EFFECT OF ROTATION ON THE ONSET OF COMPRESSIBLE VISCOELASTIC FLUID SATURATING A DARCY-BRINKMAN POROUS MEDIUM

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The effect of rotation on the onset of convection in compressible Walters' (model B') elastico-viscous fluid heated from below saturating a porous medium is considered. For the porous medium,the Brinkman model is employed and Walters' (model B') fluid used to characterize the viscoelastic fluid. By applying normal mode analysis method, the dispersion relation has been derived and solved analytically. It is observed that the medium permeability, compressibility, gravity field and viscoelasticity introduce oscillatory modes. For stationary convection,the rotation has stabilizing effect whereas Darcy number and medium permeability have destabilizing/stabilizing effect on the system under certain conditions.

Keywords: *Brinkman-porous medium, rotation, thermal convection, viscoelasticity, Walters' (model B') fluid*

1. Introduction

The problem of thermal convection in porous medium has attracted considerable interest during the last few decades because it has various applications in geophysics, food processing and nuclear reactors. A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Lapwood [2] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [3].

There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry. Bhatia and Steiner [4] studied the problem of thermal instability of a Maxwellian visco-elastic fluid in the presence of rotation and found that rotation has a destabilizing influence in contrast to its stabilizing effect on a viscous Newtonian fluid. But Maxwell's model does not describe all the characteristics of a visco-elastic fluid. Thermal instability of an Oldroydian visco-elastic fluid in porous medium has been studied by Sharma [5]. There are many visco-elastic fluids which cannot be characterized by Maxwell's constitutive relations or Oldroyd's [6] constitutive relations. One such type of fluids is Walters' (model B') elastico-viscous fluid having relevance in chemical technology and industry. Walters' [7] reported that the mixture of polymethyl methacrylate and pyridine at 25 °C containg 30.5 g of polymer per litre with density 0.98 g per litre behaves very nearly as the Walters' (model B') elastico-viscous fluid. Walters' (model B') elastico-

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viscous fluid form the basis for the manufacture of many important polymers and useful products.

The investigation in porous media has been started with the simple Darcy model and gradually was extended to Darcy-Brinkman model. A good account of convection problems in a porous medium is given by Vafai and Hadim [8], Ingham and Pop [9] and Nield and Bejan [10]. Kuznetsov and Nield [11] have studied thermal instability in a porous medium layer saturated by a nanofluid : Brinkman model.Sharma and Rana [12] have studied thermal instability of a Walters' (model B') elastico-viscous in the presence of variable gravity field and rotation in porous medium.

When the fluids are compressible, the equations governing the system become more complicated. Spiegel and Veronis [13] have simplified the set of equations governing the flow of compressible fluids under the assumption that the depth of fluid layer is much than the scale height as defined by them, if only motions of infinitesimal amplitude are considered Gupta and Aggarwal [14] studied thermal instability of compressible Walters' (model B') elastico-viscous fluid in the presence of hall currents and suspended particles whereas Rana and Kango [15] studied the effect of rotation on thermal convection in compressible Walters' (model B') elastico-viscous fluid in porous medium.

Recently, Shivakumara et al. [16] have studied the effect of thermal modulation on the onset of thermal convection in Walters' B viscoelastic fluid in a porous medium whereas Chand and Rana [17] studied the effect of rotation on thermal instability of Oldroydian visco-elastic fluid saturated by Brinkman-Darcy porous medium. Rana [18] studied the effect of suspended particles on thermal instability of Walters' (model B') fluid in a Brinkman porous medium. In the present paper, the effect of rotation on thermal convection in compressible Walters' (model B') elastico-viscous fluid in a Brinkman porous medium is studied.

