GRADUALLY VARIED TRANSPORT OF BED LOAD IN SHEET FLOW: MATHEMATICAL AND PHYSICAL MODELING

Štěpán Zrostlík*, Jan Krupička*, Tomáš Picek*, Vojtěch Bareš*, Václav Matoušek*

Intense transport of sediment is studied in flows of steep slopes. Transport- and friction formulae suitable for flows at high shear stress above eroded bed are discussed. The formulae are further incorporated to a proposed simple model for open-channel flow with gradually varied transport of sediment. Two variants of the formulae are used alternatively in the model. Tilting-flume experiments are described that provided steady-flow data suitable for a validation of the formulae and unsteady-sediment-transport data for a validation of the designed model of gradually varied transport. A comparison of experimental results with formulae predictions and model simulations shows a very reasonable agreement for both variants of the transport and friction formulae.

Keywords: *intense sediment transport, unsteady flow, steady flow, bed roughness, flume experiment, steep slope*

1. Introduction

Human activities and natural processes lead to sediment-transport related morphological changes of rivers which sometimes may have disastrous implications. A great deal of attention has been devoted to the phenomenon of sediment transport in mobile-bed rivers. A majority of experimental studies deals with the sediment transport of rather low intensity typical for usual flow conditions. The matter is relatively well understood for these conditions. On the other hand, significantly less experimental effort has been devoted to an investigation of flows for which the sediment discharge takes more than, say, 5 % of the total discharge. Transport and friction of such sediment-laden flows are subject to ongoing research, e.g. [1–3, 7–10, 13, 16]. Intense transport of coarse sediment develops in flow with free water surface if the energy grade line is steep and the flow produces high bed shear stress. This mode is typical for floods in mountain streams. To ensure a better prediction of flood events and thus a more effective protection of people and structures against floods, it is necessary to develop appropriate mathematical models.

In principle, two different approaches are employed in modeling of sediment transport in steep streams. The first one uses hydro-sedimentologic models considering sediment transport processes on catchment basin. The DHVSN model [5] is an example. The second approach uses 1D or 2D hydrodynamic simulations including sediment transport and accounting for variations in bed geometry due to erosion or deposition. There are many

* Ing. Š. Zrostlík, Ing. J. Krupička, Ph.D., Ing. T. Picek, Ph.D., Ing. V. Bareš, Ph.D., prof. Dr. Ing. V. Matoušek, Department of Hydraulics and Hydrology, Faculty of Civil Engineering, Czech Technical University in Prague, 166 29 Prague 6, CZ

models designed for a simulation of flow of water and sediment, e.g. [15], but less for simulating really intense sediment transport. Examples are the model SETRAC [4] designed for torrents and mountains streams and the software TomSed [6], which solves a one-dimensional bed load transport model for steep slopes.

Many empirical formulae have been proposed to predict the transport capacity and bed friction of sediment laden flows. Only small population of the formulae applies to specific flow conditions of transport of bed load at high shear stress, i.e. to sheet flow. The objective of this study is to test selected formulae against new experimental data for the sheet-flow conditions collected in our laboratory flume and to implement the formula into a simple model of unsteady motion of bed load at high bed shear.

2. Formulae for friction and transport of bed load

Šulc and Zrostlík [14] tested several friction and transport formulae using experimental data for bed load at high bed shear from our laboratory and from the literature [3, 13]. The most successful predictions appeared to be produced by transport- and friction formulae based on works of Rickenmann [11, 12] and used in the software TomSed [6],

$$q_\mathrm{s} = 3.1\left(\frac{d_{90}}{d_{30}}\right)^{0.2}(q_\mathrm{m}-q_\mathrm{c})\,I_\mathrm{e}^{1.5}\left(\frac{\varrho_\mathrm{s}}{\varrho_\mathrm{f}}-1\right)^{-1.5}, \tag{1a}$$

$$q_\mathrm{c} = 0.143\,g^{0.5}\,d_{90}^{1/5}\,I_\mathrm{e}^{-1.167}\left(\frac{\varrho_\mathrm{s}}{\varrho_\mathrm{f}}-1\right)^{-1.5}, \tag{1b}$$

$$\sqrt{\frac{8}{\lambda_\mathrm{b}}} = \frac{V_\mathrm{m}}{\sqrt{g\,h\,I_\mathrm{e}}} = \frac{6.5\cdot 2.5\,\dfrac{h}{d_{84}}}{\sqrt{6.5^2+2.5^2\left(\dfrac{h}{d_{84}}\right)^{5/3}}}\,. \tag{2}$$

These formulae are thus selected for a comparison with our new experimental data and our own formulae.

