NUMERICAL SIMULATIONS OF ELECTROMAGNET EXPOSED TO VIBRATION

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The article presents a numerical model of a coupled electro-magneto-mechanical system – an electromagnet exposed to vibration of a yoke. Operation of a multi-physical (an electro-magneto-mechanical) model is simulated under different working and excitation conditions and a response of the system is analyzed. Simscape, a tool of MATLAB programming environment, is used for numerical analysis of the problem. It is shown, that there exists a combination of operation parameters, which can lead to a substantial attenuation of the yoke vibration. Furthermore, there exists a critical magnitude of the current, which corresponds to a permanent attraction of the yoke to the electromagnet. An analysis of electromagnet's initialization shows an induction of high voltages in electric circuit, which can damage the electromagnet and need to be avoided by a proper choice of parameters.

Keywords: *vibratory system, electromagnet, multi-physics problem, simulation in MATLAB Simscape, shunt damping, natural frequency shift*

1. Introduction

In the paper, an interaction of the electromagnet with a vibrating dynamical system is analyzed from mechatronic viewpoint, i.e. not only electrical behaviour is of interest, but also the mechanical response. A so-called pot-type electromagnet, used in the analysis (Fig. 1), consists of a vibrating yoke, an iron core and a coil wound around the core, which is connected to the electric circuit. The air gap, a clearance between the core and yoke, serves as a variable reluctance term (variable magnetic resistance), which controls the magnetic flux and so the magnitude of the magnetic force. Due to the non-linear nature of the magnetic force the performance of electromagnet exposed to vibration of the yoke, constitutes a non-linear multidisciplinary mechatronic problem, which needs to be solved from an electrical as well as from a mechanical viewpoint.

According to the Faraday's law [1] a variation of the magnetic flux, e.g. due to a change of the air gap reluctance, induces an alternating voltage in the coil. This voltage forces alternating current to flow through a closed electric circuit. The electrical current generates in the coil an alternating magnetic field, which creates an additional magnetic force counteracting the original movement (Lenz's law) [1]. The extent of the counteracting force is proportional to the induced current intensity, which is controlled by the resistance of the electrical circuit. In this way the mechanical energy of yoke vibration is converted via the

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magnetic field into the electrical energy, and partly dissipated (in the form of heat) in the shunt resistance R_S, connected in series to a DC source (Fig. 1).

A mathematical model is derived in a lumped-parameter form and the mechatronic problem is solved numerically using a MATLAB Simscape programming environment, intended for analysis of multi-domain physical systems [2]. The computation environment is based

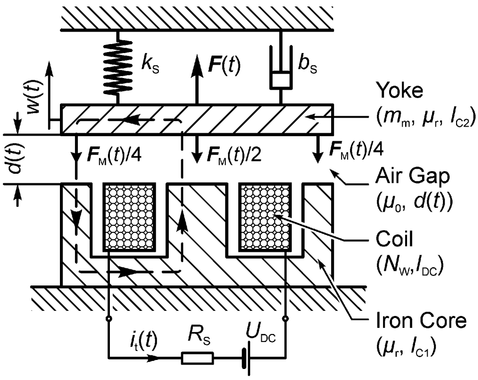

Fig.1: The analyzed electro-magneto-mechanical system

w	Displacement of the yoke
δ_{st}	Static deflection of the yoke
i_t	Total electric current, where $i_t(t) = I_{DC} + i_i(t)$
i_i	Electric current induced in the coil
t	Time
I_{DC}	DC magnetizing current
U_{DC}	DC supply voltage
R_S	Shunt electrical resistance
R_W	Winding resistance of the electromagnet's coil
N_W	Number of turns of the electromagnet's coil
S_C	Cross-sectional area of the electromagnet's core
S_G	Cross-sectional area of the air gap
λ	Magnetic flux linkage
Φ	Magnetic flux
μ_0	Permeability of the free space
μ_r	Relative permeability of the core's material
l_C	Magnetic flux line length in the core and yoke
B	Magnetic field induction
H	Magnetic field intensity
$\Re_G$	Reluctance of the air gap
$\Re_C$	Reluctance of the core
F	External mechanical excitation force
F_M	Magnetic force
d	Air gap width of the electromagnet
d_0	Initial air gap width (before the magnetic force is applied)
d_C	Scaled magnetic flux line length in the electromagnet's core, where $d_C = l_C/\mu_r$
m_m	Mass of the yoke and load
b_S	Damping coefficient of an assumed viscous damper
k_S	Passive spring stiffness

Tab.1: Summary of quantities used in the mathematical model

on the lumped parameter approach in the time domain, i.e. each system is described by a set of constants or time varying parameters. The parameters used are based on a real experimental set-up [3].

