NUMERICAL AND EXPERIMENTAL STUDIES OF LAMINAR NATURAL CONVECTION ON A HORIZONTAL CYLINDER

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This work deals with numerical and experimental studies of specific issues of natural convection flow around a horizontal cylinder in a space limited by rectangular cavity. Physical models of laminar unsteady flows have been used for numerical calculations of heat transfer coefficients of four different diameters of horizontal cylinders and for several temperature gradients. Procedures and results of the experimental studies are contained in the next section. All results were compared with different authors and different approaches. However the Nusselt numbers can be estimated due to the temperature interval, the results indicate need for further investigation.

Keywords: *natural convection, horizontal cylinder, Nusselt number, CFD, measuring*

1. Introduction

Heat exchangers and their high effectiveness are very important for storing heat energy. They are often made from horizontal or nearly horizontal circular cross section profiles where is desirable to determinate their performance as accurately as is possible. However, the task of the analytical solution of the local heat transfer coefficient on the surface of heat exchangers with complex shape of the cross section is difficult to solve together full equations of motion and energy. The angle between the gravity vector and the tangent of every point on the surface curve varies along the fluid flow direction. This can bring a lot of problems even for current numerical methods. Nevertheless the natural convection on the surface of horizontal cylinder was investigated by various authors in a wide range of parameters to obtained correlation equations. The most frequently used empirical correlation equations of the authors Morgan V.T. [1], Collis D.C. – Williams M.J. [2], Kreith F. – Black W. [3], Churchill S.W. – Chu H.H.S. [4], Jaluria Y. [5], Fand R.M. – Morris E.W. – Lum M. [6], Brdlik P.M. – Kuptsova V. S. – Malinin V.G. [7] and finally Raithby G.D. – Hollands K.G.T. [8] were chosen for comparison with the data obtained by numerical and experimental investigation of a particular case.

2. Physical description

Description of natural convection around immersed body with complex shape of the cross section could be difficult to solve however, for each fluid surrounding any body can impose certain limiting assumptions. Continuity equation for two-dimensional steady flow

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of incompressible fluid around immersed body with depth of one could be written as:

$$\frac{\partial w_{\mathrm{x}}}{\partial x} + \frac{\partial w_{\mathrm{y}}}{\partial y} = 0 \ ,$$

where w_{x} and w_{y} [m s^{-1}] are components of velocity every fluid particles in dimensions x and y of the reference system. Provided infinitely small part of immersed surface where the normal vector is perpendicular to the gravity vector the Navier-Stokes momentum equation could be written

$$w_{\mathrm{x}} \frac{\partial w_{\mathrm{y}}}{\partial x} + w_{\mathrm{y}} \frac{\partial w_{\mathrm{y}}}{\partial y} = -\beta\, g\, (T - T_{\infty}) + \nu \frac{\partial^2 w_{\mathrm{y}}}{\partial x^2} \ ,$$

where g [m s^{-2}] is the gravitational acceleration, T_{∞} [K] is the temperature in the ambient, ν [m^2 s^{-1}] kinematic viscosity of the fluid. According to the Boussinesq approximation the fluid density is dependent only on temperature of fluid thus β [K^{-1}] is thermal expansion coefficient

$$\beta = \frac{1}{\nu} \left.\frac{\partial \nu}{\partial T}\right|_{p} = -\frac{1}{\varrho} \left.\frac{\partial \varrho}{\partial T}\right|_{p} \ ,$$

where ϱ [kg m^{-3}] is density of the fluid. Fourier-Kirchhoff energy equation could be written in the differential form using of the thermal diffusivity a [m^2 s^{-1}]

$$w_{\mathrm{x}} \frac{\partial T}{\partial x} + w_{\mathrm{y}} \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial x^2} \ .$$

In this equation the component of fluid velocity w_{y} is caused by buoyancy forces in the momentum equation and therefore both equations, momentum and energetic, can not be solved separately.

