EFFECT OF SOFTENING FUNCTION TYPE IN THE DOUBLE-K FRACTURE MODEL: CONCRETE SPECIMENS WITH AND WITHOUT POLYPROPYLENE FIBRES

Ivana Havlíková*, Hana Šimonová*, Tomáš Pail*, Eva Navrátilová**, Romana Viktória Majtánová*, Zbyněk Keršner*

Cement-based composites are traditionally a commonly used material in civil engineering structures. The basic representative of this type of material is concrete, a quasi-brittle composite in which crack resistance can be achieved by the addition of fibres. The double-K fracture model can be used to calculate the fracture-mechanical parameter values of structural concrete with and without polypropylene fibres. This model combines the concept of cohesive forces acting on the crack length with a criterion based on the stress intensity factor, using a 'softening function' to determine the cohesive part of fracture toughness. In this paper, authors determine the effect of the type of this softening function on the evaluation of fracture tests performed on sets of concrete specimens with and without polypropylene fibres.

Keywords: *double-K fracture model, softening function, concrete, polypropylene fibre, fracture test*

1. Introduction

Concrete, a so-called quasi-brittle material, is a commonly used building material. Its range of applications can be extended using various additives, e.g. polypropylene fibres. Even relatively small volume quantities of these fibres in concrete mixture (1–3 %) can affect the resistance of the composite to crack propagation.

In the study of properties of existing or newly developed cement-based composites the fracture parameters (fracture toughness, fracture energy, tensile strength etc.) have to be quantified. The determination of these parameters is based on standardized fracture experiments on specimens with stress concentrators (typically the three-point bending test, performed on notched beams, or the wedge-splitting testing of compact notched specimens). Subsequently, the results of these experiments in the form of diagrams showing load-deflection or load versus crack mouth opening displacement are evaluated by direct or indirect methods using one of the many fracture models.

In this paper, the double-K fracture model (from the pilot papers [1–4] and further works up until e.g. the summarizing book [5]) is used. In principle, this model combines the concept of cohesive forces acting on the faces of the fictitious (effective) crack increment with a criterion based on the stress intensity factor. This model can determine the critical crack

* Ing. I. Havlíková, Ing. H. Šimonová, Ing. T. Pail, R. V. Majtánová, prof. Ing. Z. Keršner, PhD., Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics, Veveří 331/95, 602 00 Brno, CZ

** Ing. E. Navrátilová, Brno University of Technology, Faculty of Civil Engineering, Institute of Chemistry, Veveří 331/95, 602 00 Brno, CZ

tip opening displacement and the fracture toughness and is capable of describing different levels of crack propagation: an initiation part, which corresponds to the beginning of stable crack growth (at the level where the stress intensity factor, $K_{\mathrm{Ic}}^{\mathrm{ini}}$, is reached), and a part featuring unstable crack propagation (after the unstable fracture toughness, $K_{\mathrm{Ic}}^{\mathrm{un}}$, has been reached).

An evaluation of three-point bending tests using the double-K fracture model is presented in this paper, with a principal focus on the effect of softening function type in this model for concrete with and without polypropylene fibres.

2. Fracture testing of concrete specimens

2.1. Material

Fresh concrete mixture was prepared from heavy-weight aggregates of 0–4 mm and 4–8 mm fractions, CEM I – 42.5 R cement, fly-ash, plasticizer, water and stabilizer. The water and stabilizer were dosed by volume, the remaining components by weight. Four mixtures were made: OB_REF, OB_FF19, OB_FF38 and OB_FF54. The reference mixture (OB_REF) was made without fibres, while the mixtures OB_FF19, OB_FF38 and OB_FF54 included FORTA FERRO polypropylene fibres of 19 mm, 38 mm and 54 mm length, respectively. The composition of the fresh concrete mixtures is given in Tab. 1.