2. Mathematical modelling

A mathematical model of the mechatronic system, presented in the article, is composed of three sub-models (Fig. 2). All the quantities used in the derivations are listed in Tab. 1.

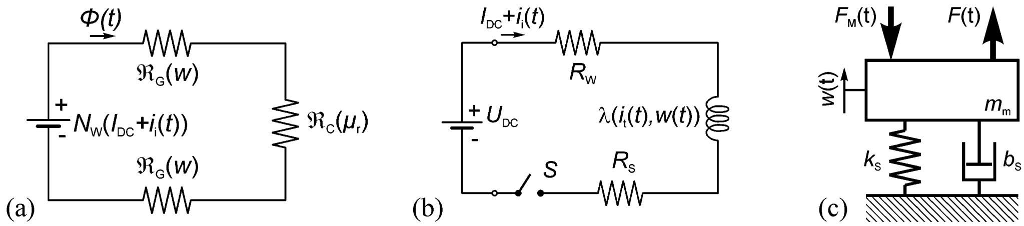

Fig.2: (a) equivalent magnetic circuit; (b) equivalent electric circuit; (c) mechanical system

2.1. The magnetic circuit

The magnetic circuit is composed of three types of lumped elements (Fig. 2a):

– *A source of the magnetic flux* – a coil of $N_{\rm W}$ turns energized by a total current $i_{\rm t}(t) = [I_{\rm DC} + i_{\rm i}(t)]$, where $I_{\rm DC}$ is the magnetizing DC current and $i_{\rm i}(t)$ is the alternating current induced in the coil.

– *An air gap reluctance* ($\Re_{\rm G} = d(t)/(\mu_0\, S_{\rm G})$) – the magnetic resistance of the air gap, where $d(t)$ is a variable width of air gap, μ_0 permeability of free space and $S_{\rm G}$ is the cross-sectional area of the air gap.

– *A core reluctance* ($\Re_{\rm C} = l_{\rm C}/(\mu_{\rm r}\, \mu_0\, S_{\rm C})$) – the magnetic resistance of the iron core and the yoke with an equivalent magnetic flux line length $l_{\rm C}$, relative permeability $\mu_{\rm r}$ and cross-sectional area $S_{\rm C}$.

From Ampere's law [4], the magnetomotive force F_Φ is expressed as:

$$F_\Phi(t) = N_{\rm W}\,[I_{\rm DC} + i_{\rm i}(t)] = \oint_C \vec{H}\cdot {\rm d}\vec{l}\ , \tag{1}$$

where $\vec{H}$ is the magnetic field intensity, $\vec{l}$ is a line vector along the closed magnetic flux line C, denoted in Fig. 1 by a dashed line. From Fig. 1 follows:

$$\oint_C \vec{H}\cdot {\rm d}\vec{l} = H_{\rm C}\, l_{\rm C} + 2\, H_{\rm G}\, d(t)\ , \tag{2}$$

where $l_{\rm C}$ is a length of magnetic flux line through the core and yoke and $d(t)$ is the variable width of the air gap (Fig. 1).

Assuming a constant cross-sectional area of the whole magnetic circuit, the cross-section of the core is the same as the one of the air gap, i.e. $S_{\rm C} = S_{\rm G}$. Noting the relationship between the magnetic flux density (magnetic induction) (B) and magnetic field intensity (H), $H_{\rm C} = B/(\mu_0\, \mu_{\rm r})$ for the core and $H = B/\mu_0$ for the air gap, and using the concept of total electric current, substitution of Eq. (2) into (1) leads to:

$$N_{\mathrm{W}}\, i_{\mathrm{t}}(t) = \left(\frac{l_{\mathrm{C}}}{\mu_{\mathrm{r}}} + 2\, d(t)\right) \frac{B(t)}{\mu_0} \ . \tag{3}$$

Then, the magnetic flux density (B) can be expressed from Eq. (3):

$$B(t) = \frac{\mu_0\, N_{\mathrm{W}}\, i_{\mathrm{t}}(t)}{2\,[d(t) + d_{\mathrm{C}}]} \ , \tag{4}$$

where $d_{\mathrm{C}} = l_{\mathrm{C}}/(2\,\mu_{\mathrm{r}})$, with relative permeability considered to be constant.