2. Mathematical model and perturbation equations

Let T_{ij}, τ_{ij}, e_{ij}, μ, μ', p, δ_{ij}, v_i, x_i and $\mathrm{d}/\mathrm{d}t$ denote respectively the total stress tensor, the shear stress tensor, the rate-of-strain tensor, the viscosity, the viscoelasticity, the isotropic pressure, the Kronecker delta, the velocity vector, the position vector and the convective derivative. Then the Walters' (model B') viscoelastic fluid is described by the constitutive relations

$$
\begin{aligned}
T_{ij} &= -p\,\delta_{ij} + \tau_{ij}\ , \\
\tau_{ij} &= 2\left(\mu - \mu'\,\frac{\mathrm{d}}{\mathrm{d}t}\right) e_{ij}\ , \\
e_i j &= \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\ .
\end{aligned}
$$

The above relations were proposed and studied by Walters' [7].

Here we consider an infinite, horizontal, compressible Walters' (model B') elastico-viscous fluid of depth d, bounded by the planes $z = 0$ and $z = d$ in an isotropic and homogeneous medium of porosity ε and permeability k_1, which is acted upon by gravity $\mathbf{g}(0, 0, -g)$ and uniform rotation $\boldsymbol{\Omega}(0, 0, \Omega)$. This layer is heated from below such that a steady adverse temperature gradient $\beta(= |\mathrm{d}T/\mathrm{d}z|)$ is maintained as shown in figure 1. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

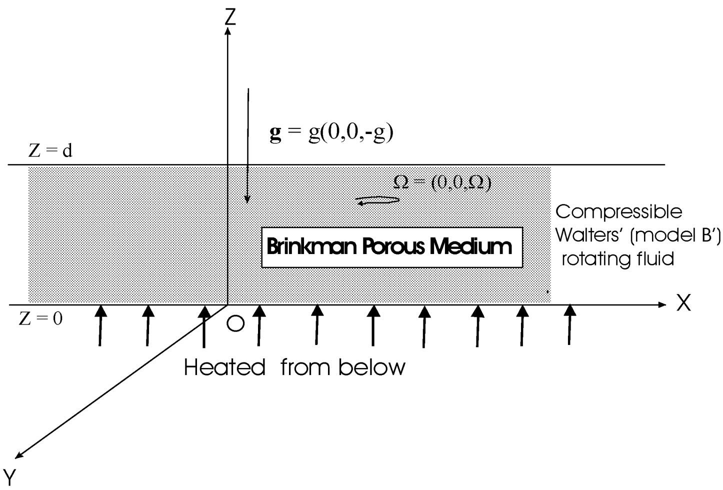

Fig.1: Schematic sketch of physical situation

Let ϱ, ν, ν', p, ε, T, α and $\mathbf{v}(0,0,0)$, denote respectively, the density, kinematic viscosity, kinematic viscoelasticity, pressure, medium porosity, temperature, thermal coefficient of expansion and velocity of the fluid.

The equations expressing the conservation of momentum, mass and temperature for Walters' (model B') elastico-viscous fluid (Chandrasekhar [1], Kuznetsov and Nield [11], Sharma and Rana [12], Rana [18]) are

$$\frac{\varrho}{\varepsilon}\left[\frac{\partial v}{\partial t}+\frac{1}{\varepsilon}(\mathbf{v}.\nabla)\,\mathbf{v}\right]=-\nabla p+\varrho\,\mathbf{g}-\frac{1}{k_1}\left(\mu-\mu'\,\frac{\partial}{\partial t}\right)\mathbf{v}+\tilde{\mu}\,\nabla^2\mathbf{v}+\frac{2\,\varrho}{\varepsilon}(\mathbf{v}\times\mathbf{\Omega})\ , \tag{1}$$

$$\frac{\partial\varrho}{\partial t}+\nabla.(\varrho\,\mathbf{v})=0\ , \tag{2}$$

$$E\,\frac{\partial T}{\partial t}+(\mathbf{v}.\nabla)\,T=\kappa\,\nabla^2 T\ . \tag{3}$$

Here

$$E=\varepsilon+(1-\varepsilon)\,\frac{\varrho_{\rm s}\,c_{\rm s}}{\varrho_0 c_{\rm f}}$$

which is constant, κ is the thermal diffusivity and $\tilde{\mu}$ is effective viscosity of porous medium; ϱ_s, $c_{\rm s}$; ϱ_0, $c_{\rm f}$ denote the density and heat capacity of solid (porous) matrix and fluid respectively.