A transport formula of the Meyer-Peter and Müller (MPM) type (Equation 3) is frequently used in the sediment-transport literature and many researchers proposed specific values of formula coefficients to fit their experimental data. Matoušek [7] extended validity of this type of formula to flows at high shear stresses (sheet flows in the upper plane bed regime) and proposed a more general form by relating the coefficients α and β to particle properties characterised by particle Reynolds number Re_P

$$\Phi = \frac{q_\mathrm{s}}{\sqrt{\left(\dfrac{\varrho_\mathrm{s}}{\varrho_\mathrm{f}}-1\right)g\,d_{50}^3}} = \alpha\,(\theta-\theta_\mathrm{c})^\beta\,, \tag{3}$$

$$\alpha = 5.4 + \frac{58}{Re_\mathrm{P}^{0.62}}\,, \qquad \beta = 1.2 + \frac{1.3}{Re_\mathrm{P}^{0.39}}\,. \tag{4}$$

A validation of the formula using very different fractions of narrow-graded solids showed that the transport formula was successful in both pressurized pipes and open flumes [9, 10].

A surface of an eroded bed is often treated as a hydraulically rough boundary, e.g. [17]. The bed can be characterised by the equivalent roughness k_s related to the size of the bed grains d_{50} and a suitable friction law (e.g. the Nikuradse formula) can be employed to calculate the bed friction coefficient,

$$\sqrt{\frac{8}{\lambda_\mathrm{b}}} = 2.5 \ln\left(\frac{11.1\, R_\mathrm{b}}{k_\mathrm{s}}\right) . \tag{5}$$

High shear stress experiments conveyed in pressurized ducts revealed that k/d_{50} is closely related to the bed Shields parameter θ [16]. Recent studies indicate that additional parameters should be taken into account to express the effect of intense transport of sediment on bed friction and thus on the roughness of the top of the eroded bed [1, 9, 10]. An introduction of the dimensionless particle velocity $V_\mathrm{t}^* = V_\mathrm{t}\,\sqrt[3]{(\varrho_\mathrm{s}/\varrho_\mathrm{f}-1)^2/(g\,\nu_\mathrm{f})}$ [1] seems to improve the correlation significantly. Hence, $k_\mathrm{s}/d_{50} = f(V_\mathrm{t}^*, \theta)$ and this function is a subject to calibration using suitable experimental data for the upper plane bed regime.

3. Mathematical model for gradually varied transport of bed load in sheet flow

Our objective is to design a simple model for gradually varied flows in which different friction- and transport formulae could be tested. The model is based on a quasi-steady solution of flow and on balancing the erosion and the deposition at the each time step. For the gradually varied flows, boundary conditions are assumed to vary with the time scale larger than the time period in which the flow became steady after the boundary condition has been changed. This assumption allows us to consider the flow as quasi-steady and to calculate a water surface profile using the standard step method at the each time. The space domain is discretized into sections by a sequence of cross-sections (Figure 1). The initial condition for the model is the longitudinal profile of the top of the bed, y_b. At each time step, the boundary conditions are 1) discharge of water, Q_f, and discharge of sediment, Q_s, in the uppermost cross-section, N; 2) the bed level in the downstream cross-section, $y_\mathrm{b}(1, j)$, and 3) the depth, h, in upstream/downstream cross-section for the supercritical/subcritical flow respectively. Iterations are required to find the bed and water surface profiles in the new time step. A flow chart of the iterative procedure is shown in Figures 2a,b. Initial estimation of the sediment discharge in the new time step is used by the first iteration (Figure 2a). Then the change of bed level is calculated for the each cross-section by balancing the erosion (or deposition) and the sediment discharges within the preceding time interval (Figure 2b). For the new bed positions, water surface profile is calculated by the step method employing selected friction formula. Then a new estimation of sediment discharge is calculated for the each section from the mean depth and slope of energy grade line, I_e. The new estimation of sediment discharge is used in the next iteration until a convergence is achieved.