2.2. The electric circuit

The source of DC voltage, U_{DC}, is assumed to be ideal (Fig. 2b), i.e. loss-less. In practice, the internal resistance of the voltage source comprises a part of the shunt resistance R_{S}. The winding resistance of the electromagnet's coil, R_{W}, needs to be considered as well. The control of an amount of dissipated energy is done via the shunt resistance R_{S}. Applying the 2$^{\mathrm{nd}}$ Kirchhoff's law for the electric circuit of Fig. 2b, the governing equation can be written as [4]:

$$U_{\mathrm{DC}} = (R_{\mathrm{S}} + R_{\mathrm{W}})\,(I_{\mathrm{DC}} + i_{\mathrm{i}}(t)) + \frac{\mathrm{d}\lambda(d(t), i_{\mathrm{t}}(t))}{\mathrm{d}t} \ , \tag{5}$$

where $\lambda[d(t), i_{\mathrm{t}}(t)] = N_{\mathrm{W}}\, \Phi(t)$ is the magnetic flux linkage. The magnetic flux $\Phi(t)$ can be expressed in terms of magnetic flux density $B(t)$ (Eq. (4)):

$$\Phi(t) = S_{\mathrm{C}}\, B(t) = \frac{\mu_0\, S_C\, N_{\mathrm{W}}\, i_{\mathrm{t}}(t)}{2\,[d(t) + d_{\mathrm{C}}]} \ . \tag{6}$$

Then the governing equation of the electrical system becomes:

$$U_{\mathrm{DC}} = (R_{\mathrm{S}} + R_{\mathrm{W}})\,(I_{\mathrm{DC}} + i_{\mathrm{i}}(t)) + \frac{\mu_0\, S_{\mathrm{C}}\, N_{\mathrm{W}}^2}{2\,[d(t) + d_{\mathrm{C}}]}\, \frac{\mathrm{d}i_{\mathrm{i}}(t)}{\mathrm{d}t} - \frac{\mu_0\, S_{\mathrm{C}}\, N_{\mathrm{W}}^2\,(I_{\mathrm{DC}} + i_{\mathrm{i}}(t))}{2\,[d(t) + d_{\mathrm{C}}]^2}\, \frac{\mathrm{d}[d(t)]}{\mathrm{d}t} \ , \tag{7}$$

i.e. the electrical system is governed by the magnetizing DC current (I_{DC}), as well as induced AC current component $i_{\mathrm{i}}(t)$. Furthermore, the air gap variation $d(t)$ couples the electrical system to the mechanical one.

2.3. The mechanical system

The mechanical system is considered to be constituted as a Single Degree of Freedom (SDOF) oscillatory system (Fig. 2c), which is composed of the yoke of mass m_{m}, hanging on a passive spring with an assumed linear stiffness k_{S} and a viscous damper, described by a damping coefficient b_{S}. The system is excited by an external mechanical force $F(t)$ and subjected to a magnetic force $F_{\mathrm{M}}(t)$, as shown in Fig. 2c. Applying e.g. D'Alembert's principle [5], the equation of motion of the system is:

$$m_{\mathrm{m}}\, \frac{\mathrm{d}^2 w(t)}{\mathrm{d}t^2} + b_{\mathrm{S}}\, \frac{\mathrm{d}w(t)}{\mathrm{d}t} + k_{\mathrm{S}}\, w(t) + F_{\mathrm{M}}(t) = F(t) \ , \tag{8}$$

where $w(t) = d(t) - d_0$ is the displacement of the yoke. The magnetic force $F_{\mathrm{M}}(t)$ is derived using a concept of the Maxwell's pulling force [6, 7]:

$$F_{\mathrm{M}}(t) = \frac{1}{\mu_0}\, B^2(t)\, S_{\mathrm{C}} \ . \tag{9}$$

Substituting the expression for $B(t)$ from Eq. (4) into Eq. (9), we obtain:

$$F_{\mathrm{M}}(d(t), I_{\mathrm{DC}}, i_{\mathrm{i}}(t)) = \frac{\mu_0\, S_{\mathrm{C}}}{4}\, \frac{N_{\mathrm{W}}^2\, i_{\mathrm{t}}^2(t)}{[d_{\mathrm{C}} + d(t)]^2} = \frac{\mu_0\, S_{\mathrm{C}}}{4}\, \frac{N_{\mathrm{W}}^2\, i_{\mathrm{t}}^2(t)}{[d_{\mathrm{C}} + d_0 + w(t)]^2}\ . \tag{10}$$

From Eq. (10), it can be concluded that the magnetic force is dependent on square of the total current $i_{\mathrm{t}}(t) = [I_{\mathrm{DC}} + i_{\mathrm{i}}(t)]$ and inversely proportional to the square of the variable width of the air gap $d(t)$.