3. Investigation

If the temperature of the surface immersed body T_{w} [K] is different than temperature of the ambient T_{∞} [K] the thermal boundary layer occurs around the surface. Due to the buoyancy forces the fluid begins to move and a momentum boundary layer occurs. When the heat transfer caused by conduction of the fluid is compared with the heat transfer caused by convection the Nusselt number is obtained. For a horizontal cylinder surface, for a dimensionless temperature gradient and for a normal length to the surface the average of Nusselt number on the cylinder surface can be written as

$$\frac{1}{2\pi} \int_{0}^{2\pi} \left. \frac{\partial \left(\dfrac{T_{\mathrm{w}} - T}{T_{\mathrm{w}} - T_{\infty}} \right)}{\partial \left(\dfrac{n}{D} \right)} \right|_{n/D=0} \mathrm{d}\theta = \frac{\alpha\, D}{\lambda_{\mathrm{f}}} = N\, u_{\mathrm{D}} \ ,$$

where n [m] is a normal length to the cylinder surface, D [m] is the cylinder diameter, θ is the angular coordinate around the cylinder. α [W m^{-2} K^{-1}] is the average value of convection coefficient and λ_{f} [W m^{-1} K^{-1}] is thermal conductivity of the fluid. The main task for an investigation of natural convection on the outside wall of a body immersed in a fluid is in determination of the heat transfer coefficient.

3.1. Empirical correlation equations

Nowadays there are many different approaches to solve the task of natural convection on a horizontal cylinder and in this section are described some of the most frequently used empirical correlation equations. According Buckingham π-theorem implies necessity two other dimensionless variables to describe the behaviour of fluid. First variable compares kinematic viscosity and thermal diffusivity is Prandtl number

$$\mathrm{Pr} = \frac{\nu}{a}\ .$$

The ratio of the buoyancy forces to viscous forces in a fluid as an average on the cylinder surface is characterised by Grashof number $\mathrm{Gr_D}$.

$$\mathrm{Gr_D} = \frac{g\,\beta\,(T_\mathrm{w} - T_\infty)\,D^3}{\nu^2}\ .$$

Product of the multiplication Prandtl and Grashof number $\mathrm{Gr_D}$ is another attribute of the dimensionless functional equations Rayleigh number $\mathrm{Ra_D}$.

As a result of the great amount of data obtained by experimental investigations, a set of recommended forms for calculating the heat transfer coefficient of a long horizontal cylinder in a large range of Rayleigh numbers was obtained by authors [1, 2]. In this work, only laminar equation for the range of Rayleigh number $10^4 \div 10^7$ was used

$$\mathrm{Nu_D} = 0.48\,\mathrm{Ra_D^{0.25}}\ .$$

According to the authors Kreith and Black [3] the correlation equation in the range of the Rayleigh number $10^4 \div 10^9$ was used

$$\mathrm{Nu_D} = 0.53\,\mathrm{Ra_D^{0.25}}\ .$$

Relatively complex solution, but with good results, is according to Churchill and Chu [4] for a really wide range of Rayleigh number for laminar and turbulent fluid flow

$$\mathrm{Nu_D} = \left[0.6 + \frac{0.387\,\mathrm{Ra_D^{1/6}}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right]^2$$

and from the same authors only for laminar fluid flow (Rayleigh number less than 10^9)

$$\mathrm{Nu_D} = 0.36 + \frac{0.518\,\mathrm{Ra_D^{1/4}}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{4/9}}\ .$$

The average Nusselt number on the cylinder diameter calculated by Jaluria [5] obtained by an integral method valid for all Prandtl numbers in the range of Grashof number $10^5 \div 10^{12}$

$$\mathrm{Nu_D} = \left[\frac{\mathrm{Pr}}{4 + 9\,\mathrm{Pr}^{1/2} + 10\,\mathrm{Pr}}\right]^{1/5} (\mathrm{Gr_D\,Pr})^{1/4}\ .$$

