PREDICTION TECHNIQUE OF STABILITY IN BLADE ASSEMBLIES AGAINST SUBSONIC FLUTTER

A. P. Zinkovskii*, V. A. Tsymbalyuk*

The basic principles of the experimental-and-computational technique for predicting the subsonic flutter stability of axial compressor blading are stated. The methodology is described for the experimental determination of non-stationary aerodynamic forces and moments acting on blades during their in-flow vibrations; the calculation of the dynamic stability of a blade assembly against flutter; the aerodynamic rig design and peculiar features of its components to perform testing of airfoil cascades. The results of testing of the developed experimental-and-computational complex are presented.

Keywords: blade assembly, modeling, airfoil cascade, subsonic flutter, aeroelastic stability prediction

1. Introduction

The most dangerous type of vibrations of axial compressor blades of the modern aircraft gas-turbine engines (AGTE) is flutter. The most susceptible to flutter are compressor blade assemblies, especially under conditions close to separation of the flow from the blades. That is why expenses related to ensuring their vibration reliability take a significant amount of general time and money, which are spent on development of AGTE.

Full-scale engine testing for blade stability against flutter is very expensive, therefore, there are many methods for the prediction of flutter of blade assemblies, which are based on finding unsteady aerodynamic loads (forces and moments) acting on blades during their vibration in a flow. Modern numerical methods based on the solution of the Reynolds averaged Navier-Stokes equations make it possible to determine the indicated loads considering the medium viscosity, however, the accuracy of their determination depends on the selection of turbulence models.

In the paper it is proposed to use the experimental and computational technique, which involves the experimental determination of aerodynamic loads acting on blades during their vibration and computation determination of the flutter stability limit by calculating the natural frequencies of coupled vibrations of blades in a flow using the obtained experimental data.

2. Unsteady aerodynamic loads measurement procedure

The most susceptible to subsonic flutter are the first flexural and torsional modes of blade vibrations. As far as such modes of vibrations are concerned, virtually all exchange of

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energy with flow is implemented in the peripheral section of the blades where the vibration amplitudes are the largest.

That is why during analyzing their dynamic stability against flutter it can be sufficient to determine aerodynamic loads for peripheral section, and these values can be regarded as constant along the blade length provided that the flow surfaces are close to cylindrical.

Considering the above-mentioned, the experimental investigations on determining aerodynamic loads have been performed using the straight cascade of airfoil models shown in Fig. 1a. To various vibration modes of the blade assembly there correspond different combinations of translational \( (x, y) \) and angular \( (\alpha) \) displacements of airfoils, while aerodynamic loads acting on them can be represented in terms of forces \( L_A, K_A \) and moment \( M_A \), as is seen in Fig. 1b, where \( \beta \) is the stagger angle; \( t_s \) is the cascade spacing, \( b \) is the airfoil chord, \( V \) is the relative flow velocity ahead of the blade assembly, \( i \) is the angle of attack.

![Fig.1 Straight cascade of airfoils (a) and scheme of acting aerodynamic loads (b)](image)

Based on the test results presented in [1], the following assumptions can be introduced:
- The influence of translational displacements \( x \) on aerodynamic loads is slight;
- The force \( K_A \) can be neglected, since it is small;
- The effect of other cascade airfoils on a given airfoil decreases very quickly as the distance to them increases.

From this it can be concluded that:
1) an airfoil can be simulated by the system with two degrees of freedom;
2) the total unsteady aerodynamic load on a chosen cascade airfoil depends on its natural vibrations and vibrations of some nearest adjacent airfoils, which are called to be determining and denoted as \( n \). By accounting for the results from [2], it is sufficient to consider the influence of no more than five determining airfoils on a given airfoil \((-2 \leq n \leq 2)\) with stalled flow regime and of no more than three determining airfoils with unstalling flow regime \((-1 \leq n \leq 1)\).

The procedure for experimental determination of aerodynamic loads, which basic statements are presented in [3, 4, 5], has been improved. The procedure described makes it possible to measure the force \( L_A \) and moment \( M_A \) simultaneously at arbitrary combinations of translational \( y \) and angular \( \alpha \) displacements of the objects under study. With this aim in view, excitation of vibrations of the elastic suspension is performed and its scheme is
shown in Fig. 2. In accordance with the Ampere law, to determine aerodynamic forces and moments, it is required to measure the current in moving coils of the vibrator both with flow and without it, moreover, the specified airfoil vibrations are maintained unchanged.