Substituting the expression of magnetic force (Eq. (10)) into Eq. (8), the governing equation of the mechanical system becomes:

$$m_{\mathrm{m}}\, \frac{\mathrm{d}^2 w(t)}{\mathrm{d}t} + b_{\mathrm{S}}\, \frac{\mathrm{d}w(t)}{\mathrm{d}t} + k_{\mathrm{S}}\, w(t) + \frac{\mu_0\, S_{\mathrm{C}}}{4}\, \frac{N_{\mathrm{W}}^2\, i_{\mathrm{t}}^2(t)}{[d_{\mathrm{C}} + d_0 + w(t)]^2} = F(t)\ . \tag{11}$$

It can be noticed, that the mechanical system is dependent also on the state-variable of the electrical circuit, the total current $i_{\mathrm{t}}(t)$. Eqs. (7) and (11) are mutually coupled, i.e. the response of mechanical system influences the electrical one and vice versa.

2.4. Limits on operating conditions

The operation of the electromagnet exposed to vibration of the yoke can be controlled by different parameters, especially the initial air gap d_0, the magnetising DC current I_{DC} and the shunt resistance R_{S}.

From the static analysis (i.e. neglecting all dynamic terms in Eq. (11) and defining static deflection $w(0) = -\delta_{\mathrm{st}}$), the equilibrium of spring elastic force and magnetic force is:

$$k_{\mathrm{S}}\, \delta_{\mathrm{st}} = \frac{\mu_0\, S_{\mathrm{C}}}{4}\, \frac{N_{\mathrm{W}}^2\, I_{\mathrm{DC}}^2}{[d_{\mathrm{C}} + d_0 - \delta_{\mathrm{st}}]^2}\ . \tag{12}$$

It means, that there exists a set of parameters for which the electromagnetic force will be larger than the spring elastic force and the yoke will be permanently attracted to the core, i.e. no oscillatory motion of the yoke would be feasible. As was analyzed in detail in [8], there exists a critical value of the DC current I_{crit}, for which the two forces are in equilibrium.

Solving the cubic equation (12), three roots of static deflection δ_{st} are obtained. In general, the roots can be either three purely real or one real and two imaginary. As analysed in more detail in [8], there can be found one double root, which leads to two real roots for $I_{\mathrm{DC}} < I_{\mathrm{crit}}$ and to two complex ones for the opposite interval. Searching for the limit value of the real roots, the critical current is found in the form:

$$I_{\mathrm{crit}}^2 = \frac{16}{27}\, \frac{(d_{\mathrm{C}} + d_0)^3\, k_{\mathrm{S}}}{\mu_0\, S_{\mathrm{C}}\, N_{\mathrm{W}}^2}\ . \tag{13}$$

Detailed discussion on the meaning of the critical current can be found in [8]. For practical applications and for the purpose of the paper $I_{\mathrm{DC}} < I_{\mathrm{crit}}$ is assumed.

3. The MATLAB Simscape model

The electro-magneto-mechanical model derived in the previous section is implemented in the MATLAB Simscape environment [2]. Each element of the model (i.e. mass, stiffness, electrical resistance, inductance, etc.) is represented by a corresponding block. The blocks of the same physical space (mechanical, electrical and magnetic) are interconnected, in order to ensure energy flow between them. The two types of blocks – 'Reluctance Force Actuator' and 'Electromagnetic converter' are used for energy transfer between the mechanical and the magnetic and the electrical and the magnetic domains, respectively (Fig. 3). The model is derived in a general form for more advanced simulations, including non-linear material properties, different mechanical excitation types, shunt resistance variation, etc.