The state variables pressure,density and temperature are expressed in the form (Spiegel and Veronis [13])

$$f(x,y,z,t)=f_{\rm m}+f_0(z)+f'(x,y,z,t)\ , \tag{4}$$

where $f_{\rm m}$ denotes for constant space distribution f, f_0 is the variation in the absence of motion,and $f'(x,y,z,t)$ is the fluctuation resulting from motion.The basic state of the system is

$$p=p(z)\ ,\qquad \varrho=\varrho(z)\ ,\qquad T=T(z)\ ,\qquad \mathbf{q}=(0,0,0) \tag{5}$$

where

$$p(z) = p_{\mathrm{m}} - g\int_0^z (\varrho_{\mathrm{m}} + \varrho_0)\,\mathrm{d}z\ , \qquad \varrho(z) = \varrho_m\left[1 - \alpha_m\,(T - T_0) + K_{\mathrm{m}}(p - p_{\mathrm{m}})\right]\ ,$$

$$T = -\beta\, z + T_0\ , \qquad \alpha_{\mathrm{m}} = -\left(\frac{1}{\varrho}\,\frac{\partial\varrho}{\partial t}\right)_{\mathrm{m}}\ , \qquad K_{\mathrm{m}} = \left(\frac{1}{\varrho}\,\frac{\partial\varrho}{\partial p}\right)_{\mathrm{m}}\ .$$

Here p_{m} and ϱ_{m} denote a constant space distribution of p and ϱ while T_0 and ϱ_0 denote temperature and density of the fluid at the lower boundary.

Let $\mathbf{v}(u, v, w)$, θ, δp and $\delta\varrho$ denote, respectively, the perturbations in fluid velocity $\mathbf{v}(0, 0, 0)$, temperature T, pressure p and density ϱ.

The change in density $\delta\varrho$ caused by perturbation θ in temperature is given by

$$\delta p = -\alpha\,\varrho_{\mathrm{m}}\,\theta\ . \tag{6}$$

The linearized perturbation equations governing the motion of fluid are

$$\frac{1}{\varepsilon}\,\frac{\partial\mathbf{v}}{\partial t} = -\frac{1}{\varrho_{\mathrm{m}}}\,\nabla\,\delta p - g\,\alpha\,\theta - \frac{1}{k_1}\left(\nu - \nu'\,\frac{\partial}{\partial t}\right)\mathbf{v} + \frac{\tilde{\mu}}{\varrho_{\mathrm{m}}}\,\nabla^2\mathbf{v} + \frac{2}{\varepsilon}\,(\mathbf{v}\times\boldsymbol{\Omega})\ , \tag{7}$$

$$\nabla.\mathbf{v} = 0\ , \tag{8}$$

$$\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{c_{\mathrm{p}}}\right) w + \kappa\,\nabla^2\,\theta\ , \tag{9}$$

where ν, ν' and w denote respectively the kinematic viscosity, kinematic viscoelasticity and the vertical fluid velocity.

3. The dispersion relation

Following the normal mode analyses, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w, \theta, \zeta] = [W(z), \Theta(z), Z(z)]\,\exp(\mathrm{i}\,l\,x + \mathrm{i}\,m\,y + n\,t)\ , \tag{10}$$

where l and m are the wave numbers in the x and y directions, $k = (l^2 + m^2)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.