Fig. 2: Scheme of the elastic suspension of the airfoil: 1 – cross beam; 2 – moving coil of the vibrator; 3 – airfoil under study

Fig. 3: Elastic suspension: 1 – airfoil, 2 – moving coils of the vibrator, 3 – cross-beam, 4 – elastic elements, 5 – displacement strain gages; 6 – aerodynamic washer; 7 – attachment points for calibration masses; 8 – small adjusting masses

Reduction of the error in determining aerodynamic loads from the difference between two measurements and enhancement of the sensitivity of the measuring device can be achieved, if the following conditions are fulfilled:

1) The center of mass of the elastic suspension should coincide with the rotation axis, i.e. \( x_m = 0 \). In this case the flexural and torsional vibrations of the elastic suspension will be mechanically uncoupled, which makes independent control of vibrations easier;

2) Natural frequencies of flexural \( \Omega_y \) and torsional \( \Omega_\alpha \) vibrations of the elastic suspension should be equal to the operating frequency of the excitation \( \omega \).
3) Mechanical damping of vibrations should be insignificant. At the same time, the elastic suspension of the given airfoil should be vibroinsulated from the bench test and elastic suspensions of other airfoils.

Apart from the above-mentioned conditions, the relation \( k_\omega = \Omega_1/\omega \), where \( \Omega_1 \) is the first natural frequency of the airfoil vibrations, which is, as a rule, flexural, should be sufficiently high to achieve similarity between vibration amplitudes along the airfoil. For instance, the relation between the maximum and minimum amplitudes of the airfoil displacement should not exceed 1.065 at \( k_\omega \geq 4 \).

The structure of elastic suspension in Fig. 3 has been designed based on the requirements to procedure of measurement of unsteady aerodynamic loads. It comprises two elastic elements 4 of different width. The auxiliary (narrow) elastic element does not impede the twisting of the main (wide) element about its own longitudinal axis, and during flexural vibrations these elements form an elastic parallelogram, which ensures the equal vibration amplitudes along the airfoil length 1 upon the fulfillment of the condition \( k_\omega \geq 4 \).

Measurement of aerodynamic loads, which act on the airfoils vibrating in a flow, is performed using the developed automated test bench, its special features are presented in Tab. 1.

<table>
<thead>
<tr>
<th>No.</th>
<th>Feature</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mach number</td>
<td>no more than 0.75</td>
</tr>
<tr>
<td>2</td>
<td>Angles of attack</td>
<td>(-15^\circ \ldots + 20^\circ)</td>
</tr>
<tr>
<td>3</td>
<td>Airfoil length</td>
<td>70.0 mm</td>
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<tr>
<td>4</td>
<td>Number of airfoils</td>
<td>7 \ldots 13</td>
</tr>
<tr>
<td>5</td>
<td>Number of active airfoils</td>
<td>max. 4</td>
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<tr>
<td>6</td>
<td>Vibration modes of active airfoils</td>
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<tr>
<td>7</td>
<td>Aerodynamic loads to be measured</td>
<td>forces and moments</td>
</tr>
<tr>
<td>8</td>
<td>Operating frequency</td>
<td>up to 250 Hz</td>
</tr>
</tbody>
</table>

Tab.1: Special features of the test bench

3. Procedure for determining the flutter stability limit using the experiment data

The linear aerodynamic loads on the initial airfoil are proportional to small vibrations of the airfoils \( y \) and \( \alpha \) and can be presented as a sum of forces and moments caused by individual components of displacements of the considered airfoil and its neighbouring airfoils:

\[
\bar{L}_A = q b \sum_{n=-2}^{2} \left( l_{ny} \frac{y_n}{b} + l_{n\alpha} \alpha_n \right), \quad \bar{M}_A = q b^2 \sum_{n=-2}^{2} \left( m_{ny} \frac{y_n}{b} + m_{n\alpha} \alpha_n \right),
\]

where \( q \) is the velocity pressure, \( l_{ny} \ldots m_{n\alpha} \) are the dimensionless coefficients of proportionality or aerodynamic influence coefficients (AIC), which are complex, i.e. they express the phase shift between the applied force (moment) and its resulting displacement.