Simscape solves the corresponding sets of differential equations (for each physical domain described by the lumped elements) numerically in the time domain. The MATLAB imbedded robust fixed-step Ordinary Differential Equation solver ode14x is used. The solver

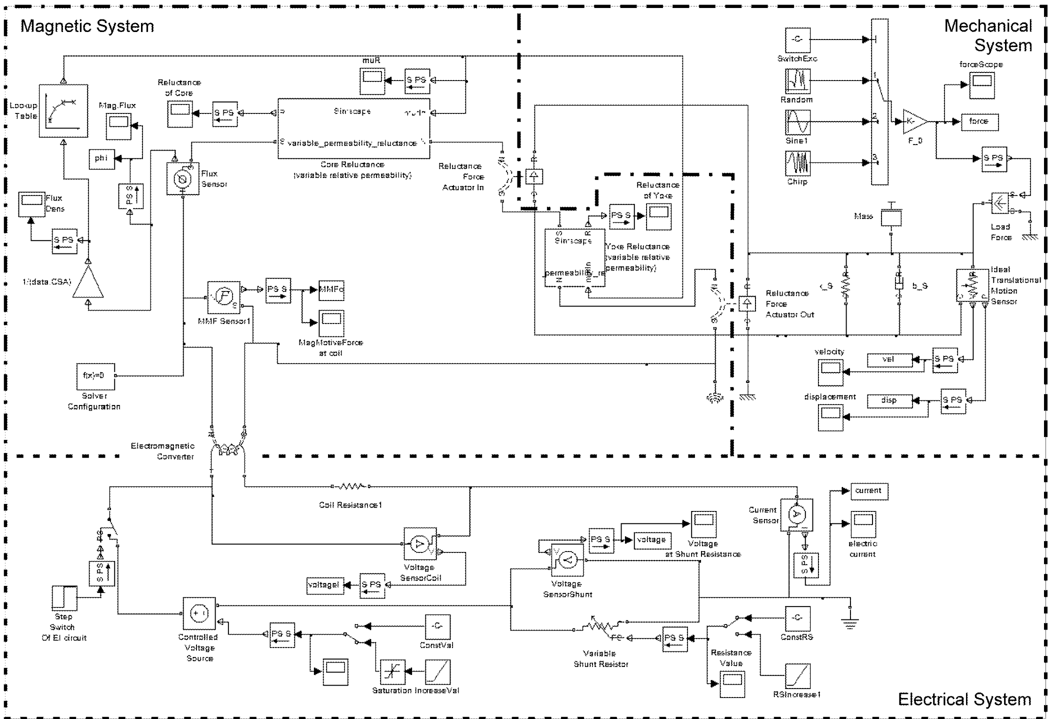

Fig. 3: MATLAB Simscape model

Number of turns	$N_\mathrm{W} = 1880$
Cross-sectional area of the electromagnet's core	$S_\mathrm{C} = 1.772\times10^{-3}\,\mathrm{m}^2$
Magnetic flux line length in the core and yoke	$l_\mathrm{C} = 156.15\times10^{-3}\,\mathrm{m}$
Relative permeability of electromagnet's core material	$\mu_\mathrm{r} = 500$
Initial air gap	$d_0 = 0.75\times10^{-3}\,\mathrm{m}$
Winding resistance of the coil of the electromagnet	$R_\mathrm{W} = 35\,\Omega$
Mass of the yoke	$m_\mathrm{m} = 51.7\,\mathrm{kg}$
Damping coefficient of an damper	$b_\mathrm{S} = 100\,\mathrm{Ns/m}$
Passive spring stiffness	$k_\mathrm{S} = 3.252\times10^{6}\,\mathrm{N/m}$
DC magnetizing supply current	$I_\mathrm{DC} = \{0.1, 0.2, 0.3\}\,\mathrm{A}$
Shunt resistance	$R_\mathrm{S} = \{1, 500, 2\times10^3, 5\times10^3, 2\times10^5\}\,\Omega$

Tab.2: Values used in numerical simulations

ode14x combines Newton's and extrapolation methods to compute the solution in the next time step [2]. The time histories of variables of interest (e.g. deflection of the yoke, current in the electric circuit, voltage across the coil, magnetic flux) are stored into a workspace and are later post-processed using a user-defined script for FFT analysis (with spectrum averaging).

From preliminary analysis [3], the natural frequency of the mechanical system is identified around 40 Hz. Since the simulations are done in the time domain and we are interested in frequency domain response, a fixed step size needs to be chosen in order to be able to do FFT transformation easily. For bandwidth 100 Hz and resolution 0.25 Hz, the time step should be at least $\Delta t = 1\times10^{-2}$ s and time record of length $t_{\mathrm{s}} = 4$ s. For better resolution in the time domain a fixed time step $\Delta t = 1\times10^{-3}$ s is used. For transient analysis simulation time $t_{\mathrm{s}} = 7$ s is used; while for the sweep excitation $t_{\mathrm{s}} = 20$ s. The parameters of the respective blocks of the Simscape model are estimated based on experiments, described in [3] and are listed in Tab. 2.