For laminar natural convection around the surface of a long cylinder with constant wall temperature according authors Fand, Morris and Lum [6] the equation is

$$\mathrm{Nu_D} = 0.474\,\mathrm{Ra_D^{0.25}}\,\mathrm{Pr}^{0.47}\ ,$$

for range of Rayleigh number $3\times10^2\div2\times10^7$ and Prandtl number $0.7\div3090$. For laminar fluid flow with a constant heat flux from the surface wall of the horizontal cylinder is according authors Brdlik, Kuptsova and Malinin [7]

$$\mathrm{Nu_D} = 0.563\,\mathrm{Ra_D^{0.2}}\,\mathrm{Pr^{0.04}}\ , \qquad \mathrm{Gr_D} = 10^3\div10^8\ , \qquad \mathrm{Pr} = 0.01\div100\ .$$

Using the thin boundary layers method by authors Raithby and Hollands [8] for laminar and turbulent fluid flow around horizontal cylinder surface with constant temperature of the wall the equation was obtained as

$$\mathrm{Nu_D^{3.337}} = \left[\frac{2}{\ln\left(1+\frac{\pi}{1.294^{3/4}}\,C_1\,\mathrm{Ra_D^{1/4}}\right)}\right]^{3.337} + (0.72\,C_2\,\mathrm{Ra_D^{1/3}})^{3.337}$$

where C_1 and C_2 are factors depended on Prandtl number

$$C_1 = 0.5\left[1+\left(\frac{0.49}{\mathrm{Pr}}\right)^{9/16}\right]^{-4/9}\ , \qquad C_2 = [0.14\,\mathrm{Pr^{0.084}}, 0.15]\ .$$

In the factor C_2 is chosen the left part in the case of $0.14\,\mathrm{Pr^{0.084}} \le 0.15$ and the right side in the case of $0.14\,\mathrm{Pr^{0.084}} > 0.15$.

3.2. Numerical investigation

Four different diameters of horizontal cylinder commonly available in our marked were used for numerical investigations. For an investigation of the problem by CFD (Computational Fluid Dynamics) the program Fluent version 6.3 was used. Physical 3D model representing real case was placed in gravitational field defined by gravitational acceleration given by $g = -9.807$ [m s^{-2}] in the direction of y-dimension. The model represents the lower parts of stratification builds with limited space around a heat exchanger tube. The basic geometry of the model with boundary conditions is shown in Fig. 1.

An incompressible flow of fluid with physical properties of water was considered with an unsteady implicit algorithm related to the Boussinesq approximation. On the surface

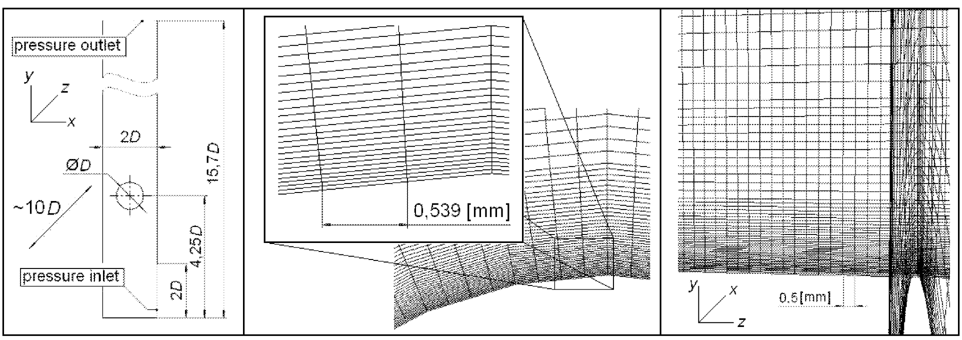


Fig.1: The basic geometry of the model with boundary conditions related to the diameter of the cylinder and with preview of the grid on the cylinder wall (dimensions are chosen for diameter of cylinder $D = 0.022$ [m])

Heat flux $[\mathrm{W\,m^{-2}}]$		ΔT [K]					
		0.5	2	5	10	20	40
D [mm]	20	122.4	696.4	5464.1	5464.1	12692.6	30262.7
	22	119.6	680.0	5160.9	5160.9	12393.7	29550.2
	35	106.5	605.5	4595.3	4595.3	11035.5	26311.7
	50	97.4	553.8	4345.4	4345.4	10094.1	24067.1

Tab.1: Heat fluxes as a boundary condition on the cylinder surface for four different cylinder diameters and six approximate values of thermal differences between every cylinder surface and ambient

of every diameter of horizontal cylinder constant heat fluxes were considered to obtain six approximate values for thermal differences between every cylinder surface and the ambient as is seen in Tab. 1. A viscous model of the fluid flow was computed in laminar regime. Momentum and energetic equations were solved with discretization of second order.

