FRACTURE ANALYSIS OF SANDWICH BEAM LOADED IN TORSION

Viktor Rizov*

Fracture behavior of foam core composite sandwich Split Cantilever Beam (SCB) loaded in torsion was analyzed theoretically using the apparatus of linear-elastic fracture mechanics. A loading method is proposed to create two transverse forces of identical magnitude, which load the beam in torsion. In this way, mixed-mode II/III crack loading conditions were induced. A three-dimensional finite element model was developed for simulating the mechanical response of the sandwich SCB. The strain energy release rate was used as a fracture characterizing parameter. The analysis by virtual crack closure technique revealed that the strain energy release rate mode components were distributed non-uniformly along the crack front through the width of the sandwich SCB. The relation between the fracture behavior and the sandwich core material was studied. For this purpose, three foam core composite sandwich SCB configurations (with three different rigid cellular foams used as core material) were simulated. It was found that the fracture behavior of the sandwich beam can be improved by using foams with higher density as a core material.

Keywords : *foam core sandwich beam, mixed-mode II/III crack, linear-elastic fracture mechanics, strain energy release rate*

1. Introduction

The increasing use of load-bearing sandwich structures is facilitated by a superior stiffness and strength per weight in comparison with conventional metal structures [1, 2, 3, 4, 5]. The development and the design of sandwich structures require an extensive knowledge of the mechanical properties of sandwich components (face sheet and core material). In particular, the attention of engineers and researches is focused on the core material, which is the weakest and most critical constitutive. Rigid foams are commonly used as a core material in sandwich structures. The load-bearing capacity of these structures is strongly influenced by the strength and the reliability of the foam. For instance, the foam may contain various defects as flaws and voids which act as stress concentrators. A crack usually initiates at a stress concentration point in the foam material. The propagation of a crack can drastically reduce the load-bearing capacity of the sandwich structure. Therefore, an understanding of the fracture mechanisms is of significant relevance to practical engineering applications.

Studies on mechanical properties of rigid foams used as a core material in sandwich structures are well documented [6, 7, 8, 9]. However, relatively little work has been published on fracture behavior of rigid foams. Ashby et al. have studied static fracture behavior of rigid cellular foams [10]. Bažant et al. have investigated static fracture in closed-cell foam under mode I crack loading conditions [11]. Farshad and Flüeler have used an anti-clastic plate

* prof. Dr. V. Rizov, Department of Technical Mechanics, University of Architecture, Civil Engineering and Geodesy, 1 Chr. Smirnensky blvd., 1046 – Sofia, Bulgaria

bending test method for experimental characterization of mode III static fracture toughness of rigid cellular foam materials [12]. Almost a pure mode III crack loading conditions have been induced in the foam plate specimen using a special loading device. Prassad and Carlsson have conducted an experimental and numerical investigation of debonding in foam core sandwich structures under static loading [13, 14]. Double cantilever beam and shear sandwich specimens have been used for experimental characterization of debonding fracture behavior. Linear-elastic fracture mechanics concepts have been applied for analysis of the test data. For this purpose, two-dimensional finite element models have been developed. Comparisons between the simulations and the experimental measurements have been performed in order to verify the finite element models. A static delamination crack along the face sheet/core interface in a foam core composite sandwich structure subjected to transverse loading has been investigated by Goswami and Becker using the methods of linear-elastic fracture mechanics [15]. Two-dimensional finite element simulations have been carried-out. The influence has been evaluated of different structural parameters on the face sheet/core delamination fracture behavior of the sandwich structure. Noble and Lilley have applied the Paris' law equation in order to characterize fatigue crack growth in rigid foam material [16]. Yau and Mayer have studied environmental effects on the fatigue fracture behavior of Polycarbonate foam material [17].

The aim of the present paper was to investigate theoretically the fracture in foam core composite sandwich SCB loaded in torsion using the methods of linear-elastic fracture mechanics. It was expected that by loading the sandwich beam in torsion, mixed-mode II/III crack loading conditions will be induced (the present paper was motivated by the fact that no mixed-mode II/III fracture studies of sandwich beams were found in the literature). Fracture behavior was studied in terms of the strain energy release rate using the virtual crack closure techniques in conjunction with three- dimensional finite element simulations of the sandwich SCB. The effect of the sandwich core material was evaluated.