The simulations are done under three types of mechanical excitation:

- *Random noise* – to identify and locate the resonances.
- *Tonal sine wave* – i.e. pure sine excitation – for simulations of a constant machine operation.
- *Chirp/sweep sine* – i.e. the sine wave with frequency linearly increased in time – for run-up analysis.

4. Discussion of the numerical simulations

The numerical simulations are performed using specific parameters estimated from the set-up analyzed in [3], and listed in Tab. 2. For such a set of parameters, the critical current (i.e. limit in magnetizing DC current defined by Eq. (13)) is $I_{\mathrm{crit}} = 0.427$ A. To avoid permanent attraction during oscillatory motion, the simulations are executed for the current values up to $I_{\mathrm{DC}} = 0.3$ A.

4.1. Random noise excitation

As shown in previous articles using analytical tools [9, 10], the electromagnetic controller behaves as a spring element with variable stiffness (i.e. it is capable of detuning the original natural frequency of the mechanical system) and as a damper (i.e. reducing the amplitude of the vibration in resonance). In order to investigate this behaviour and identify location of resonances, the model is excited by mechanical forcing in the form of stationary random (white) noise. As can be seen from transfer functions plotted in Fig. 4a, increasing the value of the DC current energizing the coil (parameter I_{DC}), more damping is introduced. Furthermore, detuning, i.e. a change in the natural frequency of the mechanical system, is observed as well. From Fig. 4b, it can be concluded that depending on operation conditions, three operation regimes can be distinguished [9, 10]:

- *Pure damping* (low values of resistance – $R_{\mathrm{S}} < 2\,\mathrm{k}\Omega$) – the amplitude of vibration in resonance is reduced. Furthermore, a negligible shift of the system's natural frequency is observed as well.
- *Detuning and damping* (optimal value of resistance – approx. $R_{\mathrm{S}} \approx 2\,\mathrm{k}\Omega$) – a reduction in amplitude of vibration in resonance as well as shift in position of the resonance (damped natural frequency) are introduced.

– *Pure detuning* (large value of resistance – $R_S > 2\,\mathrm{k\Omega}$) – a reduction of amplitude of vibration in resonance is negligible. However, a significant shift in natural frequency is noticed.

Depending on practical applications, the optimal shunt resistance can be chosen accordingly. Following analysis is focused into a region of maximal damping. Therefore, the case of very high resistance is not considered and the shunt resistance values up to the order of $\mathrm{k\Omega}$, interesting from practical view point, are used.

4.2. Harmonic excitation

To simulate the initialization of the controller during periodic operation of e.g. a rotating machine, the yoke is exposed to harmonic force excitation. In time $t = 5\,\mathrm{s}$ the controller is initialized and its response is observed. As seen from Fig. 5a, the shunt resistance controls not only amplitude of vibration of steady state (damping) but also 'smoothness' of the initialization. Note a slight departure of the middle position in the downward direction after switching on the DC current due to the constant magnetic force caused by the I_{DC} component in Eq. (10).

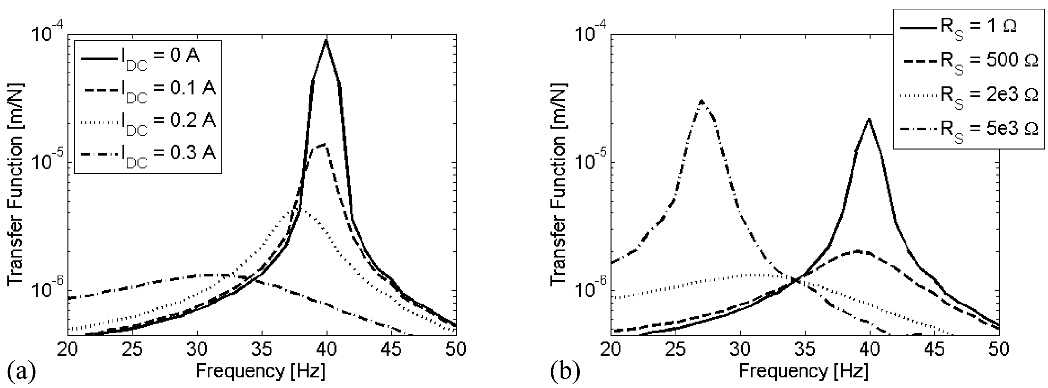

Fig.4: Transfer function for random noise excitation : (a) variation of current, assuming constant shunt resistance $R_S = 2{\times}10^3\,\Omega$; (b) varying resistance, for constant magnetizing current $I_{DC} = 0.3\,A$