Figure 2c shows a schematic cross section of the flow above deposit and indicates how the flow cross section area is divided into the sub-area associated with the top of the deposit and the sub-area associated with the channel walls. The sub-areas are used to determine the bed hydraulic radius R_b and the wall hydraulic radius R_w required for a determination of the Shields parameter, θ, and the bed roughness, k_s, (see Figure 2a) in the model.

4. Experimental work

4.1. Physical model

The experimental work was carried out in the tilting flume of the Laboratory of Water Engineering of Czech Technical University in Prague. It is a recirculating flume and it can

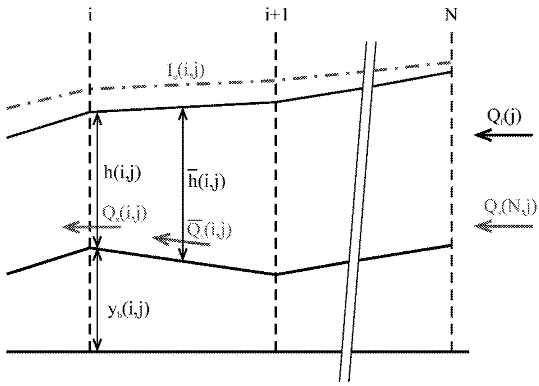


Fig.1: Scheme of numerical discretization used in model of unsteady sediment motion

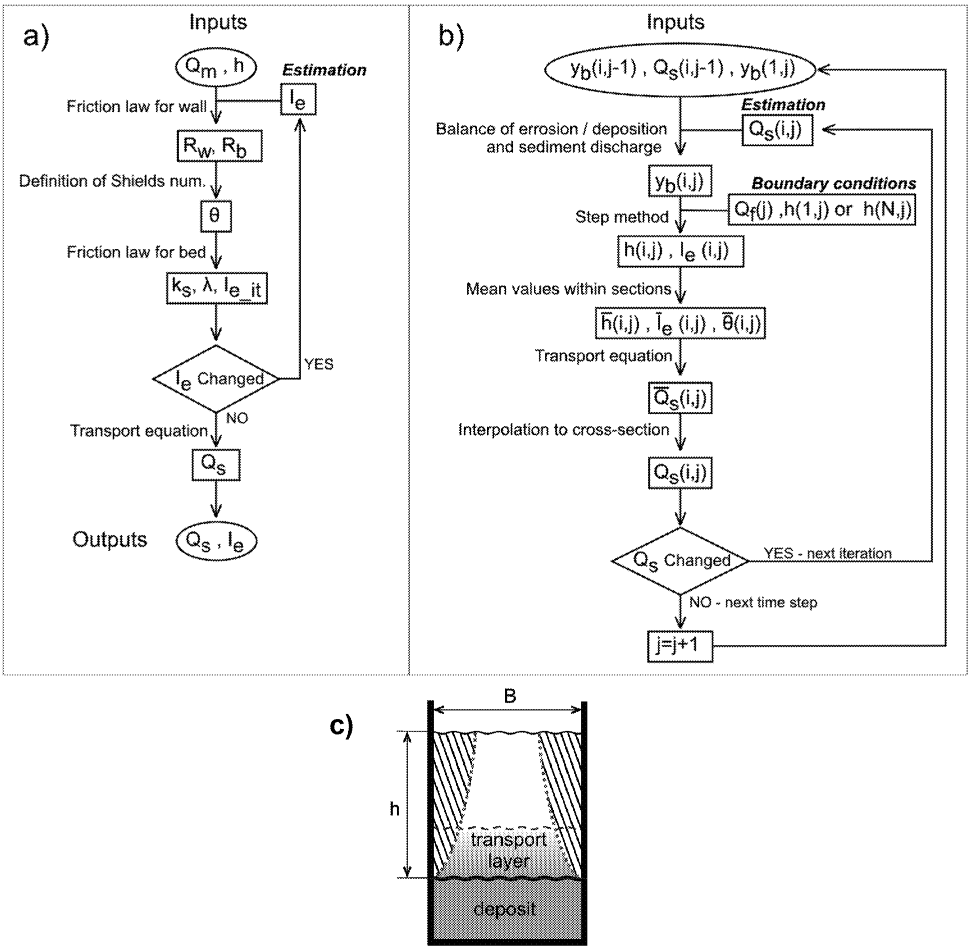


Fig.2: a) Flow chart of iterative evaluation of hydraulic gradient followed by calculation of sediment discharge; b) Flow chart of calculation of bed and water surface profile in one time step of model of unsteady sediment motion; c) Schematic cross section of modelled flow above deposit

be tilted to steep slopes. The flume is 0.2 m wide and 8 m long, additional dimensions are in Figure 3.

