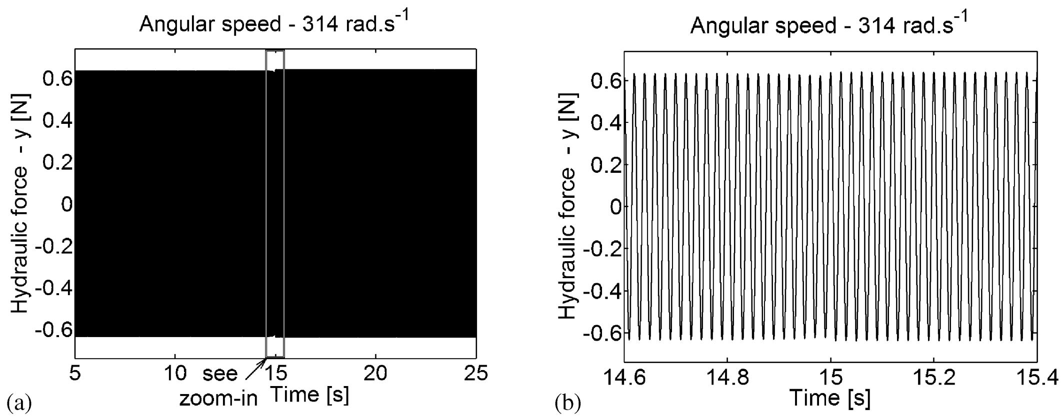


*Fig.11: Time history of the y-component of the hydraulic force (a) and its detail at time 15 s (b) ($k_\mathrm{p} = 5$, $k_\mathrm{d} = 1\,\mathrm{s}$, $\omega = 314\,\mathrm{rad\,s^{-1}}$, $e_\mathrm{T} = 16.5\,\mu\mathrm{m}$)*

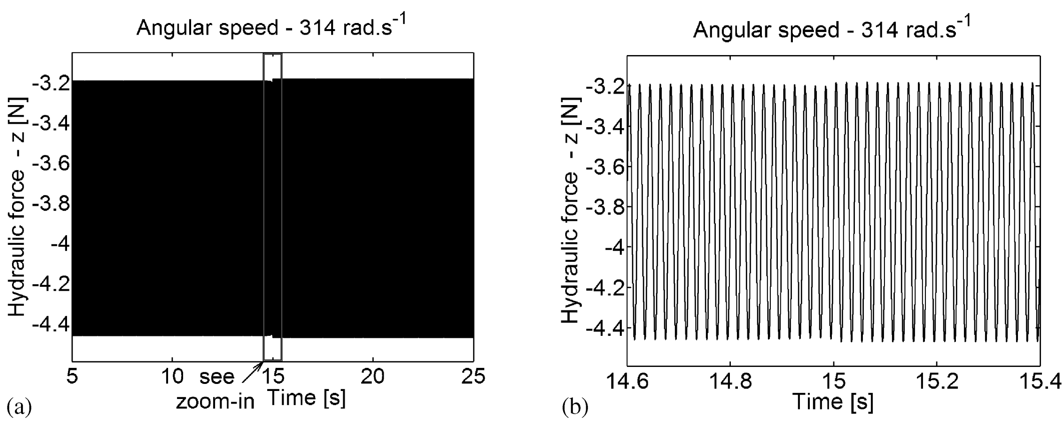


*Fig.12: Time history of the z-component of the hydraulic force (a) and its detail at time 15 s (b) ($k_\mathrm{p} = 5$, $k_\mathrm{d} = 1\,\mathrm{s}$, $\omega = 314\,\mathrm{rad\,s^{-1}}$, $e_\mathrm{T} = 16.5\,\mu\mathrm{m}$)*