2. Simulations of fracture behavior

The foam core composite sandwich SCB configuration analyzed in the present paper is shown in Fig. 1. The sandwich construction consists of two composite face sheets with thick-

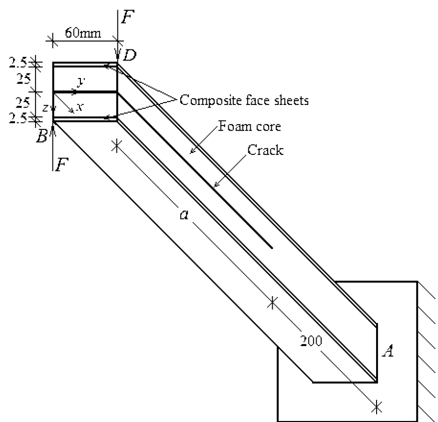

Fig.1: Sandwich SCB geometry, dimensions, and loading (the beam is clamped in section A)

ness of 2.5 mm each adhesively bonded to a light-weight foam core with thickness of 50 mm. The overall dimensions of the sandwich SCB configuration are 55 mm × 60 mm × 500 mm. In the mid-plane of the beam, there is a longitudinal crack, a, with length of 300 mm. The beam is clamped in section A. Two transverse forces of identical magnitude, F, are applied to the crack arms at the free end of the beam in order to induce torsion moment, $T = F\,b$, where b is the sandwich beam width (Fig. 1).

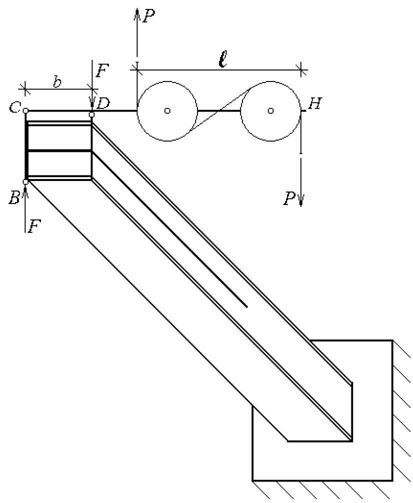

Fig.2: Loading method

A loading method is proposed here to create the two transverse forces (Fig. 2). The method uses a special rig, BCDH, which is connected to the lower and the upper crack arms in points B and D, respectively. The rig is loaded by a moment, $M = P\,l$, created by the force, P, in a wire, which runs from the test machine through rollers as shown in Fig. 2. Thus, the transverse force is $F = P\,l/b$.

The fracture behavior was studied theoretically applying the concepts of linear-elastic fracture mechanics. For this purpose, a three-dimensional finite element model of the sandwich SCB was developed using the ANSYS program system. The geometry, the dimensions, and the loading of the model corresponded to these shown in Fig. 1. Three-dimensional continuum brick finite element SOLID45 was used to mesh the model. This element is defined by 8 nodes (one at each vertex) having three degrees of freedom per node (translation in the nodal x, y, and z directions). A total of 4920 elements were used. The mesh was refined in the crack front area to allow for a more accurate analysis of the strain energy release rate mode components distribution. In order to prevent penetration of the crack surfaces, high stiffness normal springs were incorporated between the crack surfaces in the finite element model. It should be mentioned that before carrying out further simulations, a mesh sensitivity study was performed with respect to the element number in order to ensure that the mesh was fine enough to give reliable results. The mesh used in the finite element analysis is depicted in Fig. 3. The deformed state of the model is displayed in Fig. 4.

