NUMERICAL MODELING OF THE REINFORCEMENT CORROSION OF RC T-BEAM

Peter Koteš*, Miroslav Brodňan*

The reinforcement corrosion is the phenomenon that highly affects the reliability and durability of reinforced concrete structures. From that reason, a lot of researchers in Slovakia and in the world pay their attention to reinforcement corrosion. In the frame of the research work, the reinforced concrete girder bridges were diagnosed and observed. These bridges are influenced by reinforcement corrosion of main girders. The paper is concerned with detection and simulation of corrosion of steel reinforcement in the reinforced concrete. The cracking response of the reinforced concrete beams due to the corrosion effect of the steel reinforcement was analyzed. The effect of corrosion was simulated by the nonlinear numerical analysis using the program ATENA – 2D and 3D module.

Keywords: reinforcement, corrosion, cracks, T-beam, numerical modeling, concrete

1. Introduction

Reinforced concrete is a versatile, economical and successful construction material. Usually, it is durable and resistant material, performing well throughout its service life. However, sometimes it does not perform adequately as it is expected. It is due to poor design, construction, inadequate materials selection and more severe environment than anticipated or a combination of those factors [1, 2].

The corrosion of reinforcing steel in concrete, due to severe environment, is the phenomenon that highly affects the reliability and durability of reinforced concrete structures. In the frame of the research work of Department of Structures and Bridges at the University of Žilina, reinforced concrete girder bridges were diagnosed and observed. Simultaneously, the bridges were evaluated and their remaining lifetimes were estimated.

In the paper, just two bridges are mentioned. The considered bridges are similar and they are situated in the villages Kolárovice and Topolčianky (Fig. 1). The bridge’s structural system is created by the reinforced concrete single span girder with theoretical span of 10.006 m (Kolárovice) or 13.60 m (Topolčianky). The width of road is 7.51 m and the overall width of bridge is 9.51 m (Kolárovice) and in the case of bridge in Topolčianky, the road width is 6.00 m and the overall width is 7.80 m. The bridge obliqueness is 45° and 80°, respectively.

In the case of bridge near Kolárovice, from the results of the bridge diagnostics follows that the concrete has quality of C30/37 and the beams are reinforced by rebar of the type A (10210) in two layers (5φA30 in the lower layer and 2φA30 in the upper layer). Accordingly, the reinforcement corrosion was indicated. The corrosion caused the diameter loss from the
initial value of 30 mm to the actual average value of 29.3 mm (the minimal measured value is 28.7 mm) and also caused the dropping out the concrete cover (Fig. 2). The concrete cover thickness is 30 mm.

Alike, in the case of bridge near Topolčianky, from the results of the bridge diagnostics follows that the concrete has quality of C16/20 and the beams are reinforced by rebar of the type C (10452) also in two layers (5φC35 in the lower layer and 2φC35 in the upper layer). The corrosion caused the diameter loss from the initial value of 35 mm to the actual average value of 33.99 mm (the minimal measured value is 33.50 mm) and also caused the dropping out the concrete cover. The concrete cover thickness is just 15 mm.

![Fig.1: Diagnosed bridges in villages Kolárovice (a) and Topolčianky (b)](image)

From the bridge evaluation results follows that the remaining lifetime of bridges is about 20 years [3, 4]. From those results, the urgency of bridge reconstruction or strengthening in order to increase its load-carrying capacity is evident.

The reinforcement corrosion of main girders influenced the remaining lifetime severely. The corrosion does not only decrease the reinforcement cross-section, but also causes the cracks and dropping out the concrete cover. It means that the flexural stiffness is decreasing [5, 6, 7]. The corrosive reduction rate (rust) causes the pressure at surrounding concrete by increasing its volume. So, the tension stresses are appearing. The micro cracks are created after exceeding of concrete tension strength. The micro cracks are getting connected into longitudinal cracks by subsequent increasing of corrosive reduction rate volume. The cracks with corrosive reduction rate are able to cause the decreasing of bond between concrete and reinforcement and following dropping out of concrete cover. The same effect occurs also at some places of spandrel beams (Fig. 1).
For the purpose of the better comprehension of crack formation and development, the numerical model of reinforcement corrosion in concrete cross-section was created in computer program ATENA [8].

2. Numerical models of reinforcement corrosion

2.1. 2D numerical model

The girder bridge near Kolárovice was used for numerical modeling. Firstly, the simple numerical model of reinforcement corrosion was created in the 2D module ATENA. Only the cross-section of the T-girder with real dimensions was modeled (Fig. 3).