Component	Unit	Quantity per 1 m^3
Cement 42.5 R	kg	340
Fly ash (Třinec)	kg	80
DTK 0–4 mm (Zaječí)	kg	784
HTK 4–8 mm (Tovačov)	kg	920
Plasticizer Sika Viscocrete 1035	kg	5
Water	l	180
Stabilizer (0.4 % of cement dosage)	l	1.6
Fibres Forta Ferro selected lengths; REF	kg	9; 0

Tab.1: Composition of fresh concrete mixture

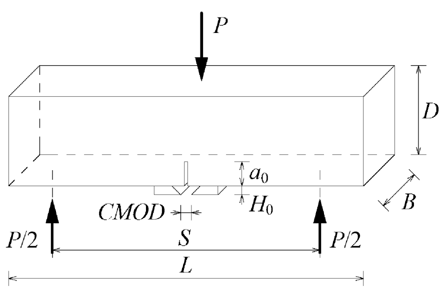

Fig.1: Three-point bending fracture test geometry

2.2. Concrete specimens, three-point bending tests

Three-point bending tests were performed on a total of twelve beams (comprising three specimens fabricated from each concrete mix) with a central edge notch to obtain the data described below. The nominal dimensions of the specimens were 100×100×400 mm, the depth of the central edge notch was about 1/3 of the depth of the specimen, and the loaded span was equal to 300 mm. A notch was cut before testing. Specimen age was 28 days.

The geometry of a specimen used in the three-point bending tests is shown in Fig. 1, where D is specimen depth, B is specimen width, L is specimen length, S is span; a_0 is the initial notch length. The output of the performed measurements was a set of load versus crack mouth opening displacement (*P–CMOD*) diagrams – a set of these diagrams for reference specimens (Fig. 2, left), for specimens with polypropylene fibres of 19 mm (Fig. 2, right), 38 mm (Fig. 3, left) and 54 mm (Fig. 3, right) length.

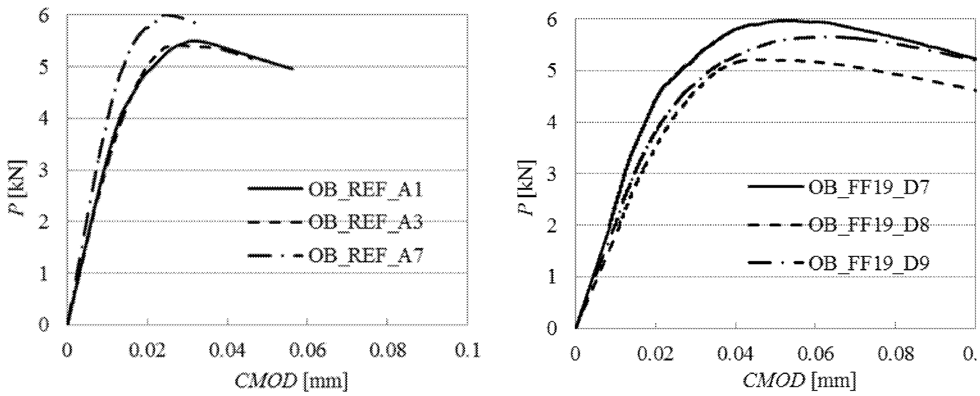

Fig.2: P–CMOD diagrams for reference specimens (left) and for specimens with polypropylene fibres of 19 mm length

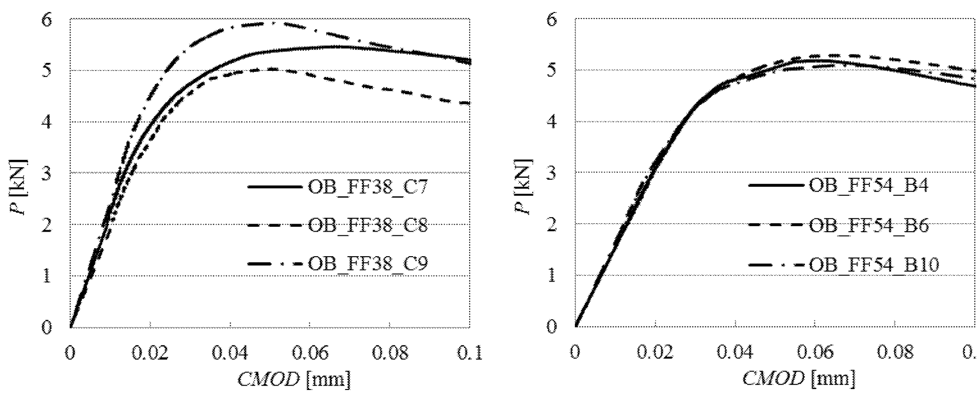

Fig.3: P–CMOD diagrams for specimens with polypropylene fibres of 38 mm (left) and 54 mm length