Using expression (10) in equations (7)–(9) become

$$\begin{aligned}\frac{n}{\varepsilon}\left[\frac{\mathrm{d}^2}{\mathrm{d}z^2} - k^2\right] W = -g\,k^2\,\alpha\,\theta - \frac{1}{k_1}\,(\nu - \nu'\,n)\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - k^2\right) W\,+ \\ +\frac{\tilde{\mu}}{\varrho_0}\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - k^2\right)^2 W - \frac{2\,\Omega}{\varepsilon}\,\frac{\mathrm{d}Z}{\mathrm{d}z}\ ,\end{aligned} \tag{11}$$

$$\frac{n}{\varepsilon}\,Z = -\frac{1}{k_1}\,(\nu + \nu'\,n)\,Z + \frac{2\,\Omega}{\varepsilon}\,\frac{\mathrm{d}W}{\mathrm{d}z} + \frac{\tilde{\mu}}{\varrho_0}\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - k^2\right) Z\ , \tag{12}$$

$$E\,n\,\Theta = \left(\beta - \frac{g}{c_{\mathrm{p}}}\right) W + \kappa\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - k^2\right)\,\Theta\ . \tag{13}$$

Equations (11) to (13) in non-dimensional form, become

$$\left[1+\left(\frac{P_l}{\varepsilon}-F\right)\sigma-D_A(D^2-a^2)\right](D^2-a^2)\,W+\frac{g\,a^2\,d^2\,P_l\,\alpha\,\Theta}{\nu}+\frac{2\,P_l\,\Omega\,d^3}{\varepsilon\,\nu}\,D\,Z=0\ ,\quad (14)$$

$$\left[1+\left(\frac{P_l}{\varepsilon}-F\right)\sigma-D_A\,(D^2-a^2)\right]Z=\frac{2\,\Omega\,d}{\varepsilon\,\nu}\,P_l\,D\,W\ ,\qquad (15)$$

$$[D^2-a^2-E\,P_r\,\sigma]\,\Theta=-\frac{\beta\,d^2}{\kappa}\,\frac{G-1}{G}\,W\ ,\qquad (16)$$

where we have put $a=k\,d$, $\sigma=n\,d^2/\nu$, $F=\nu'/d^2$ and $P_l=k_1/d^2$ is the dimensionless medium permeability, $P_r=\nu/\kappa$ is the thermal Prandtl number, $G=(c_p/g)\,\beta$ is the compressibility and $D_A=(\tilde{\mu}\,k_1)/(\mu\,d^2)$ is the Darcy number modified by the viscosity ratio.

Eliminating Θ and Z between equations (14) to (16), we obtain

$$\begin{aligned}&\left[1+\left(\frac{P_l}{\varepsilon}-F\right)\sigma-D_A(D^2-a^2)\right](D^2-a^2)\,(D^2-a^2-E\,P_r\,\sigma)\,W-\\&-R\,a^2\,P_l\,\frac{G-1}{G}\,W+\frac{T_A\,P_l^2\,(D^2-a^2-E\,P_r\,\sigma)}{1+\left(\frac{P_l}{\varepsilon}-F\right)\sigma-D_A\,(D^2-a^2)}\,D^2\,W=0\ ,\end{aligned}\qquad (17)$$

where $R=(g\,\alpha\,\beta\,d^4)/(\nu\,\kappa)$ is the thermal Rayleigh number and $T_A=((2\,\Omega\,d^2)/(\varepsilon\,\nu))^2$ is the modified Taylor number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non-conducting. The boundary conditions appropriate to the problem are (Chandrasekhar [1])

$$W=D^2\,W=D^4\,W=D\,Z=\Theta=0\quad\text{at}\quad z=0\text{ and }1\ .\qquad (18)$$

The case of two free boundaries,though a little artificial is the most appropriate for stellar atmospheres. Using the boundary conditions (18),we can show that all the even order derivatives of W must vanish for $z=0$ and $z=1$ and hence the proper solution of W characterizing the lowest mode is