To determine the AIC from Eqs (1) it is required to measure the aerodynamic force \( \bar{L}_A \) and moment \( \bar{M}_A \) for only several linearly-independent combinations of vibrations \( y \) and \( \alpha \). To calculate the stability of the blade assembly, it is required to obtain the AIC matrix for different angles of attack \( i \) and reduced frequencies \( K = \omega b/V \), where \( \omega \) is the frequency of airfoil vibrations.
As a rule, the AIC values are reduced to the axis of angular displacements, which is at the middle of the chord. It is known that flexural mode of vibrations of twisted blade with asymmetric airfoil is accompanied by rotation of its cross section, relative to the point \( O \) as it is shown in Fig. 4.

\[ \bar{m}_{n\alpha} = m_{n\alpha} + m_{ny} \frac{X_t}{b} + l_{na} \frac{X_t}{b} + l_{ny} \left( \frac{X_t}{b} \right)^2. \]  
\[ (2) \]

However, if to consider that \( Y_t = 0 \), recalculation of the AIC values can be performed for vibrations and can be reduced relative to the point \( O \), as it is shown in [1]:

\[ \tilde{m}_{n\alpha} = m_{n\alpha} + m_{ny} \frac{X_t}{b} + l_{na} \frac{X_t}{b} + l_{ny} \left( \frac{X_t}{b} \right)^2. \]  
To determine the stability limit of the blade assembly against flutter, the following assumptions are made:
- there is no mechanical coupling between the blades;
- the inertia and elastic forces of a blade are much higher than the aerodynamic forces, i.e. vibration modes in a flow and in vacuum differ slightly;
- each blade has one degree of freedom and the blade assembly consisting of \( N \) blades has \( N \) degrees of freedom.

The equation of motion of the chosen discrete model of the blade assembly can be written as:

\[ I_k \varrho_k \ddot{\alpha}_k + g_{k\alpha} \dot{\alpha}_k + k_{k\alpha} \alpha_k = \tilde{M}_k , \quad k = 1, 2, \ldots, N. \]  
\[ (3) \]

Here, \( I_k, g_{k\alpha}, k_{k\alpha} \), are the generalized volume moment of inertia relative to the point \( O \) and coefficients of mechanical resistance and stiffness of the blades, respectively; \( \varrho_k \) is the density of the blade material; \( \alpha_k \) are the complex generalized displacements of blades that correspond to the displacements of their finite cross sections; \( k \) is the blade number in the assembly; \( \tilde{M}_k \) is the generalized aerodynamic moment acting on each blade in the assembly during their vibration:

\[ \tilde{M}_k = q_h b_h^2 \sum_{n = k-2}^{k+2} \tilde{m}_{(n-k)\alpha} \alpha_n , \]  
\[ (4) \]

where the generalized AIC obtained considering the vibration mode \( \varphi \), that characterizes the distribution of amplitudes along the blade, i.e. along the axis \( z \) in vacuum, are as follows:

\[ \tilde{m}_{(n-k)\alpha} = \frac{1}{q_h b_h^2} \int_0^h q(z) b^2(z) \tilde{m}_{(n-k)\alpha}(z) \varphi^2(z) \, dz . \]  
\[ (5) \]

The index \( h \) denotes that velocity pressure and chord relate to the finite section of the blade \( z = h \), where \( h \) is the blade airfoil portion height.
The eigenvalues $\lambda$ make it possible to determine the stability coefficient $\delta$ for the blade assemblies against flutter:

$$
\delta = \max \text{Im}(\lambda).
$$

(6)

If the inequality $\delta < 0$ is true, the blade assembly is stable against flutter and its stability limit meets the condition $\delta = 0$.

4. Implementation of the experimental and computational complex for predicting the stability of blade assemblies against subsonic flutter

As an example, Fig. 5 provides the results of determining the stability limits of blades for GTE compressor wheel. At first, the dependencies of the stability factor on the reduced frequency of vibrations $K$ were determined. Then, using these dependencies critical values of $K$ were obtained, at which $\delta = 0$, i.e. they correspond to the stability limit of the blade assembly.

From the given data it can be concluded that with flexural vibrations operating conditions of the compressor under study are in the region of stability far from the flutter limit. At large angles of attack (both positive and negative) stability limit shifts in the direction of higher frequencies of vibrations.

The operation frequency decreases with an increase in the angle of attack and approaches the stability limit, i.e. with an increase in the angles of attack stability margin of the torsional mode of vibration of the blade assembly decreases.

![Fig.5: Dependence of critical reduced frequency of vibrations $K$ on the angle of attack $i$ and for positions of axis of rotation of the blade sections $X_i/b = 3$ ($\Delta$); □ – dependence of the reduced frequency for flexural mode of vibrations of full-scale blade under operating conditions of GTE](image)

5. Conclusions

The test bench, which has been developed at the G. S. Pisarenko Institute for Problems of Strength, makes it possible to determine aerodynamic loads (forces and moments) acting on blades in a flow by implementing unique experimental and computational complex and
based on their use to predict stability of AGTE compressor blade assemblies against subsonic flutter over a wide range of variation of their mechanical parameters and flow characteristics. Moreover, it should be noted that the developed complex is a fully automated system with modern measurement and computer facilities.

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