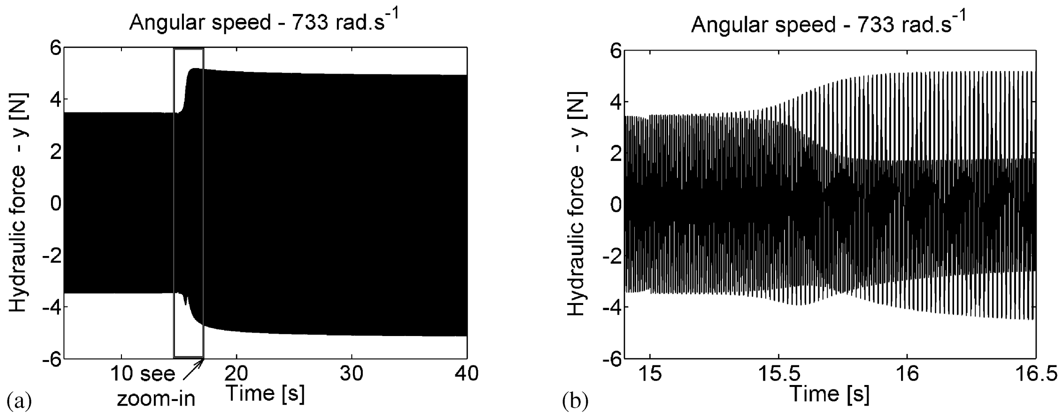


*Fig.13: Time history of the y-component of the hydraulic force (a) and its detail at time 15.7 s (b) ($k_{\mathrm{p}} = 5$, $k_{\mathrm{d}} = 1\,s$, $\omega = 733\,rad\,s^{-1}$, $e_{\mathrm{T}} = 16.5\,\mu m$)*

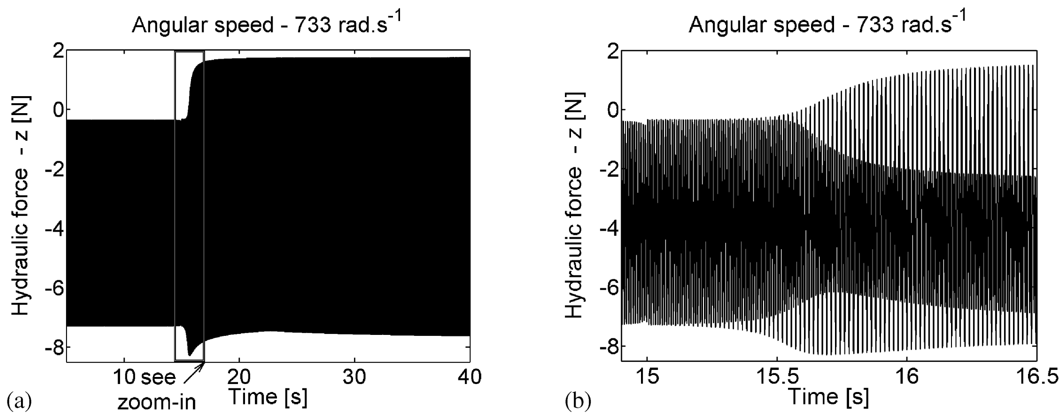


*Fig.14: Time history of the z-component of the hydraulic force (a) and its detail at time 15.7 s (b) ($k_{\mathrm{p}} = 5$, $k_{\mathrm{d}} = 1\,s$, $\omega = 733\,rad\,s^{-1}$, $e_{\mathrm{T}} = 16.5\,\mu m$)*

The lower rotational velocity corresponds to the case when the frequency of excitation and of the rotor response are the same and the higher one is related to the operating conditions when the rotor performs self-excited vibrations. Switching the controller off arrives at increase of the oscillation amplitude for both velocities. In both cases the initial controlled orbits are very small so that they appear only as points in Figs. 5 and 7. If the rotor rotates at the velocity of $314\,\mathrm{rad\,s^{-1}}$ and the controller is switched off, the orbit size slightly rises and the rotor continues to exhibit forced vibrations (Figs. 5 and 6) with unchanged frequency. But if it turns at the velocity of $733\,\mathrm{rad\,s^{-1}}$, amplitude of the vibration rapidly rises (Fig. 8) until the orbit fulfils the whole bearing gap (Fig. 7) and the rotor starts to exhibit vibrations with a double period as evident from the time histories of the rotor z-displacement (Fig. 9) and from the corresponding Fourier transform drawn in Fig. 10. Switching the controller off changes also magnitude of the hydraulic forces acting between the journals and the bearing housings. While this change is negligible if the rotor turns at the speed of $314\,\mathrm{rad\,s^{-1}}$ (Figs. 11 and 12), the hydraulic force components rise by about 70 % if the rotor rotates at the speed of $733\,\mathrm{rad\,s^{-1}}$ as shown in Figs. 13 and 14.