The elastic properties of the composite face sheets used in the finite element simulations of the sandwich SCB are summarized in Table 1. The effect of the core material on the fracture in the sandwich beam was studied. For this purpose, it was presumed that the core is

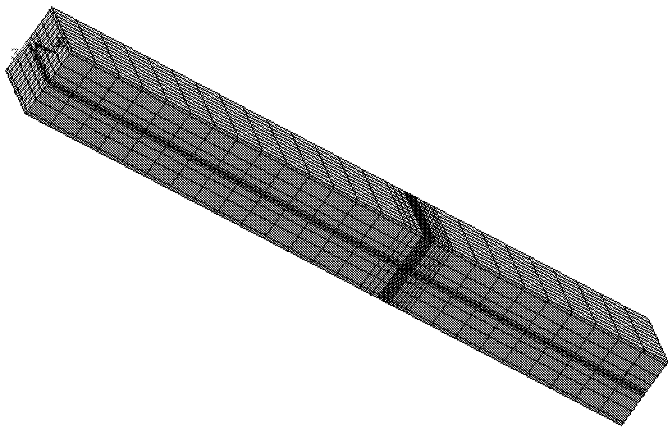

Fig.3: Finite element model of the sandwich SCB

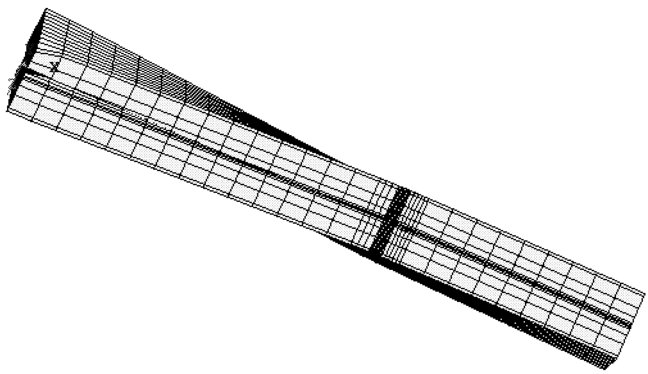

Fig.4: Deformed shape of the model (the displacements are exaggerated for clarity)

made by three different rigid cellular foams (Divinycell H60, Divinycell H100, and Divinycell H200), i.e., three sandwich SCB configurations were simulated. Divinycell polyvinylchloride (PVC) foams are fully cross-linked rigid foams with closed-cell structure that are frequently used as core material in sandwich structures. The manufacturing process of these foams basically consists of mixing the chemical polymer components together and carrying-out the thermal expansion of the polymer mass in hot water. In the finite element modeling the foam core was regarded as a linear-elastic material (the corresponding elastic properties are reported in Table 2).

E_{xx} (GPa)	E_{yy} (GPa)	E_{zz} (GPa)	G_{xy} (GPa)	G_{yz} (GPa)	G_{zx} (GPa)	ν_{xy}	ν_{yz}	ν_{zx}
45.000	12.000	12.000	4.500	4.500	4.400	0.3	0.3	0.3

Tab.1: Elastic properties of the composite face sheets used in the finite element analysis of the sandwich SCB; the subscripts refer to the axes x, y, and z of the coordinate system shown in Fig. 1

The strain energy release rate was evaluated by the virtual crack closure technique. By this technique the strain energy release rate components associated with the three basic nodes of crack growth can be calculated separately [18, 19, 20, 21, 22]. The main advantage

	E (GPa)	ν	γ (kg/m^3)
Divinycell H60	0.060	0.300	60
Divinycell H100	0.105	0.310	100
Divinycell H200	0.293	0.332	200

Tab.2: Elastic properties and densities of the rigid cellular foams used in the finite element simulations of the foam core composite sandwich SCB

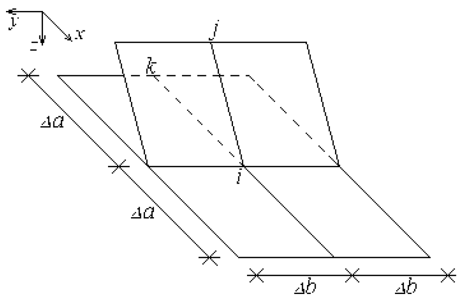

Fig.5: Schematic of the crack front in the vicinity of node i with nodal notations

of the virtual crack closure technique is that it requires only one analysis by the finite element model of the sandwich SCB for the actual crack length. The nodal forces at the crack front and the nodal displacements behind the crack front determined by the finite element solution are needed to compute the strain energy release rate mode components. The nodal notations at the crack front are shown in Fig. 5. The virtual crack closure technique assumes that the work done to closure the crack by one element is equal to the strain energy released when the crack grows by one element length. Thus, at node i in Fig. 5 the strain energy release rate mode components can be approximated by