![Cross-section of T-girder and numerical model](image)

The material model of concrete ‘Concrete-SBETA Material’ with compression strength $f_{cu} = 40.0 \text{Nmm}^{-2}$, derived from CEB-FIP MC 90, was applied for concrete. The basic properties of this material model are: tensile strength, fracture energy and the equivalent uniaxial law. This material model provides objective results due to formulations based on energetic principles and its dependency on the finite element mesh is negligible. The main reinforcement in the RC girder was modeled as a 3D Bilinear Steel von Mises element with yield strength of $f_y = 200.0 \text{Nmm}^{-2}$. Others characteristics are calculated by software ATENA on the basis of redefined formulae. The rigid contacts between concrete and reinforcement were considered.

The concrete part of the cross-section was divided into two parts (two macroelements) – the upper part was divided into elements with dimensions $20 \times 20 \text{mm}$ (the influence of corrosion on cracks was not expected) and the bottom part was divided into elements with dimensions $2 \times 2 \text{mm}$. The reinforcement was also divided into elements with dimensions $2 \times 2 \text{mm}$.

The reinforcement corrosion modeling was very important part of the problem. It was chosen in such a way in modeling that the increase of reinforcement volume acts as load on the T-girder cross-section. In general, the reinforcement cross-section area is decreased due to corrosion, but the corrosion product (rust), on the contrary, increases its volume, what means the increase of the cross-section area of reinforcement.

The increase of reinforcement volume can be given by percentage. The increasing percentage $p$ depends on corrosion type and its products and it reaches the values from 4% to 12%. The value $p = 8\%$ was chosen in this case. However, this is just increasing percentage of the corrosion rate area. Thus, it was needed to find the total percentage of growth.
of corroded reinforcement area and corrosion rate together. The corrosion – increase of full reinforcement cross-section – was modeled using the function ‘shrinkage’ with minus sign (swelling). This way of corrosion modeling was verified in [9, 10].

If the corrosion rate area \( A_{s1,\text{diff}}(t) \) is considered be increasing with time for about percentage ‘\( p \)’, the entire reinforcement area including corrosion rate area (Fig. 4) is equal to

\[
A_{s1,\text{full}}(t) = A_{s1}(t) + A_{s1,\text{diff}}(t) (1 + p) = \frac{\pi}{4} (\phi^2 (1 + p) - \phi^2(t)p) ,
\]

where \( \phi \) is the reinforcement diameter, \( \phi(t) \) is the changed reinforcement diameter in time, and vice versa, it is also possible to derive the new reinforcement diameter taking into account area increasing

\[
\phi^*(t) = \sqrt{\phi^2 (1 + p) - \phi^2(t)p} .
\]

Fig. 4: Change of reinforcement cross-section area due to corrosion

The percentage increase ‘\( p^* \)’ taking into account changing of the whole reinforcement area is given by

\[
A_{s1,\text{full}}(t) = A_{s1}(t) (1 + p^*) \Rightarrow p^* = \frac{A_{s1,\text{full}}(t)}{A_{s1}} - 1 .
\]

The changing of the whole reinforcement area means that the remaining carrying part of reinforcement \( A_{s1}(t) \) decreases and the corrosion rate area \( A_{s1,\text{diff}}(t) \) increases. The initial value of diameter \( \phi = 30.0\text{mm} \), the reduced diameter in time \( \phi(t) = 28.7\text{mm} \) and the percentage \( p = 8\% \) were considered in the numerical model. So, the final increase of reinforcement area is \( p^* = 0.678\% \). For better observing of cracks formation and development, the value \( p^* \) was divided into ten loading steps.

2.2. 3D numerical models with full contact between reinforcement and concrete

The created 2D model is not perfect because it considers the transverse cracks creation and development only. The cracks across the longitudinal axis due to vertical loads cannot be modeled in 2D – combination of cracks due to vertical loads and due to reinforcement corrosion. Thus, the 3D model was created in ATENA to obtain better understanding of the crack formation and development.

The two 3D models were created: model 1 – without transverse stiffeners (just main reinforcement) and model 2 – with transverse stiffeners (using stirrups and reinforcement in a flange). The 3D models of the half length of the girder are shown in Fig. 5.
In this case, the material models of concrete ‘CC3DNonLinCementitious2’ with compression strength $f_{cu} = 40.0 \text{ N mm}^{-2}$ was applied for concrete. The main reinforcement in the RC girder was modeled again as a 3D Bilinear Steel von Mises element with yield strength of $f_{y} = 200 \text{ N mm}^{-2}$. Other characteristics are calculated by software ATENA on the basis of redefined formulae. The rigid contacts between concrete and reinforcement were again considered. The list of used materials is given in Tab. 1.

In the 3D model, the concrete part of cross-section was divided into 11 parts (macroelements) – the upper part was divided into elements with dimensions $200 \times 200 \text{ mm}$ (the influence of corrosion on cracks was not expected) and the bottom 10 parts were divided into elements with dimensions $20 \times 20 \text{ mm}$. The reinforcement was also divided into elements with maximal dimensions $20 \times 20 \text{ mm}$. The reinforcement was modeled as multi-side column (6-side column) because it is not possible to model the circular cross-section in 3D module of Atena.