In the connecting pipes of the recirculating system, the discharge of mixture is measured using the magnetic flow meter (symbol Q in Figure 3) and the delivered concentration of sediment is determined from the measured pressure differences in the upgoing and downcoming vertical pipes (P in Figure 3). In the flume itself, measurements are carried out in four measuring cross sections (I, II, II, IV in Figure 3) over 4 m length of the flume. The water level is measured using ultrasonic probes (H in Figure 3). The position of the top of the bed and the position of the top of the shear layer are observed visually. Furthermore, the slope of the flume is measured.

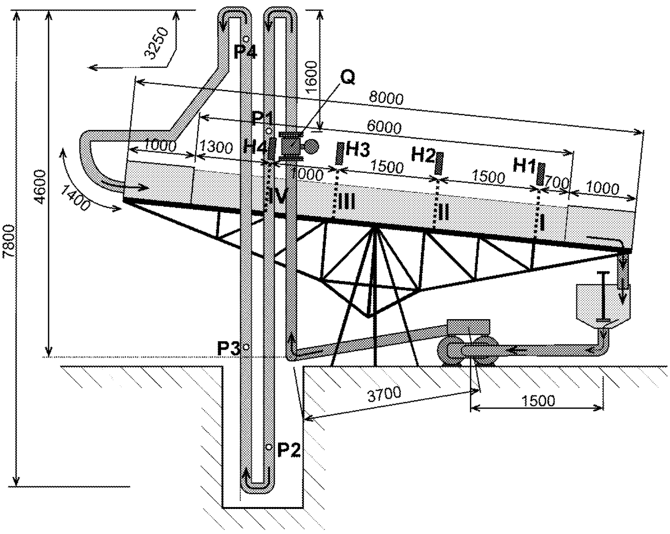


Fig.3: Tilting flume with recirculating system in Laboratory of Water Engineering of Czech Technical University in Prague; legend: H – ultrasonic gauges for water surface measurement, P – pressure sensors, Q – flow meter

4.2. Experiments with steady-state transport

The major objective of the sheet flow experiments under steady-state condition was to validate the transport- and friction formulae for the model of the gradually varied transport. The experiments were carried out with two narrow-graded fractions of glass beads (the finer fraction with $d_{50} = 1.49\,\mathrm{mm}$, $\varrho_\mathrm{s} = 2480\,\mathrm{kg/m^3}$ and the coarser fraction with $d_{50} = 3.00\,\mathrm{mm}$, $\varrho_\mathrm{s} = 2500\,\mathrm{kg/m^3}$).

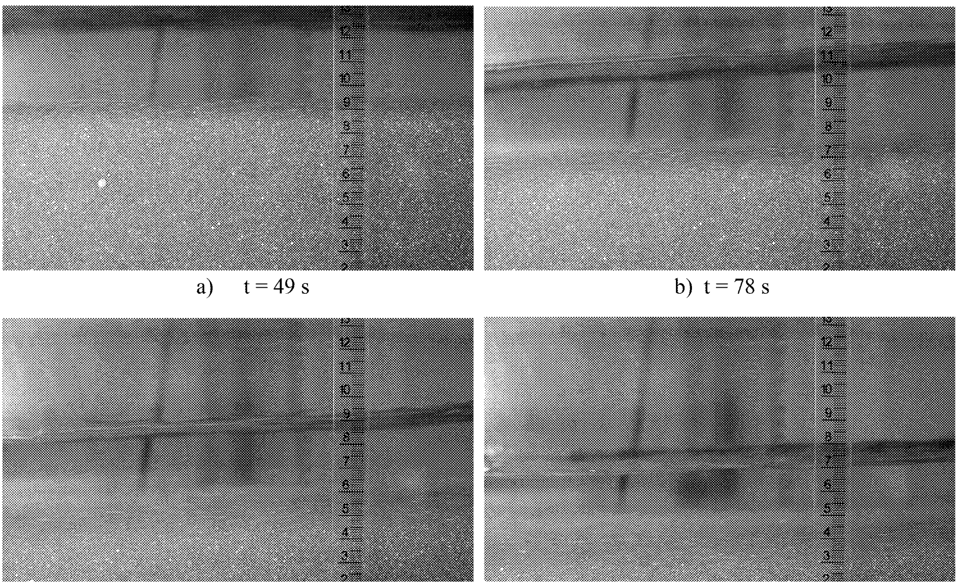
A typical test run is carried out for an installed constant mixture discharge (controlled by a pump speed) and a constant sediment discharge (controlled by positioning the overshot weir and inclination of flume). It leads to a development of the equilibrium longitudinal slope of the top of the bed and the flow depth in the flume.