3. Application of the double-K fracture model

The measured *P–CMOD* diagrams are used to determine the fracture parameters of the double-K model. The unstable fracture toughness $K_{\mathrm{Ic}}^{\mathrm{un}}$ is numerically determined first, followed by the cohesive fracture toughness $K_{\mathrm{Ic}}^{\mathrm{c}}$. When both of these values are known, the following formula can be used to calculate the initiation fracture toughness $K_{\mathrm{Ic}}^{\mathrm{ini}}$:

$$K_{\mathrm{Ic}}^{\mathrm{ini}} = K_{\mathrm{Ic}}^{\mathrm{un}} - K_{\mathrm{Ic}}^{\mathrm{c}} \; . \tag{1}$$

The unstable fracture toughness $K_{\mathrm{Ic}}^{\mathrm{un}}$ is defined as the critical stress intensity factor created by the maximum load $P_{\max}$ at the effective crack tip and can be expressed as the resistance to unstable crack propagation. To evaluate this parameter one can use the following linear elastic fracture mechanics formula:

$$K_{\mathrm{Ic}}^{\mathrm{un}} = \frac{M_{\max}}{W} \sqrt{a_{\mathrm{c}}}\, F_1(\alpha_{\mathrm{ck}}) \ , \quad \text{where} \quad \alpha_{\mathrm{ck}} = \frac{a_{\mathrm{c}}}{D} \ , \tag{2}$$

$$M_{\max} = \frac{(q\, L + P_{\max})\, S - \frac{1}{2}\, q\, L^2}{4} \ , \tag{3}$$

$$F_1(\alpha_{\mathrm{ck}}) = \frac{1.99 - \alpha_{\mathrm{ck}}\, (1 - \alpha_{\mathrm{ck}})\, (2.15 - 3.93\, \alpha_{\mathrm{ck}} + 2.7\, \alpha_{\mathrm{ck}}^2)}{(1 + 2\, \alpha_{\mathrm{ck}})\, (1 - \alpha_{\mathrm{ck}})^{3/2}} \ , \tag{4}$$

where a_{c} is the critical effective crack length, D, L, are the specimen dimensions (depth, length), S is span (according to Fig. 1), q is self-weight of specimen and W is section modulus given by following equation:

$$W = \frac{1}{6}\, B\, D^2 \ . \tag{5}$$

To evaluate Eq. (2) respectively (4) it is necessary to evaluate the critical effective crack length a_{c} at the unstable loading condition ($P_{\max}$) by solving the following nonlinear equation:

$$CMOD_{\mathrm{c}} = \frac{P_{\max}\, S\, a_{\mathrm{c}}}{W\, E}\, V_1(\alpha_{\mathrm{c}}) \ , \quad \text{where} \quad \alpha_{\mathrm{c}} = \frac{a_{\mathrm{c}} + H_0}{D + H_0} \ , \tag{6}$$

$$V_1(\alpha_{\mathrm{c}}) = 0.76 - 2.28\, \alpha_{\mathrm{c}} + 3.87\, \alpha_{\mathrm{c}}^2 - 2.04\, \alpha_{\mathrm{c}}^3 + \frac{0.66}{(1 - \alpha_{\mathrm{c}})^2} \ , \tag{7}$$

where $CMOD_{\mathrm{c}}$ is the critical crack mouth opening displacement due to the maximum load $P_{\max}$, H_0 represents the thickness of the edge of the holder clip on the extensometer and E is Young's modulus.

To calculate the cohesive part of fracture toughness $K_{\mathrm{Ic}}^{\mathrm{c}}$ it is necessary to accept the assumption of the distribution of the cohesive stress σ along the fictitious crack. Generally, the relation between this cohesive stress σ and the fictitious crack opening displacement w is termed the cohesive stress function $\sigma(w)$.

To simplify in double-K model is considered the linear distribution of the cohesive stress along effective crack length and linear course of crack opening displacement along its length. At the maximum load $P_{\max}$ the crack becomes unstable and the corresponding opening at the tip of the stress free crack (the origin of the fictitious crack) is termed the critical crack tip opening displacement $CTOD_{\mathrm{c}}$ and it is expressed by following formula:

$$CTOD_{\mathrm{c}} = CMOD_{\mathrm{c}} \left(\left(1 - \frac{a_0}{a_{\mathrm{c}}}\right)^2 + \left(1.081 - 1.149\, \frac{a_{\mathrm{c}}}{D}\right) \left(\frac{a_0}{a_{\mathrm{c}}} - \left(\frac{a_0}{a_{\mathrm{c}}}\right)^2 \right) \right)^{1/2} \ . \tag{8}$$