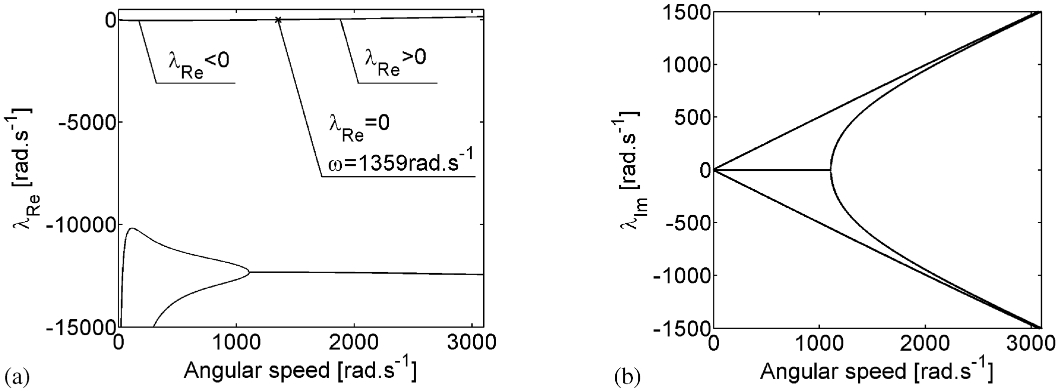


*Fig.15: Real $\lambda_{\mathrm{Re}}$ (a) and imaginary $\lambda_{\mathrm{Im}}$ (b) parts of the eigenvalues of the uncontrolled rotor system*

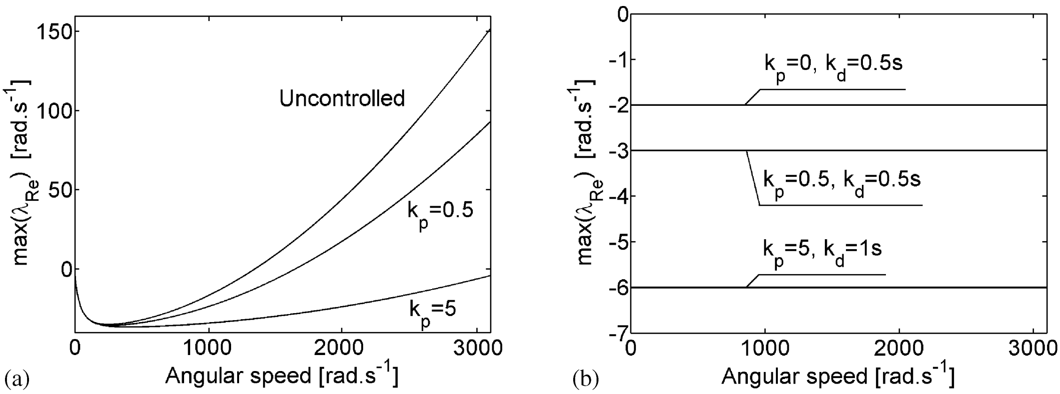


*Fig.16: The largest real part $\lambda_{\mathrm{Re}}$ of the eigenvalues of the rotor system with the P (a) and PD (b) feedback controller*

Dependences of the real ($\lambda_{\mathrm{Re}}$) and imaginary ($\lambda_{\mathrm{Im}}$) parts of eigenvalues of the linearized rotor system on its angular speed of rotation referred to the uncontrolled system are drawn in Fig. 15. The rotor equilibrium position becomes unstable if any real part takes a positive value. One can see that for lower angular velocities two system eigenvalues are real (their imaginary parts are zero) and the second two ones are complex conjugate. At the speed of $1108\,\mathrm{rad\,s^{-1}}$ a bifurcation occurs and both real eigenvalues become complex. Fig. 15a also shows that after exceeding the velocity of $1359\,\mathrm{rad\,s^{-1}}$ the maximum real part becomes positive, which implies that the rotor equilibrium position gets unstable. As evident from Fig. 16a if the P controller is used, the increasing proportional gain stabilizes the equilibrium position and extends the velocity region in which all the system eigenvalues have negative real parts. Application of the PD controller results in the stable equilibrium position in the whole extent of the rotor investigated speeds (Fig. 16b) independently of magnitude of the proportional gain constant.

The loss of stability of the equilibrium position indicates that the angular velocity of the rotor reaches the value at which the self-excited vibration induced by the hydrodynamic bearings can set on. But the detailed investigations show that because of nonlinear properties of the fluid film bearings its occurrence is a process taking place in a limited speed interval. The self-excited vibration can start at the angular speed that is something lower than those at which the equilibrium position becomes unstable. The direct integration of the equations of motion provides more accurate results.

## 6. Conclusions

The presented article deals with analysis of a control strategy for suppression of the forced and self-excited vibration of the unbalanced rotor supported by two hydrodynamic bearings. The feedback controller of a PD type governs the sliding motion (position and velocity) of the bearing shells with the aim to reduce amplitude of the rotor vibration and to increase the rotor angular speed at which the self-excited vibration starts.

The computational simulations proved that after exceeding a critical speed a self-excited vibration of the rotor induced by the fluid film bearings set on. Application of a P or PD controller enabled to shift the critical angular velocity to higher values. A PD controller made it possible to suppress the self-excited vibration in the whole extent of the investigated speeds independently on the values of the controller gains. This effect was achieved by a controlled kinematic excitation of the bearings shells. The simulations proved that the used control strategy was able to prevent occurrence of the rotor self-excited vibration or to move it to the region of higher angular velocities.

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