$$W=W_0\sin\pi\,z\ ;\qquad W_0\text{ is a constant}\ .\qquad (19)$$

Substituting equation (19) in (17),we get

$$\begin{aligned}R_1\,x\,P=\frac{G}{G-1}\Bigg\{&\left[1+\left(\frac{P}{\varepsilon}-\pi^2\,F\right)i\,\sigma_1+D_{A_1}\,(1+x)\right](1+x)\,(1+x+E\,P_r\,i\,\sigma_1)+\\&+\frac{T_{A_1}\,P^2\,(1+x+E\,P_r\,i\,\sigma_1)}{1+\left(\frac{P}{\varepsilon}-\pi^2\,F\right)i\,\sigma_1+D_{A_1}\,(1+x)}\Bigg\}\ .\end{aligned}\qquad (20)$$

where we have put $R_1=R/\pi^4$, $T_{A_1}=T_A/\pi^4$, $D_{A_1}=(D_A/\pi^2)\,x=a^2/\pi^2$, $i\,\sigma_1=\sigma/\pi^2$, $P=\pi^2\,P_l$.

Equation (20) is required dispersion relation accounting for the onset of thermal convection in Walters' (model B') elastico-viscous fluid under rotation in a Brinkman porous medium.

4. Oscillatory modes

Here we examine the possibility of oscillatory modes, if any, in Walters' (model B') elastico-viscous fluid due to the presence of rotation, viscoelasticity, medium permeability and gravity field. Multiply equation (14) by W^* the complex conjugate of W, integrating over the range of z and making use of equations (15) and (16) with the help of boundary conditions (18), we obtain

$$\left[1+\left(\frac{P_1}{\varepsilon}-F\right)\sigma\right] I_1 + D_\mathrm{A}\,(I_2 + d^2\, I_3) + \frac{g\,a^2\,\alpha\,\kappa\,P_1}{\nu\,\beta}\,\frac{G}{G-1}\,(I_4 + E\,P_\mathrm{r}\,\sigma^*\,I_5) + \\ + d^2\left[1+\left(\frac{P_1}{\varepsilon}+F\right)\sigma^*\right] I_6 = 0 \;, \tag{21}$$

where

$$I_1 = \int_0^1 (|D\,W|^2 + a^2\,|W|^2)\,\mathrm{d}z \;,$$

$$I_2 = \int_0^1 (|D\,W|^4 + 2\,a^2\,|D\,W|^2 + a^4\,|W|^2)\,\mathrm{d}z \;,$$

$$I_3 = \int_0^1 (|D\,Z|^2 + a^2\,|Z|^2)\,\mathrm{d}z \;,$$

$$I_4 = \int_0^1 (|D\,\Theta|^2 + a^2\,|\Theta|^2)\,\mathrm{d}z \;,$$

$$I_5 = \int_0^1 |\Theta|^2\,\mathrm{d}z \;,$$

$$I_6 = \int_0^1 |Z|^2\,\mathrm{d}z \;.$$

The integral part I_1–I_6 are all positive definite. Putting $\sigma = \mathrm{i}\,\sigma_\mathrm{i}$ in equation (21), where σ_i is real and equating the imaginary parts, we obtain

$$\sigma_i\left[\left(\frac{P_l}{\varepsilon}-F\right)(I_1 - d^2\,I_6) - \frac{g\,a^2\,\alpha\,\kappa\,P_1}{\nu\,\beta}\,\frac{G}{G-1}\,E\,P_\mathrm{r}\,I_5\right] = 0 \;, \tag{22}$$

Equation (22) implies that $\sigma_\mathrm{i} = 0$ or $\sigma_\mathrm{i} \neq 0$ which mean that modes may be non-oscillatory or oscillatory. The oscillatory modes introduced due to presence of viscoelasticity, compressibility and medium permeability which were non-existent in their absence.

5. The stationary convection and discussion

For stationary convection, putting $\sigma = 0$ in equations (20), we obtain

$$R_1 = \frac{G}{G-1}\,\frac{1+x}{x}\left[\frac{1+x}{P} + \frac{D_{\mathrm{A}_1}\,(1+x)^2}{P} + \frac{T_{\mathrm{A}_1}\,P}{1+D_{\mathrm{A}_1}\,(1+x)}\right] \;, \tag{23}$$

Equation (23) expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1}, D_{A_1}, P and Walters' (model B') elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ.