Subsequently, the linear distribution of the cohesive stress along effective crack length can be expressed using this equation:

$$\sigma(COD) = \sigma(CTOD_{\mathrm{c}}) + \frac{x - a_0}{a_{\mathrm{c}} - a_0}\, (f_{\mathrm{t}} - \sigma(CTOD_{\mathrm{c}})) \ , \tag{9}$$

where $0 \le COD \le CTOD_{\mathrm{c}}$ and $a_0 \le x \le a_{\mathrm{c}}$.

Designation $\sigma(CTOD_\mathrm{c})$ is the cohesive stress at the tip of the initial notch length a_0 at the critical state and can be obtained from the softening curve. Four types of softening curve are used in the following text and calculations: linear, bilinear, and two exponential variants by Reinhardt (exp_R) and Karihaloo (exp_K).

If a linear softening curve is used, the value $\sigma(CTOD_\mathrm{c})$ can be calculated as follows:

$$\sigma(CTOD_\mathrm{c}) = \frac{f_\mathrm{t}\,(w_\mathrm{c} - CTOD_\mathrm{c})}{w_\mathrm{c}} \;, \tag{10}$$

where tensile strength f_t and critical crack tip opening displacement w_c are parameters of the softening curve. As indicated, $CTOD_\mathrm{c}$ is critical crack tip opening displacement (see e.g. [5]). In this paper, w_c is a constant value (0.16 mm) for all the softening curves. The tensile strength value is estimated using the measured compression strength value f_cu using the following relationship (by [6]):

$$f_\mathrm{t} = 0.24\, f_\mathrm{cu}^{\frac{2}{3}} \;. \tag{11}$$

When using a bilinear softening curve there are two cases:

In case I, $(CTOD_\mathrm{c} \leq w_\mathrm{s})$ can be obtained as a $\sigma(CTOD_\mathrm{c})$ value according to the formula:

$$\sigma(CTOD_\mathrm{c}) = f_\mathrm{t} - (f_\mathrm{t} - \sigma_\mathrm{s})\,\frac{CTOD_\mathrm{c}}{w_\mathrm{s}} \;, \tag{12}$$

where σ_s and w_s are respectively the ordinate and abscissa at the point of slope change of the bilinear softening curve. According to [7], these values can be considered using the following formulas:

$$\sigma_\mathrm{s} = \frac{1}{3}\, f_\mathrm{t} \;, \quad \text{and} \quad w_\mathrm{s} = \frac{2}{9}\, w_\mathrm{c} \;. \tag{13}$$

In case II, $(w_\mathrm{s} \leq CTOD_\mathrm{c} \leq w_\mathrm{c})$ can be calculated as a $\sigma(CTOD_\mathrm{c})$ value using the following equation:

$$\sigma(CTOD_\mathrm{c}) = \frac{\sigma_\mathrm{s}}{w_\mathrm{c} - w_\mathrm{s}}\,(w_\mathrm{c} - CTOD_\mathrm{c}) \;. \tag{14}$$

When using the exponential softening curve by [8] a $\sigma(CTOD_\mathrm{c})$ value can be obtained using the expression:

$$\sigma(CTOD_\mathrm{c}) = f_\mathrm{t} \left\{ \left[1 + \left(\frac{c_1\, CTOD_\mathrm{c}}{w_\mathrm{c}} \right)^2 \right] \exp\!\left(\frac{-c_2\, CTOD_\mathrm{c}}{w_\mathrm{c}} \right) - \frac{CTOD_\mathrm{c}}{w_\mathrm{c}}\,(1 + c_1^3)\, \exp(-c_2) \right\} , \tag{15}$$

where c_1 and c_2 are the material constants. For normal concrete these dimensionless parameters are the following: $c_1 = 3$ and $c_2 = 6.93$.

In the case when the exponential softening curve by [9] is used the $\sigma(CTOD_\mathrm{c})$ value can be calculated using the following formula:

$$\sigma(CTOD_\mathrm{c}) = f_\mathrm{t}\, \exp\left(-\mu\, \frac{CTOD_\mathrm{c}}{w_\mathrm{c}} \right) \;, \tag{16}$$

where μ is a material constant with the assumed value $\mu = 4.6052$ for $\sigma = 0.01\, f_\mathrm{t}$.