4.3. Experiment with gradually-varied transport

To test the performance of the designed model for the gradually varied transport, an experiment was carried out with unsteady transport of sediment (the finer fraction of glass beads; $d_{50} = 1.49\,\mathrm{mm}$) in the flume under the condition of the upper plane bed regime. During the experiment, the flow was super-critical and intense transport of bed load through a shear layer developed gradually above the plane bed. The discharge of mixture was maintained approximately constant during the entire experiment.

The experiment took of about 5 minutes. A video sequence was taken of the entire event, the camera shot the flow near the last measuring cross section of the flume (Cross section I in Figure 3). The position of the bed was observed visually and the position of water surface was sensed by the ultrasound gauges at four cross-sections along the flume. A set of the position data was read out visually each 10 second together with the measured discharge of mixture and delivered concentration of solids.

The flow was uniform and steady at the beginning of experiment. The unsteady condition for the transport of sediment was introduced by a gradual lowering of the overshot weir in the outlet section of the flume (it took approximately 60 second to complete the manipulation). During the manipulation and for some time after its completion, the bed slope increased due to more intense erosion near the weir in the flume, the thickness of the shear layer and the discharge of the sediment increased as well (Figure 4). Because of the recirculation of the sediment through the system, the increased erosion in the output section of the flume resulted in an increase of the sediment discharge at the inlet to the flume. At the end of the experiment, a new equilibrium between the longitudinal slope of the bed and the discharge of sediment was established in the flume in approximately 4.5 minutes after the start of the weir manipulation.



a) t = 49 s

b) t = 78 s

c) t = 98 s

d) t = 130 s

Fig.4: Development of eroded bed, flow depth and shear layer during experiment with gradually varied transport of glass beads; legend: time from the start of the experiment

5. Results and discussion

5.1. Steady-state transport – comparison of formulae and experiments

Our transport formula of the MPM type (Equations 3 and 4) was calibrated with experimental data from pressurized pipes, but proves to perform well also for sheet flows carrying narrow-graded sediments in a flume (Figure 5a). For both glass bed fractions, the formula performed better than the formula used in TomSed which tended to underestimate the sediment flow rate.

The experimental results confirmed that $k_\mathrm{s}/d_{50} = f(V_\mathrm{t}^*, \theta)$ is able to cover the effects of different sediment properties (as grain size) and friction conditions. A calibration using the new experimental data produced

$$\frac{k_\mathrm{s}}{d_{50}} = 0.63\, {V_\mathrm{t}^*}^{1.40}\, \theta^{0.74} \tag{6}$$

and Figure 5b shows a good agreement between the experimental data and the predicted values using this correlation (except for the data with the most intense transport of sediment, i.e. for $k_\mathrm{s}/d_{50} > 15$). The TomSed friction formula seems to be quite insensitive to changes in friction conditions and tends to underestimate the equivalent roughness of the eroded bed, particularly at very high shear stresses where a thick shear layer develops.