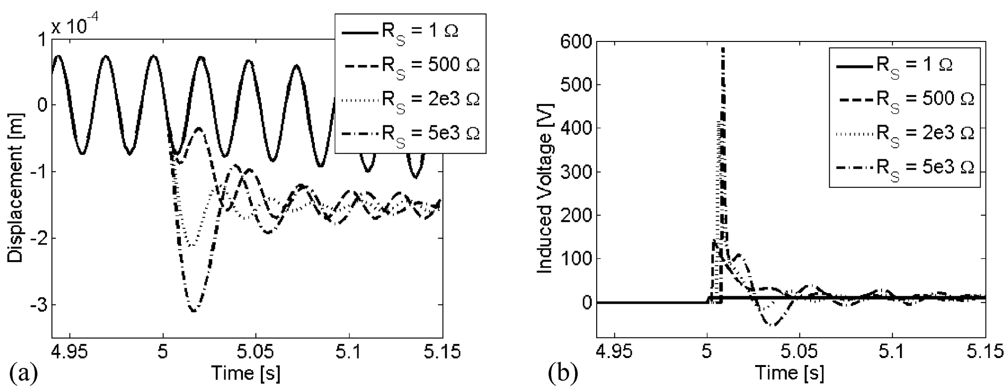

Fig.5: Harmonic excitation, initialization of the controller at $t = 5\,s$ for $I_{DC} = 0.3\,A$ and a variable shunt resistance : (a) displacement response; (b) voltage induced in the coil

From application viewpoint, it is very important to analyze the induced voltage surge at the sudden initialization. As presented in Fig. 5b, an incorrect design of shunt resistance circuit (especially a high value of R_S), can lead to the coil damage caused by excessive peak of induced voltage. This result points out to practical limits in a choice of shunt circuit properties.

4.3. Sweep sine excitation

As discussed in Sec. 4.2, a sudden initialization of the controller can cause dangerous transients especially in the electrical circuit (Fig. 6a and c). In order to avoid this problem, a continuous change of operation parameters during a sine sweep excitation (e.g. by electric current variation) can be applied as shown in Fig. 6b and d. The transient voltage induced in the electric circuit during initialization is distributed more smoothly in time when continuous change of operation conditions is applied (Fig. 6d). No voltage surges are present. This is a less hazardous way of switching-in the electromagnetic system than its sudden engagement.

In Fig. 6b, the two phenomena analyzed in the previous section can be seen – i.e. damping (amplitude reduction as current value I_{DC} and/or shunt resistance R_S increases) and detuning (the peak of vibratory amplitude occurs in different time, i.e. at different excitation frequency). The simulations using sweep sine excitation also show how the electromagnet can