Let the non-dimensional number G accounting for compressibility effect is kept as fixed, then we get

$$\bar{R}_{\mathrm{c}} = \frac{G}{G-1}\, R_{\mathrm{c}} \;, \tag{24}$$

where $\bar{R}_{\mathrm{c}}$ and R_{c} denote, respectively, the critical number in the presence and absence of compressibility. Thus, the effect of compressibility is to postpone the instability on the onset of thermal instability. The cases $G = 1$ and $G < 1$ correspond to infinite and negative values of Rayleigh numbers due to compressibility which are not relevant to the present study.

To study the effects of rotation, Darcy number and medium permeability, we examine the behavior of $\partial R_1/\partial T_{\mathrm{A}_1}$, $\partial R_1/\partial D_{\mathrm{A}_1}$ and $\partial R_1/\partial P$ analytically.

From equation (23), we get

$$\frac{\partial R_1}{\partial T_{\mathrm{A}_1}} = \frac{G}{G-1}\,\frac{1+x}{x}\,\frac{P}{1 + D_{\mathrm{A}_1}(1+x)} \;, \tag{25}$$

which is positive.Hence rotation has stabilizing effect on the thermal convection of Walters' (model B') elastico-viscous fluid in a Brinkman porous medium.This stabilizing effect is an agreement of the earlier work of Sharma [5], Sharma and Rana [12] in the absence of Brinkman model.

From equation (23), we get

$$\frac{\partial R_1}{\partial D_{\mathrm{A}_1}} = \frac{1+x}{x}\,\frac{G}{G-1}\left[\frac{(1+x)^2}{P} - \frac{(1+x)\,P\,T_{\mathrm{A}_1}}{\{1 + D_{\mathrm{A}_1}(1+x)\}^2}\right] \;, \tag{26}$$

Thus Darcy number has destabilizing effect when

$$\frac{(1+x)^2}{P} < \frac{(1+x)\,P\,T_{\mathrm{A}_1}}{\{1 + D_{\mathrm{A}_1}(1+x)\}^2}$$

and stabilizing effect when

$$\frac{(1+x)^2}{P} > \frac{(1+x)\,P\,T_{\mathrm{A}_1}}{\{1 + D_{\mathrm{A}_1}(1+x)\}^2}$$

on the thermal convection of Walters' (model B') elastico-viscous fluid in a Brinkman porous medium. In the absence of rotation, $\partial R_1/\partial D_{\mathrm{A}_1}$ is always positive implying thereby the stabilizing effect of Darcy number which is good agreement of the earlier result derived by Rana [18].

It is evident from equation (23) that

$$\frac{\partial R_1}{\partial P} = -\frac{1+x}{x}\,\frac{G}{G-1}\left[\frac{1+x}{P^2} + \frac{D_{\mathrm{A}_1}(1+x)^2}{P^2} - \frac{T_{\mathrm{A}_1}}{\{1 + D_{\mathrm{A}_1}(1+x)\}^2}\right] \;, \tag{27}$$

From equation (34), we observe that medium permeability has destabilizing effect when

$$\frac{1+x}{P^2}+\frac{D_{A_1}(1+x)^2}{P^2}>\frac{T_{A_1}}{\{1+D_{A_1}(1+x)\}^2}$$

and stabilizing effect when

$$\frac{1+x}{P^2}+\frac{D_{A_1}(1+x)^2}{P^2}<\frac{T_{A_1}}{\{1+D_{A_1}(1+x)\}^2}$$

on the thermal convection of Walters' (model B') elastico-viscous fluid in a Brinkman porous medium. In the absence of rotation, $\partial R_1/\partial P$ is always negative implying thereby the destabilizing effect of medium permeability. This destabilizing/stabilizing effect is an agreement of the earlier work of Sharma [5], Sharma and Rana [12], Gupta and Aggrawal [14], Rana [18]. The dispersion relation (23) is analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics.