Subsequently, the cohesive fracture toughness $K_{\mathrm{Ic}}^{\mathrm{c}}$ can be calculated by integrating the following expression:

$$K_{\mathrm{Ic}}^{\mathrm{c}} = \int\limits_{a_0/a_\mathrm{c}}^{1} \frac{2\sqrt{a_\mathrm{c}}}{\sqrt{\pi}}\,\sigma(U)\,F(U,V)\,\mathrm{d}U \ , \tag{17}$$

where

$$\begin{aligned} F(U,V) = {} & \frac{3.52\,(1-U)}{(1-V)^{3/2}} - \frac{4.35 - 5.28\,U}{(1-V)^{1/2}} + \\ & + \left(\frac{1.30 - 0.30\,U^{3/2}}{(1-U^2)^{1/2}} + 0.83 - 1.76\,U\right)[1-(1-U)\,V] \ . \end{aligned} \tag{18}$$

In Eq. (17), respectively (18), the substitutions $U = x/a_\mathrm{c}$ and $V = a_\mathrm{c}/D$ are used; $\sigma(U)$ is the cohesive stress defined for variable U according to formula (9) and $F(U,V)$ is the characteristic Green function. To evaluate Eq. (17) a special numerical integration method is necessary to handle the singularity at the integral boundary.

Finally, the initiation fracture toughness $K_{\mathrm{Ic}}^{\mathrm{ini}}$ is calculated using Eq. (1) and according to Eq. (19) is determined the value of the load P_{ini}. This value can be defined as the load level at the beginning of stable crack propagation from an initial crack/notch and can be obtained using the expression:

$$P_{\mathrm{ini}} = \frac{4\,W\,K_{\mathrm{Ic}}^{\mathrm{ini}}}{S\,F_1(\alpha)\,\sqrt{a_0}} \ . \tag{19}$$

where W is section modulus determined using Eq. (5) and $F_1(\alpha_0)$ is geometry function given by following equation:

$$F_1(\alpha_0) = \frac{1.99 - \alpha_0\,(1-\alpha_0)\,(2.15 - 3.93\,\alpha_0 + 2.7\,\alpha_0^2)}{(1+2\,\alpha_0)\,(1-\alpha_0)^{3/2}} \ . \tag{20}$$

where α_0 is ratio a_0/D.

4. Results

The relative mean values of selected material properties (compressive strength, modulus of elasticity, effective crack elongation, and unstable fracture toughness) are introduced in Tab. 2: the 100 % value for each material parameter represents the values of those parameters for the reference concrete without fibres OB_REF (these findings lead to the assumption that the mixes are not properly designed). The figures show the arithmetic mean, standard deviation and coefficient of variation values of selected parameters: compressive strength (Fig. 4, left), elasticity modulus (Fig. 4, right), effective crack elongation (Fig. 5, left), and unstable fracture toughness (Fig. 5, right).

	Concrete			
Parameter	OB_REF	OB_FF19	OB_FF38	OB_FF54
f_c	100.0	95.3	78.1	83.4
E	100.0	53.5	57.6	43.1
$a_\mathrm{c} - a_0$	100.0	114.4	134.6	117.0
$K_{\mathrm{Ic}}^{\mathrm{un}}$	100.0	102.6	107.5	97.4

Tab.2: Relative mean values of selected material parameters in %

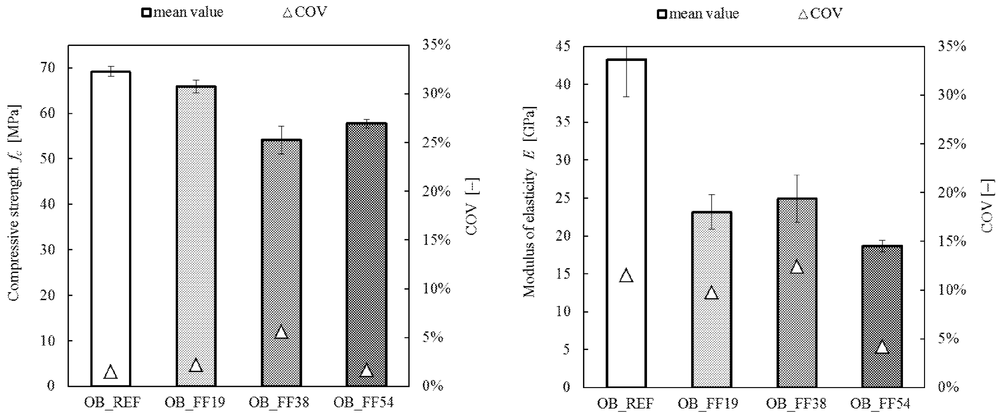

Fig.4: Compressive strength f_{c} (left) and modulus of elasticity E for the four concretes