$$G_{\mathrm{I}} = \frac{1}{2\,\Delta a\,\Delta b}\,Z_i\,(w_j - w_k)\ , \tag{1}$$

$$G_{\mathrm{II}} = \frac{1}{2\,\Delta a\,\Delta b}\,X_i\,(u_j - u_k)\ , \tag{2}$$

$$G_{\mathrm{III}} = \frac{1}{2\,\Delta a\,\Delta b}\,Y_i\,(v_j - v_k)\ , \tag{3}$$

where X, Y, and Z are the nodal force components, and u, v, and w are the nodal displacement components in the x, y, and z directions, respectively. The subscripts in Eqs. (1)–(3) denote the corresponding nodes in Fig. 5.

Although the virtual crack closure technique is widely used, it is an approximate method for calculation of the strain energy release rate. Thus, the results obtained by Eqs. (1)–(3) were verified by the crack closure technique, which is a direct method for computation of the strain energy release rate. The application of the crack closure techniques requires two analyses by the finite element model of the sandwich SCB. The first analysis, which is carried-out just prior to crack growth, yields the nodal forces at the crack front. The crack advance is simulated in the second analysis by releasing the crack front nodes. The second analysis provides the corresponding nodal displacements needed to compute the strain energy release rate mode components. The crack closure technique assumes that the nodal forces obtained in the first analysis, are the forces needed to closure the crack. The work done during the process of crack closure can be calculated by multiplying one half of the nodal forces with corresponding nodal displacements. The strain energy release rate mode components at node i in Fig. 4 can be expressed as

$$G_{\mathrm{I}} = \frac{1}{2\,\Delta a\,\Delta b}\, Z_i\,\delta w_i \ , \tag{4}$$

$$G_{\mathrm{II}} = \frac{1}{2\,\Delta a\,\Delta b}\, X_i\,\delta u_i \ , \tag{5}$$

$$G_{\mathrm{III}} = \frac{1}{2\,\Delta a\,\Delta b}\, Y_i\,\delta v_i \ , \tag{6}$$

where δu, δv, and δw are the differences in the nodal displacement components.

The strain energy release rate mode components in the sandwich SCB loaded in torsion were calculated using both techniques. The difference between the results obtained by Eqs. (1)–(3) and Eqs. (4)–(6) was within 2 %, which was an indication for the good accuracy of the analysis.

The strain energy release rate mode components distribution along the crack front was investigated by applying Eqs. (1)–(3) for each node at the crack front. The average values of the strain energy release rate mode components along the crack front were calculated as

$$G_{\mathrm{I}}^{\mathrm{a}} = \frac{1}{b}\int_0^b G_{\mathrm{I}}(y)\,\mathrm{d}y \ , \tag{7}$$

$$G_{\mathrm{II}}^{\mathrm{a}} = \frac{1}{b}\int_0^b G_{\mathrm{II}}(y)\,\mathrm{d}y \ , \tag{8}$$

$$G_{\mathrm{III}}^{\mathrm{a}} = \frac{1}{b}\int_0^b G_{\mathrm{III}}(y)\,\mathrm{d}y \ , \tag{9}$$

where b is the sandwich beam width, G_{I}, G_{II}, and G_{III} are the strain energy release rate distributions along the crack front computed by Eqs. (1)–(3). The average value of the total strain energy release rate was obtained as

$$G^{\mathrm{a}} = G_{\mathrm{I}}^{\mathrm{a}} + G_{\mathrm{II}}^{\mathrm{a}} + G_{\mathrm{III}}^{\mathrm{a}} \ . \tag{10}$$