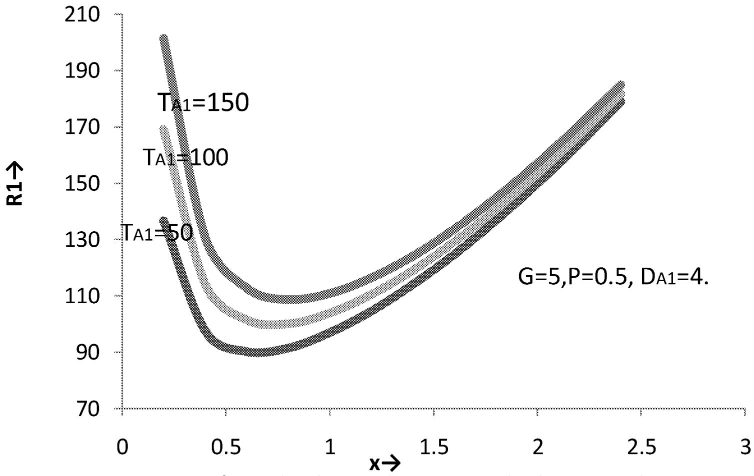

Fig.2: Variation of Rayleigh number R_1 with dimensionless wave number x for different values of Taylor number T_{A_1}

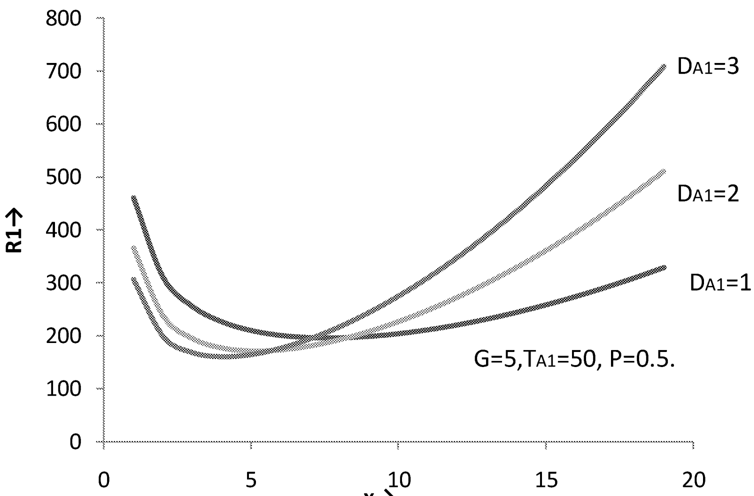

Fig.3: Variation of Rayleigh number R_1 with dimensionless wave number x for different values of Darcy number D_{A_1}

In fig. 2, Rayleigh number R_1 is plotted against dimensionless wave number x for various values of Taylor number T_{A_1}. This shows that rotation has a stabilizing effect as R_1 increases with the increase of Taylor number. In Fig. 3, Rayleigh number R_1 is plotted against dimensionless wave number x for various values of Darcy number D_{A_1}. This shows that Darcy number D_{A_1} has a destabilizing/stabilizing effect on the system as the Rayleigh number decreases/increases as the Darcy number D_{A_1} increases.

In fig. 4, Rayleigh number R_1 is plotted against dimensionless wave number x for various values of medium permeability P. Since Rayleigh number decreases/increases as the medium permeability P increases. Thus, medium permeability has a destabilizing/stabilizing effect on the system.

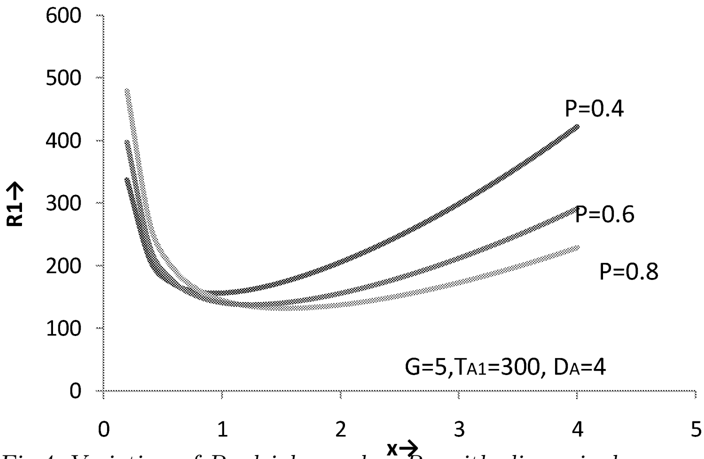

Fig.4: Variation of Rayleigh number R_1 with dimensionless wave number x for different values of medium permeability P