In this case, the corrosion was considered just in the middle of the span at length 2.0 m, not along the full length. The contact between reinforcement and concrete was modeled as ‘perfect bond’ – full adhesion.

As it was later shown, this model was not also so accurate and eloquent. In this model, the cracks across the longitudinal axis have appeared, but that was also caused due to model of corrosion – using the function ‘shrinkage’. There is the volume increase in all

<table>
<thead>
<tr>
<th>Material</th>
<th>Material element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete C30/37</td>
<td>3D Nonlinear Cementitious 2</td>
</tr>
<tr>
<td></td>
<td>$f_{cu} = 40.0 \text{ N mm}^{-2}$ (C30/37)</td>
</tr>
<tr>
<td>Main reinforcement 5φA30</td>
<td>3D Bilinear Steel Von Mises</td>
</tr>
<tr>
<td></td>
<td>$E = 210 \times 10^3 \text{ N mm}^{-2}$; $f_{y} = 200 \text{ N mm}^{-2}$</td>
</tr>
<tr>
<td>Stirrups φA8 and reinforcement in slab φA10</td>
<td>Reinforcement, bilinear</td>
</tr>
<tr>
<td></td>
<td>$E = 210 \times 10^3 \text{ N mm}^{-2}$; $f_{y} = 200 \text{ N mm}^{-2}$</td>
</tr>
<tr>
<td>Steel plate</td>
<td>3D Elastic Isotropic</td>
</tr>
<tr>
<td></td>
<td>$E = 210 \times 10^3 \text{ N mm}^{-2}$; $f_{y} = 210 \text{ N mm}^{-2}$</td>
</tr>
</tbody>
</table>

Tab. 1: Review of used material characteristics – 3D model
directions \((x, y, z)\) in this model. It means that the volume of reinforcement increases also in longitudinal direction \((x)\), so the cracks across the longitudinal axis have appeared due to corrosion. But real corrosion causes the volume increase just across the longitudinal axis, not along.

### 2.3. 3D numerical models with ‘3D interface’ contact between reinforcement and concrete

Next important part of the work was also modeling the contact between concrete and reinforcement. The material model ‘3D Interface’ was used in Atena. This model is used for contact modeling between two various elements.

The interface material model ‘3D Interface’ is based on Mohr-Coulomb criterion with tension cut off. The constitutive relation for a general three-dimensional case is given in terms of tractions on interface planes and relative sliding and opening displacements and it is given by formula

\[
\begin{bmatrix}
\tau_1 \\
\tau_2 \\
\sigma
\end{bmatrix} =
\begin{bmatrix}
K_{tt} & 0 & 0 \\
0 & K_{tt} & 0 \\
0 & 0 & K_{nn}
\end{bmatrix}
\begin{bmatrix}
\Delta v_1 \\
\Delta v_2 \\
\Delta u
\end{bmatrix},
\]

where \(\tau\) is the shear stress in direction \(x\) and \(y\), \(\sigma\) is the normal stress, \(\Delta v\) is the relatively displacement on surface, \(\Delta u\) is the relatively opening of contact, \(K_{tt}\) is the initial elastic shear stiffness, \(K_{nn}\) is the initial elastic normal stiffness.

The initial failure surface corresponds to Mohr-Coulomb condition with tension cut off

\[
|\tau| \leq c + \sigma \phi \quad \text{for} \quad \sigma \leq f_t,
\]

\[
\tau = 0 \quad \text{for} \quad c > f_t,
\]

where \(c\) is the cohesion, \(\phi\) is the coefficient of friction, \(f_t\) is the tension strength on surface.

After stresses violate this condition, the surface collapses to a residual surface, which corresponds to dry friction (Fig. 6).

![failure_surface][1]

The cohesion \(c\) is equal to surface stresses \(\sigma_{surf}\). From the contact character follows that the contact should assure the slip between concrete and reinforcement to avoid the cracks formation and development – the cracks across the longitudinal axis due to corrosion. Next, the contact should assure the compression between concrete and reinforcement due to corrosion (function shrinkage), but not tension. So, the value of cohesion is equal to zero \((c = 0 \text{ MN m}^{-2})\) and the value of tensile strength is also equal to zero \((F_t = 0 \text{ MN m}^{-2})\).
Moreover, the values of initial elastic normal and shear stiffness are estimated from formulae

\[ K_{nn} = \frac{E}{t} \quad \text{and} \quad K_{tt} = \frac{G}{t} \]  \hspace{1cm} (7, 8)

where \( E \) is minimal elastic modulus, \( G \) is minimal shear modulus, \( t \) is width of interface zone.