3.3. Post-processing

As results of numerical investigation were obtained data intended for post-processing. A time average of area weighted mean temperature on the horizontal cylinder surface was calculated according

$$\bar{T}_{\mathrm{w}} = \frac{1}{\tau\, S} \int_0^{\tau} \left(\sum_{i=0}^{n} T_i(\tau)\, |S_i| \right) \mathrm{d}\tau \ ,$$

where τ [s] is time, S [m^2] total area of the tube, n is the total number of the grid elements on the tube surface, T_i [K] is temperature of every element and S_i [m^2] is area of every element and where the fluctuation should be equal to zero $\bar{T}'_{\mathrm{w}} = 0$. Under this assumption in every time we can get area-weighted mean of temperature difference between surface of the tube and fluid in ambient

$$\Delta T = T_{\mathrm{w}} - T_{\infty} \ .$$

Fluctuations of area weighted mean of temperature difference in calculation are very well apparent in Fig. 2. They are caused by unsteady fluid flow in the near surrounding of the

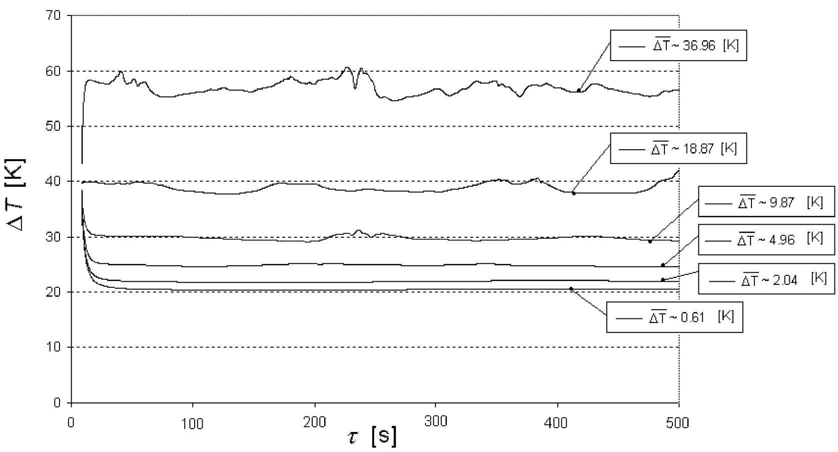


Fig.2: Curves of the area weighted mean of the temperature difference between surface of the tube and fluid in ambient for different heat fluxes (results are chosen for diameter of cylinder $D = 0.022$ [m])

cylinder surface. This has an influence on the local heat transfer coefficients that with boundary condition of constant heat flux affects the local temperature on the surface.

Time averaged of mean Nusselt numbers related to temperature difference as results of the numerical investigation of laminar natural convection on the horizontal cylinder with diameter 0.035 [m] in laminar regime are shown in Fig. 3.

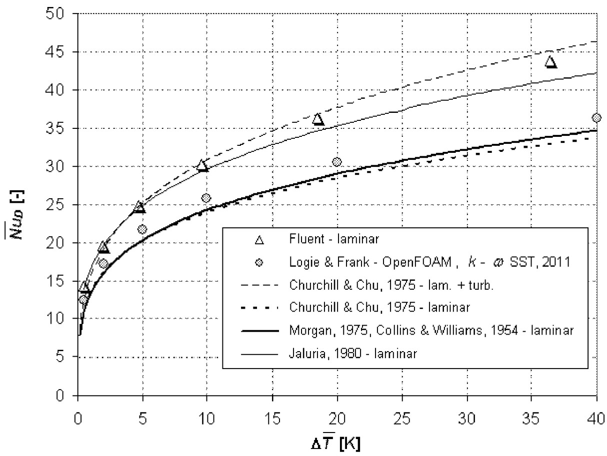


Fig.3: Results of numerical investigations of mean Nusselt numbers relative to temperature difference are compared with correlation equations of chosen correlation equations (results for diameter cylinder of 0.035 [m])