6. Conclusions

The effect of rotation on the onset of convection in compressible Walters' (model B') elastico-viscous fluid heated from below in a Brinkman porous medium has been investigated. The dispersion relation, including the effects of rotation, Darcy number and medium permeability on the thermal instability in Walters' (model B') fluid is derived. From the analysis,the main conclusions are as follows:

(i) For the case of stationary convection, Walters' (model B') elastico-viscous fluid behaves like an ordinary Newtonian fluid as elastico-viscous parameter F vanishes with σ.

(ii) The expressions for $\partial R_1/\partial T_{\mathrm{A}_1}$, $\partial R_1/\partial D_{\mathrm{A}_1}$ and $\partial R_1/\partial P$ are examined analytically and it has been found that the rotation has stabilizing effect whereas Darcy number and medium permeability have a destabilizing/stabilizing effect on the system. However, Darcy number has stabilizing effect and medium permeability has destabilizing effect on the system in the absence of rotation.

(iii) The effects of rotation, Darcy number and medium permeability on thermal convection have also been shown graphically in figures 2, 3 and 4 respectively.

(iv) The oscillatory modes introduced due to presence of viscoelasticity, rotation, gravity field and medium permeability which were non-existent in their absence.

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Nomenclature

d .... depth of fluid layer, m
$F = \nu'/d^2$ .... dimensionless kinematic viscoelasticity
$G = (c_p/g)\beta$ .... dimensionless compressibility
$P_l = k_1/d^2$ .... dimensionless medium permeability
g .... gravitational acceleration, $\mathrm{m\,s^{-2}}$
$\mathbf{g}$ .... gravitational acceleration vector
k_1 .... medium permeability
$D_\mathrm{A} = (\tilde{\mu}\,k_1)/(\mu\,d^2)$ .... modified Darcy number
$T_\mathrm{A} = ((2\,\Omega\,d^2)/(\varepsilon\,\nu))^2$ .... modified Taylor number
p .... pressure, Pa
$R = (g\,\alpha\,\beta\,d^4)/(\nu\,\kappa)$ .... thermal Rayleigh number
P_r .... thermal Prandtl number
v .... velocity of fluid, $\mathrm{m\,s^{-1}}$
k .... wave number of disturbance,m-1
β .... adverse temperature gradient, $\mathrm{K\,m^{-1}}$
$\tilde{\mu}$ .... effective viscosity of the porous medium, $\mathrm{kg\,m^{-1}\,s^{-1}}$
$\boldsymbol{\varrho}$ .... fluid density, $\mathrm{kg\,m^{-3}}$
μ .... fluid viscosity, $\mathrm{kg\,m^{-1}s^{-1}}$
μ' .... fluid viscoelasticity, $\mathrm{kg\,m^{-1}\,s^{-1}}$
ν .... kinematic viscosity, $\mathrm{m^2\,s^{-1}}$
ν' .... kinematic viscoelasticity, $\mathrm{m^2\,s^{-1}}$
ε .... medium porosity
δ .... perturbation in respective physical quantity
θ .... perturbation in temperature, K
$\mathbf{\Omega}$ .... rotation vector having components $(0, 0, \Omega)$, $\mathrm{m\,s^{-1}}$
κ .... thermal diffusitivity, $\mathrm{m^2\,s^{-1}}$
α .... thermal coefficient of expansion, $\mathrm{K^{-1}}$
ζ .... z-component of vorticity

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