The \( K_{nn} \) and \( K_{tt} \) get extremely high values (approaching to infinity) because the width of interface zone \( t \) between concrete and GFRP girder is approaching to zero value. According to [8], it is not recommended to use so high values because they would lead to numerical instabilities during calculation. From this reason, the values \( K_{nn} = 3.0 \times 10^6 \text{ MN m}^{-2} \) (to assure the compression between concrete and reinforcement) and \( K_{tt} = 1.0 \times 10^{-3} \text{ MN m}^{-2} \) (to assure the slip between concrete and reinforcement) were considered.

3. Results of numerical models

3.1. 2D model

In the 2D model, the cracks formation and development were observed in the T-girder cross-section. For this reason, the monitoring points were situated in various places of cross-section. The parameters like cracks width or strains in direction of \( x \) or \( y \) were observed in these points. There were 21 monitoring points in cross-section of numerical model (Fig. 7).

![Monitoring points in cross-section](image)

The crack formation and development in cross-section are shown in Fig. 8. There is possible to see the consecutive cracks development from the bulk towards the edges of the figure. The majority of cracks, with maximum width, occurred just inside. The cracks were connected into edge cracks causing the concrete cover dropping out. Moreover, they caused large failure of concrete inside the concrete cross-section and therefore the bond between concrete and reinforcement was decreased. The main compressive and tensile stresses are shown in Fig. 9.

![Cracks formation and development in loading steps](image)
The crack width at the two monitoring points 15 and 12 in the 2D model is shown in Fig. 10. It can be seen in the figure that the crack width at the monitoring point 15 is monotonically increasing, whereas at the monitoring point 12 the crack width decreases, after a small initial increase. This is due to repartitioning of the compression and tension stresses in the cross-section.

3.2. 3D models

Once again, the monitoring points were situated in various places of the cross-section or in macroelements. The parameters like cracks’ widths or strains and deformations in directions $x$, $y$ or $z$ were observed at these points. In this case, it is also possible to monitor the maximum width of all cracks in one macroelement.

The crack formation and development in cross-section in the middle of the girder of both models are shown in Fig. 11. It is possible to observe the consecutive cracks development in middle of girder – perfect bond.
from the bulk towards the edges like in the 2D model. The majority of cracks with maximum width again occurred just inside. This 3D model also confirms that the cracks are getting connected into edge cracks causing the concrete cover dropping out. The crack pattern is similar to the 2D model (Fig. 8, 11, 12). The crack width development of both models is shown in Fig. 13, 14.

The results also show that 3D model using 3D interface contact is more accurate than the 2D model. The cracks formation and development in 3D model with 3D interface contact is close to 2D model in cross-section, but it is also possible to observe the cracks due to vertical loading.

Using the 3D interface contact between concrete and reinforcement results in decreasing the cracks width (Fig. 13, 14). The difference between cracks width is marked in model 1 (model without stirrups).

**Fig.12: Cracks formation and development – cross-section in middle of girder – 3D interface contact**

**Fig.13: Cracks development – 3D models – perfect bond**

**Fig.14: Cracks development – 3D models – 3D interface contact**
4. Conclusions

The results concerning the reinforcement corrosion numerical modeling are presented in the paper. The influence of reinforcement corrosion on the crack formation and propagation were observed in the cross-section of the T-girder. In the paper was shown that already a small corrosion (percentage of corroded reinforcement area) caused the micro crack formation and propagation inside the cross-section near reinforcement.

Small differences between the 2D and 3D models are probably due to repartitioning of the compression and tension stresses, not only in the cross-section (directions $y, z$ – 2D model), but also in the full girder volume (directions $x, y, z$ – 3D models).

From the results of the 3D models (model 1 - without transverse stirrups, model 2 – with transverse stirrups) follows that the stirrups (transverse stiffeners) did not influence greatly the crack pattern (Fig. 11, 12) at cross-section – in both models were achieved approximately the same crack development. However according to expectation, the stirrups did influence the crack width (Fig. 13, 14). Using the stirrups causes the crack width to decrease in longitudinal direction for about 11% (direction $x$). Nevertheless, the limit crack width was exceeded in both models without using vertical loading induced bending stresses. The crack width in the vertical direction (direction $z$) is not markedly changed.

The micro cracks are getting connected into edge cracks due to corrosion increase, which can lead to concrete cover dropping out. In that case, the sufficient strength and bonding of concrete cover is not ensured. Consequently, using some types of strengthening (e.g. gluing of FRP materials on concrete cover) is limited or is not possible to apply.

Practically, this means that the need to insist on better diagnostics exists, to check the degree of failure of concrete cover and to control the bonding between concrete and reinforcement. Based on correct diagnostics, it is recommended to decide if the existing concrete cover was better to retain or was it preferable to replace it by the new concrete cover.

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References


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