According to the specified boundary condition of the constant heat flux on the surface, heat transfer coefficient can be defined as a ratio of the heat flux and temperature difference between mean area weighted average of surface temperature and the ambient fluid and it can be written as

$$\overline{\mathrm{Nu}_\mathrm{D}} = \frac{1}{\tau}\int_0^\tau \mathrm{Nu}_\mathrm{AW}(\tau)\,\mathrm{d}\tau = \frac{1}{\tau}\int_0^\tau \frac{\dot{q}\,D}{\lambda_\mathrm{f}\,(T_\mathrm{w}(\tau)-T_\infty)}\,\mathrm{d}\tau \ ,$$

where $\mathrm{Nu}_\mathrm{AW}(\tau)$ is an area weighted mean of Nusselt number on the cylinder surface and $\dot{q}$ [$\mathrm{W\,m^{-2}}$] is the defined boundary condition of the heat flux.

Results are compared with correlation equations according [1], [4] and [5] and with results given by authors Logie and Frank [9] who calculated similar case for diameter of horizontal cylinder 0.035 [m] immersed in unlimited space. The physical model was calculated in OpenFOAM using unsteady implicit formulation with two-equation model of turbulent viscosity k-ω SST for low velocity of fluid flow with zero turbulence intensity of the cylinder surface. Similarly they used constant heat flux as a boundary condition on the surface.

3.4. Experimental investigation

In this section is a very brief description of the measurement procedure. For heating of the fluid was used a horizontal copper tube commonly available in our markets manufactured according the standard EN 1057 with outer diameter 0.022 [m] and wall thickness 0.001 [m]. The temperature measurements were realized by five thermocouples (2×0.2 [mm], type 'K') built inside of the tube wall Fig. 4.

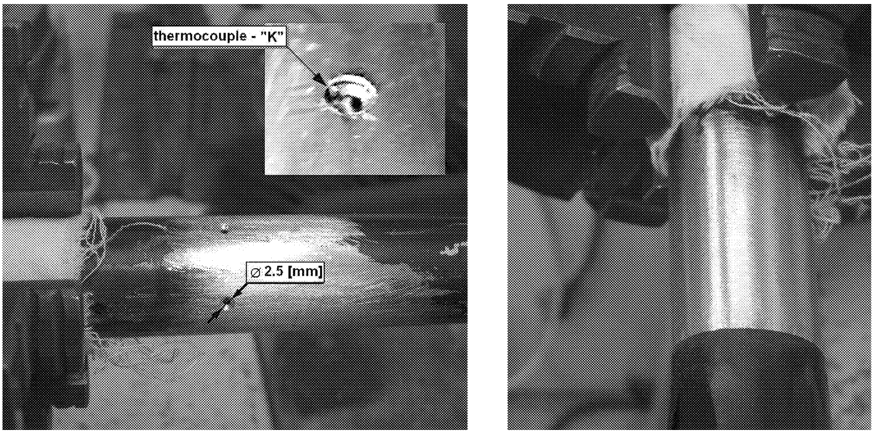


Fig.4: Thermocouples built into the wall of commonly available cooper tube manufactured under standard EN 1057 with outer diameter 0.022 [m] and wall thickness 0.001 [m]