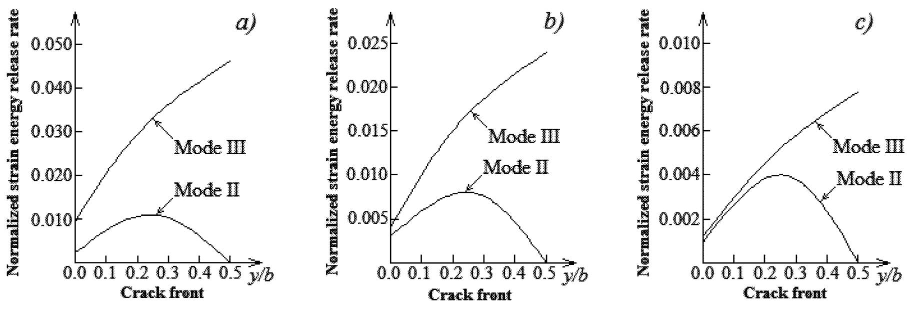

Fig.6: Distribution of the normalized strain energy release rate mode components along the crack front in the sandwich SCB with foam core made of: (a) Divinycell H60, (b) Divinycell H100, and (c) Divinycell H200. Only half the crack front is plotted, because the distribution is symmetrical about the crack front centre. The horizontal axis is defined such that $y/b = 0.0$ is at the sandwich beam edge; thus, $y/b = 0.5$ is at the crack front centre

The distribution of the normalized strain energy release rate mode components along the crack front in the sandwich SCB with different foam core is illustrated in Fig. 6 (the normalization is performed using the formula $G_i^{\mathrm{N}} = G_i\, b/F$, where b is the width of the sandwich beam cross-section, F is the load, and i = II and III). As expected, the strain energy release rate distribution is non-uniform. Since the distribution is symmetric with respect to the crack front center, only half of the crack front is plotted. Figure 6 indicates strong edge effects. The mode III component has maximum at the crack front centre and quickly decreases in the zones near the edges of the beam. The mode II component of the strain energy release rate is zero at the crack front centre. The absolute maximum of the mode II component is at one quarter of the beam width. In the zones close to the beam edges, the mode II component quickly decreases (Fig. 6). The computations yielded no mode I component of the stain energy release rate. The diagrams in Fig. 6 indicate that the fracture is mode III dominated. The mixed-mode ratios of the average values of the strain energy release rate mode components along the crack front, calculated by formulae (7)–(10), are $G^{\mathrm{a}}_{\mathrm{II}}/G^{\mathrm{a}} = 0.124$ and $G^{\mathrm{a}}_{\mathrm{III}}/G^{\mathrm{a}} = 0.876$ (the integrals were solved numerically).

One can see in Fig. 7 that the strain energy release rate decreases when the foam modulus of elasticity increases. Having in mind that the modulus of elasticity increases with increase of the foam density (Table 2), it can be summarized that the strain energy release rate decreases when the foam density increases.

3. Conclusions

A theoretical investigation of the mixed-mode II/III fracture in foam core composite sandwich SCB configuration loaded in torsion was performed. A loading method was proposed to create two transverse forces of identical magnitude, which applied on the crack arms, load the sandwich beam in torsion. In this way, mixed-mode II/III crack loading conditions were generated. The study was performed using the concepts of linear-elastic fracture mechanics. The fracture behavior was analyzed in terms of the strain energy release rate. In this relation, a three-dimensional finite element model of the sandwich SCB was developed using the ANSYS software. The strain energy release rate mode components distribution along the crack front was simulated by the virtual crack closure technique. The results obtained were conformed by the crack closure technique. The simulations revealed that the strain energy release rate mode components were distributed non-uniformly along the crack front. It was found that the mode III component has maximum at the crack front centre and progressively decreases towards the beam lateral edges. At the crack front centre, the mode II component is zero. The absolute maximum of the mode II component is at one quarter of the beam width. In the zones close to the beam lateral edges, the mode II component quickly decreases. No mode I component of the stain energy release rate was yielded by the analysis. The mixed-mode ratios of the average values of the strain energy release rate mode components along the crack front were $G^{\mathrm{a}}_{\mathrm{II}}/G^{\mathrm{a}} = 0.124$ and $G^{\mathrm{a}}_{\mathrm{III}}/G^{\mathrm{a}} = 0.876$, i.e. the fracture was mode III dominated. The effect of the sandwich core material on the fracture behavior was evaluated. For this purpose, it was presumed that the sandwich core was made by three different rigid cellular foams. The simulations revealed that the stain energy release rate decreases when the foam density increases. Thus, the mixed-mode II/III fracture performance of the sandwich beams can be significantly improved by using foams with higher density as core material.