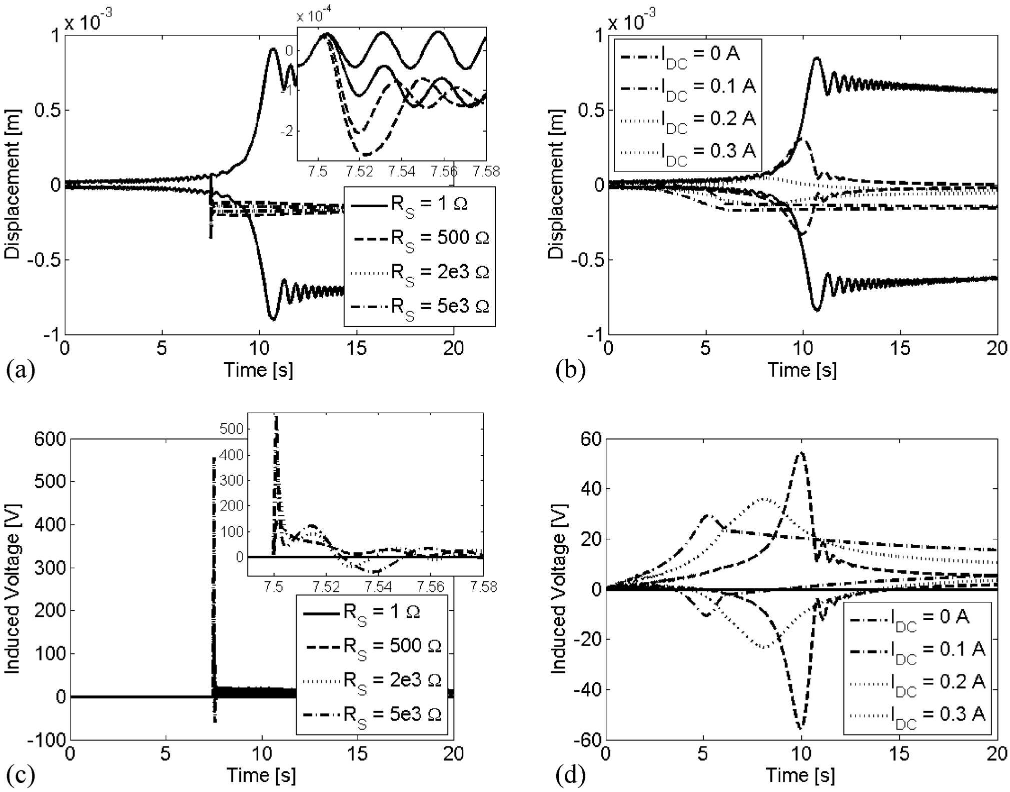

Fig.6: Amplitude envelopes for sweep sine excitation (frequency change of sine excitation from 20–50 Hz within 20 s): (a) and (c) a sudden initialization of the controller; (b) and (d) smooth, continuous increase of the supply voltage, where for (a) and (c) $I_{DC} = 0.3\,A$ is kept constant and in (b) and (d) a $R_S = 1{\times}10^3\,\Omega$

be effectively used to suppress the vibration during machine start-up and reduce the amplitude of vibration when passing resonance, which is interesting from the practical viewpoint.

5. Conclusions

A lumped parameter model of a mechatronic system – an electromagnet with a yoke exposed to vibratory motion – is derived and analyzed numerically using MATLAB Simscape environment.

It is shown that the multi-physical numerical modelling in the time domain confirms predictions obtained using analytical tools, with their inherent simplification and linearisation [9, 10]. There exists a combination of parameters (air gap width d_0, magnetising DC current I_{DC}, shunt resistance magnitude R_S) for which the attenuation of vibratory motion is maximal.

Furthermore, the numerical analysis of the electromagnet switching-in during yoke vibration demonstrates that a large voltage surge in the electric circuit may evolve, if the parameters are chosen in an improper way. It can damage the winding isolation and lead to a failure of the electromagnet. Thus, the Simscape simulation can be used to investigate the limitations of real set-up, as well as its transients prior to the hardware design.

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References

[1] Cheng D.K.: Field and Wave Electromagnetics, Addison Weseley 1989, Reading
[2] Mathworks: web page of MATLAB Simulink Simscape, on-line available at: http://www.mathworks.com/products/simscape/
[3] Darula R.: Multidisciplinary analysis and experimental verification of electromagnetic SAVC, Master's thesis, Aalborg University, 2008, Aalborg
[4] Krause P.C., Wasynczuk O.: Electromechanical motion devices, McGraw-Hill 1989, New York
[5] Rao S.S.: Mechanical Vibrations, 4^{th} ed. Pearson 2004, Upper Saddle River
[6] Furlani E.P.: Permanent Magnet and Electromechanical Devices: Materials, Analysis, and Applications. Academic Press 2001, San Diego
[7] Mayer D., Ulrych B.: Elektromagnetické aktuátory (Electromagnetic actuators – in Czech), BEN 2009, Prague
[8] Stein G.J., Darula R., Chmurny R.: A Clamped-Clamped Beam Sag Limits Under Influence of Static Magnetic And Electric Force, Engineering MECHANICS 18 (2011), No. 5/6, p. 323–329
[9] Stein G.J., Darula R., Sorokin S.: Control of transversal vibrations of a clamped-clamped beam by a permanent magnet and shunt circuit, In: Proceeding of the 8th Int. Conf. on Structural Dynamics 'EURODYN 2011', KUL 2011, Leuven, p. 1735–1741
[10] Darula R., Stein G.J., Sorokin S: An Application of Electromagnetic Induction in Vibration Control, In: Proceeding of the 10th Int. Conf. on Vibration Problems, Springer Science, 2011 Prague – London, p. 447–453

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