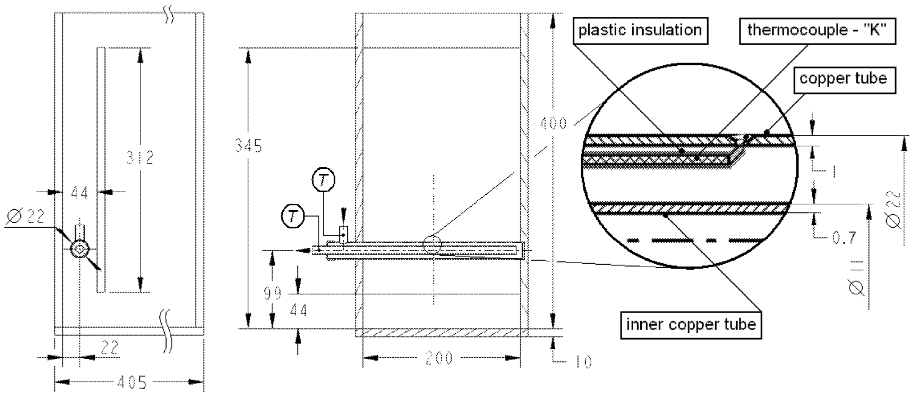


Fig.5: The geometry of the model and the measurement set-up used for experimental determination of the heat transfer coefficient during free convection near the horizontal cylinder

Basic geometry of the experimental device is shown in the Fig. 5. Thermocouples were located on the whole perimeter of the tube and in the middle of the model depth.

Another water circuit was used for the heating of the tube with an inlet and an outlet according the Fig. 5, where other two thermocouples and a rotameter ensured the data for performance counting.

3.5. Used experimental devices

All the thermocouples were calibrated to an accuracy of one tenth of a degree using the calibration device AMETEK-Jofra. To determine the total thermal performance transferred by natural convection from the surface of the cylinder the inlet and outlet temperature of the heating fluid was measured in the centres of inlet and outlet tubes, as is shown in Fig. 5 in the middle. These thermocouples were connected to a universal measuring system ADAM module 5018-A1. Electronic flow meter for heating of the tube FVA915VTHM was connected to the data logger ALMEMO 2390-8. During the measurement the data from flow meter were compared with rotameter OMEGA. As a source of thermal energy was used calorimeter HAAKE K35, the bath temperature was controlled by an accurate mercury thermometer.

4. Results

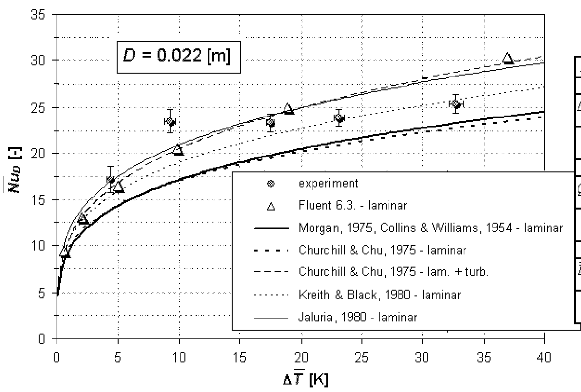
Time average of the area weighted mean Nusselt numbers relative to the horizontal cylinder surface as results of experimental investigation were determined according

$$\overline{\mathrm{Nu}}_\mathrm{D} = \frac{1}{\tau}\int\limits_0^\tau \frac{\dot q\, D}{\lambda_\mathrm{f}\,(T_\mathrm{w}(\tau) - T_\infty)}\,\mathrm{d}\tau \quad\Rightarrow\quad \frac{1}{\tau}\sum_{i=\Delta\tau}^{\tau} \frac{\dot Q_i}{\pi\, l\, \lambda_\mathrm{f}\,(T_\mathrm{w}(\tau)_i - T_{\infty i})}\ ,$$

where τ [s] is time of the process, $\dot Q$ [W] is the heat flux from the cylinder surface, l [m] is length of the tube, $T_\mathrm{w}(\tau)$ [K] is the mean value of the temperature on cylinder surface in every time-step. Thus the time average of Nusselt number could be described as

$$\overline{\mathrm{Nu}}_\mathrm{D} = \frac{\dot m\, c_\mathrm{p}\,(T_\mathrm{in} - T_\mathrm{out})}{\lambda_\mathrm{f}\,\pi\, l\,(\bar T_\mathrm{w} - \bar T_\infty)}\ ,$$

where $\dot m$ [$\mathrm{kg\,s^{-1}}$] is mean value of mass flow of the heating water, c_p [$\mathrm{J\,kg^{-1}\,K^{-1}}$] is the specific heat capacity and T_in and T_out [K] mean inlet and outlet temperatures of the heating water. Experimental results with relative and total errors are summarised in Fig. 6 in the table on the right side. Time average of mean Nusselt number for a diameter of tube 0.022 [m] relative to temperature difference was compared as results of numerical and experimental investigation with chosen correlation equations.