References

[1] Triantafillou T.C., Gibson L. J.: Debonding in Foam Core Sandwich Panels, Materials and Structures 15 (1989) 266–276
[2] Nemes J.A., Simmonds K.E.: Low-Velocity Impact Response of Foam-Core Sandwich Composites, Journal of Composite Materials 26 (1992) 500–519
[3] Akil Hazizan Md., Cantwell W.J.: The low velocity impact response of foam-based sandwich structures, Composites: Part B 33 (2002) 193-204.
[4] Steeves C.A., Fleck N.A.: Failure modes in sandwich beams with composite face-sheets and PVC foam cores, ASME ICEME'2000 Conference in Orlando, November 5–10, ASME AERO AMD AD vol. 62/AMD vol. 245; 2000 p. 21–28
[5] Soden P.D.: Indentation of Composite sandwich beams, J. Strain Analysis 31 (1996) 353–360
[6] Hilyard N.C., Cunningham A.: Low Density Cellular Plastics, Chapman&Hall, 1994, London
[7] Gibson L.J., Ashby M.F.: Cellular Solids: Structure and Properties, Cambrige University Press, 1997, Cambrige
[8] Shim V.P.W., Tu Z.H., Lim C.T.: Two-dimensional response of crushable polyurethane foam to low velocity impact, Int. J. Impact Engng. 24 (2000) 703–731
[9] Vaz M.F., Fortes M.A.: Characterization of deformation bonds in the compression of cellular materials, J. Mater. Sci. Lett. 12 (1993), 1408–1410
[10] Ashby M.F., Gibson L.J, Maiti S.K.: Fracture Toughness of Brittle Cellular Solids, Scr. Metall. 18 (1984) 213–217
[11] Bažant Z.P., Zhou Y., Zi G., Daniel I.: Size effect and asymptotic matching analysis of fracture of closed-cell polymeric foam, International Journal of Solids and Structures 40 (2003) 7197–7217
[12] Farshad M., Flüeler P.: Investigation of mode III fracture toughness using an anti-clastic plate bending method, Engineering Fracture Mechanics 60 (1998) 597–603.
[13] Prassad S., Carlson L.A.: Debonding and crack kinking in foam core sandwich beams – I. Analysis of fracture specimens, Engng. Fracture Mech. 47 (1994) 813-824
[14] Prassad S., Carlson L.A.: Debonding and crack kinking in foam core sandwich beams – II. Experimental investigation, Engng. Fracture Mech. 47 (1994) 825–841
[15] Goswami S., Becker W.: The effect of face sheet/core delamination in sandwich structures under transverse loading, Composite Structures 54 (2001) 515–521
[16] Noble F.W., Lilley J.: Fatigue Crack Growth in Polyurethane Foam, Journal of Materials Science 16 (1981) 800–1808
[17] Yau S.S., Mayer G.: Fatigue Crack Propagation in Polycarbonate Foam, Journal of Materials Science and Engineering 78 (1986) 111–114
[18] Shivakumar K.N., Tan P.W., Newman J.C.: A virtual crack-closure technique for calculation of stress intensity factors for cracked three-dimensional bodies, Int. J. Fract. 36 (1988) R 43–R 50.
[19] Szekrenyes A., Uj J.: Comparison of some improved solutions for mixed-mode composite delamination coupons, Composite Structures 72 (2006) 321–329
[20] Suemasu H., Kumagai T., Gozu K.: Compressive behavior of multiply delaminated composite laminates Part finite element analysis, AIAA J. 36 (1998) 1286–1290
[21] Hellen T.K.: On the Method of virtual crack extension, International Journal for Numerical methods in Engineering 9 (1975) 187–207
[22] Narayana K.B., George S., Dattaguru B., Ramanurty T.S., Vijyakumar K.: Modified crack closure integral (MCCI) for 3D problems using 20 noded brick elements, Fatigue Fract. Eng. Mater. Struct. 17 (1994) 145–157

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