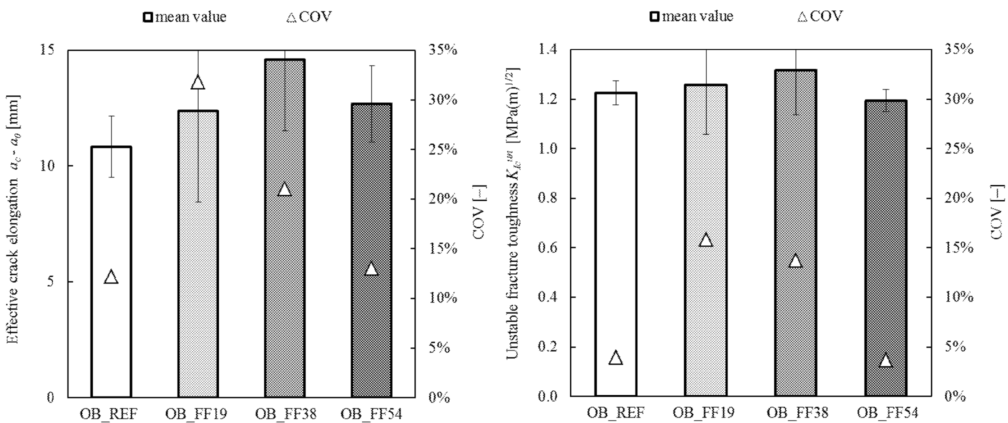

Fig.5: Effective crack elongation $a_{\mathrm{c}} - a_0$ (left) and fracture toughness $K_{\mathrm{Ic}}^{\mathrm{un}}$ for the four concretes

	Concrete			
Softening function	OB_REF	OB_FF19	OB_FF38	OB_FF54
linear	100.0 | 100.0	100.0 | 98.8	100.0 | 106.9	100.0 | 103.3
bilinear	112.0 | 100.0	127.4 | 112.3	127.3 | 121.5	130.3 | 120.2
exp_R	126.6 | 100.0	142.1 | 110.9	140.1 | 118.4	142.7 | 116.4
exp_K	117.6 | 100.0	131.5 | 110.4	130.4 | 118.6	132.9 | 116.7

Tab.3: Relative mean values of ratio $K_{\mathrm{Ic}}^{\mathrm{ini}}/K_{\mathrm{Ic}}^{\mathrm{un}}$ in %

Relative mean values of ratio $K_{\mathrm{Ic}}^{\mathrm{ini}}/K_{\mathrm{Ic}}^{\mathrm{un}}$ are introduced in Tab. 3; 100 % represents: (i) the value of ratio $K_{\mathrm{Ic}}^{\mathrm{ini}}/K_{\mathrm{Ic}}^{\mathrm{un}}$ for the linear softening curve for the appropriate concrete, (ii) the value of ratio $K_{\mathrm{Ic}}^{\mathrm{ini}}/K_{\mathrm{Ic}}^{\mathrm{un}}$ for the reference concrete OB_REF for each type of softening curve. Arithmetic mean, standard deviation and coefficient of variation values of the ratio $K_{\mathrm{Ic}}^{\mathrm{ini}}/K_{\mathrm{Ic}}^{\mathrm{un}}$ are introduced in Fig. 6 for all considered softening curves.