Experimental results:

ΔT [K]	5	10	20	25	40
$\Delta\bar T$ [K]	4.3 ± 0.3	9.3 ± 0,4	17.4 ± 0.3	23.1 ± 0.3	32.7 ± 0.3
Relative error	6.93%	4.78%	1.82%	1.60%	0.92%
$\dot Q$ [W]	28.1 ± 1.3	83 ± 3	153 ± 6	208 ± 9	312 ± 14
Relative error	4,52%	4.09%	4.06%	4.24%	4.33%
$\overline{Nu}_D$ [-]	17.2 ± 0.8	23.3 ± 1.3	23.3 ± 1.0	23.9 ± 1.2	25.4 ± 1.1
Relative error	8.24%	5.49%	4.12%	4.19%	4.10%

Fig.6: The average Nusselt number relative to the temperature difference between cylinder surface and ambient water with results of the experimental investigation

For better overview of results the Nusselt numbers of numerical and experimental investigations were related to the Rayleigh number in the laminar regime $10^4 \leq \mathrm{Ra_D} \leq 10^8$ as is shown in Fig. 7. The graph in Fig. 7 is easy to see an agreement of the results obtained by numerical investigation cases with different diameters of cylinder and different heat flows with the results of correlation equations of authors Churchill and Chu [4] and Jaluria [5].

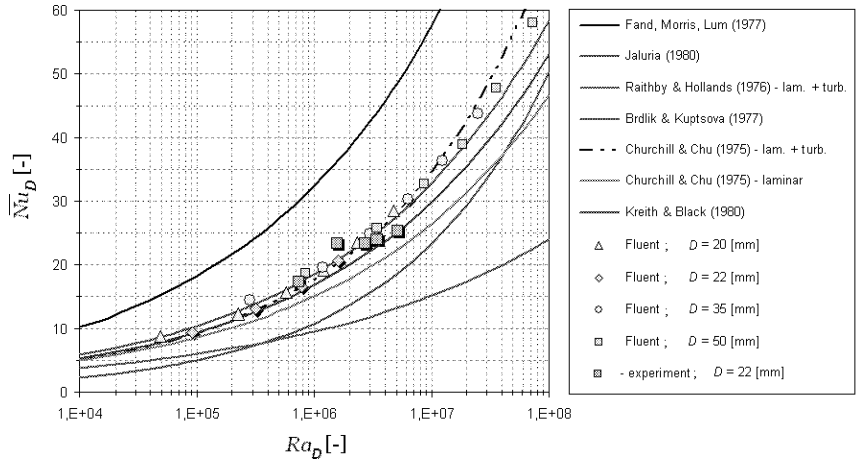


Fig.7: The data of mean Nusselt numbers obtain by the numerical and experimental investigations relative to Rayleigh numbers compared with correlation equations of different authors

5. Discussion and Conclusion

In the numerical investigations of the physical model twenty-four laminar flow calculations have been made for four different diameters of the horizontal cylinder. The experimental investigation has been made for five approximate temperature differences between temperature on the cylinder surface and the temperature of ambient water. In the Fig. 7 are shown results of numerical and experimental investigations compared with correlation equations of chosen authors. Obtained results are within the laminar range for $\mathrm{Ra_D} < 10^8$. All these empirical correlation equations of various authors compute with a constant temperature of the surface, only Brdlik – Kuptsova [7] consider constant heat flux. It should be noted that numerical and experimental results have been obtained for the space limited by rectangular cavity with width corresponding two cylinder diameters. This limitation of space around horizontal cylinder has a significant influence on fluid flow. In experimental investigation have not been processed temperature differences lower than 4 K with regard to the increasing of relative errors. Therefore only measuring of five approximate temperature differences between the cylinder surface and the ambient water has been processed.

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