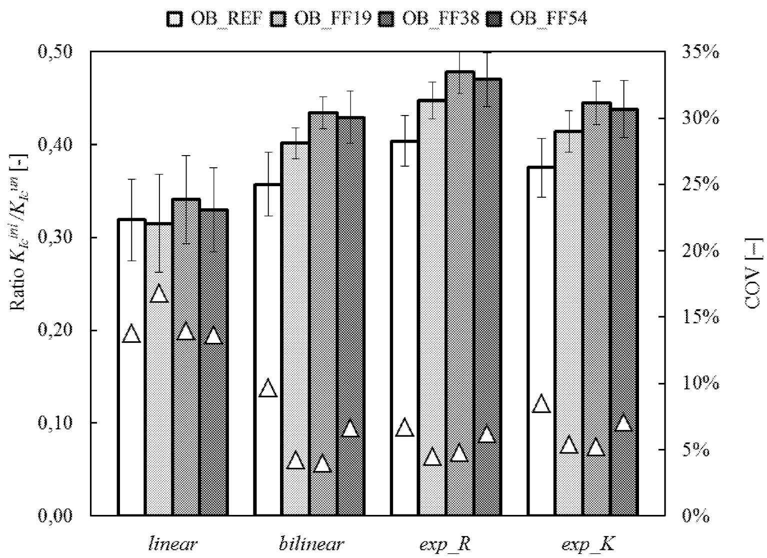

Fig.6: Comparison of the effects of softening function types on calculated $K_{\mathrm{Ic}}^{\mathrm{ini}}/K_{\mathrm{Ic}}^{\mathrm{un}}$ ratio for the four concretes

5. Conclusions

The conclusions can be divided into two parts: the first relating to the evaluation of concrete with/without fibres and second relating to the effect of the applied softening curve type on calculated results.

The presence of polypropylene fibres in the composite caused a reduction in the compressive strength values of 5 to 22 percent, and modulus of elasticity values were reduced by 46 to 57 percent. The largest reduction in compressive strength values was exhibited by concrete OB_FF38; in the case of elasticity modulus it was composite OB_FF54 that showed the largest fall. The effective crack elongation values of composites with fibres were from 14 to 35 percent higher in comparison with the reference concrete, the largest being in the case of concrete OB_FF38. The presence of fibres had no significant effect on the unstable fracture toughness values (composite OB_FF38 showed the largest relative increase, which was of less than 8 percent). In terms of resistance to stable crack propagation the addition of fibres appears to be a positive step – the highest relative increase in this resistance (over 20 percent) was reported by OB_FF38 concrete. These findings lead to the assumption that the mixes are not properly designed.

Using the selected softening curve has a significant effect on the determination of the resistance against stable crack growth for all investigated composites. Compared to a linear softening function, using a bilinear softening function leads to an increase in resistance of 12 to 30 percent, Karihaloo's exponential softening curve produced an increase of about 18 to 33 percent, and the highest increase was seen for Reinhardt's exponential softening curve: it was about 27 to 43 percent.

Acknowledgement

This outcome has been achieved with the financial support of the specific research programme at Brno University of Technology, project No. FAST-J-13-2078. The authors greatly thank Dr. Jaromír Laník for providing the results of fracture tests and Ing. Jan Bedáň for providing supports for special calculations; both are members of the Faculty of Civil Engineering at Brno University of Technology.

References

[1] Reinhardt H.W., Xu S.: Crack extension resistance based on the cohesive force in concrete, Engineering Fracture Mechanics 64 (1999) 563

[2] Xu S., Reinhardt H.W.: Determination of double-K criterion for crack propagation in quasi-brittle fracture, Part I: Experimental investigation of crack propagation, International Journal of Fracture 98 (1999) 111

[3] Xu S., Reinhardt H.W.: Determination of double-K criterion for crack propagation in quasi-brittle fracture, Part II: Analytical evaluating and practical measuring methods for three-point bending notched beams, International Journal of Fracture 98 (1999) 151

[4] Xu S., Reinhardt H.W.: Determination of double-K criterion for crack propagation in quasi-brittle fracture, Part III: Compact tension specimens and wedge splitting specimens, International Journal of Fracture 98 (1999) 179

[5] Kumar S., Barai S.: Concrete Fracture Models and Applications, Springer 2011, Berlin

[6] Červenka V., Jendele L., Červenka J.: ATENA Program documentation – Part 1: theory, Červenka Consulting 2012, Prague

[7] Petersson P.E.: Crack growth and development of fracture zone in plain concrete and similar materials, Report No. TVBM-1006, Lund Institute of Technology 1981

[8] Reinhardt H.W., Cornelissen H.A.W., Hordijk D.A.: Tensile tests and failure analysis of concrete, Journal of Structural Engineering 112 (1986) 2462

[9] Karihaloo B.L.: Fracture Mechanics and Structural Concrete, Longman Scientific & Technical 1995, New York

Received in editor's office: July 24, 2013
Approved for publishing: February 21, 2014