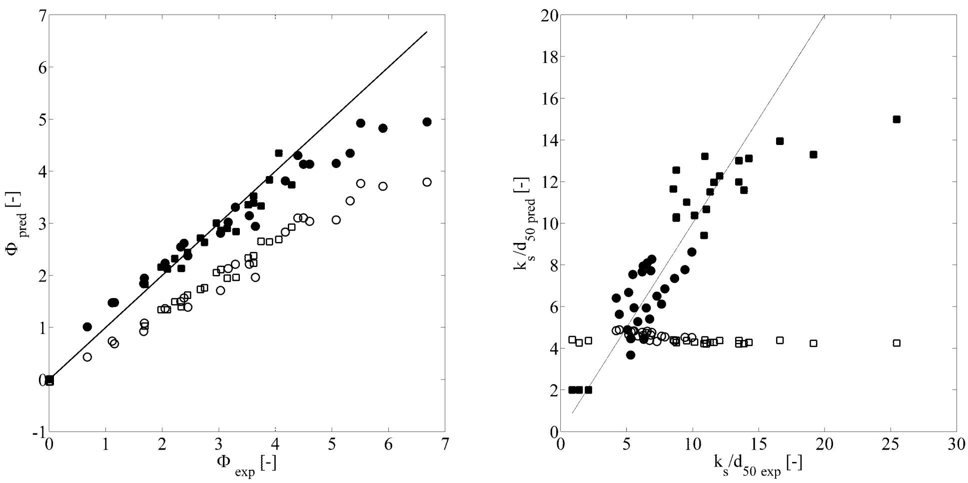


Fig.5: Comparison of measured and predicted parameters of bed load transport in upper plane bed regime; a) Dimensionless discharge of sediment; b) Dimensionless equivalent roughness of bed; legend: circle: $d_{50} = 1.5$ mm, square: $d_{50} = 3.0$ mm, blank points = Eqs. 1a–2, black points = Eq. 3–6

The friction- and transport formulae can be used in the model of steady-state uniform flow to predict the slope of the energy grade line (the hydraulic gradient) and the delivered concentration of the sediment (the delivered concentration is defined as ratio of the sediment discharge and the total discharge, $C_\mathrm{vd} = Q_\mathrm{s}/Q_\mathrm{m}$), see the flow chart in Figure 2a. Figure 6 reveals the results of the predictions using our formulae (Equations 3 to 6) and the formulae used in TomSed (Equations 1a to 2). It is not surprising that the predictions using Equations 3–6 exhibit better agreement with the data as both the transport- and friction formulae show closer match to the data than Equations 1a–2. It is interesting, however, that the difference in the accuracy is quite small for the predicted slopes of the energy grade line, at least at low values of the hydraulic gradient (Figure 6a). It indicates a quite small sensitivity of the I_e prediction to the bed roughness k_s/d_{50} at low values of the hydraulic gradient.

An important advantage of Equation 2 is that the hydraulic gradient is calculated directly from the flow velocity and the flow depth. Formulae employing the Shields parameter (as Equation 6) have to be solved in combination with the friction law and the Darcy-Weisbach equation to produce the hydraulic gradient. Because the Shields parameter itself is related to the hydraulic gradient, the solution is iterative (Figure 2a). The iterative solution often suffers with convergence issues – the phenomenon discussed recently [2]. In the light of these findings, an application of θ-based correlations for k_s/d_{50} seems rather questionable, at least in case of open channel flows.

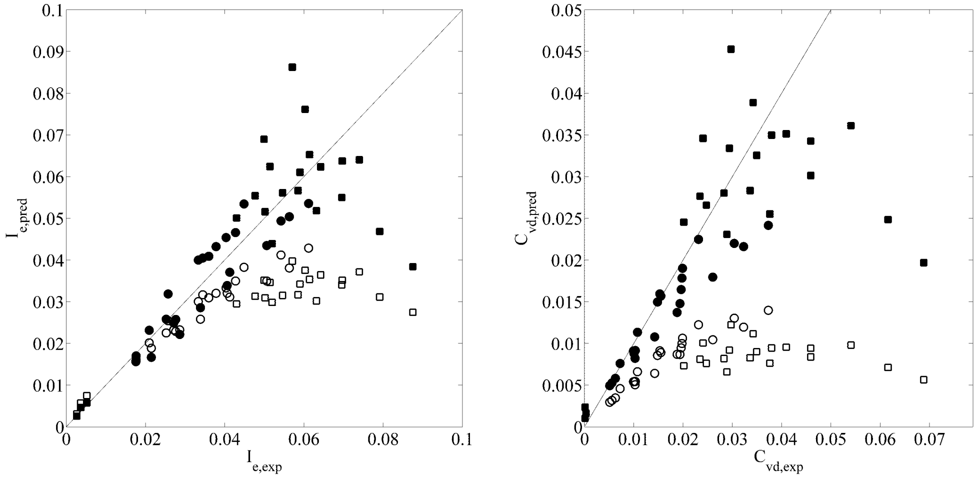


Fig.6: Comparison of measured and predicted parameters of bed load transport in upper plane bed regime; a) Slope of energy grade line; b) Delivered concentration of sediment; legend: circle: $d_{50} = 1.5$ mm, square: $d_{50} = 3.0$ mm, blank points = Eqs. 1a–2, black points = Eq. 3–6

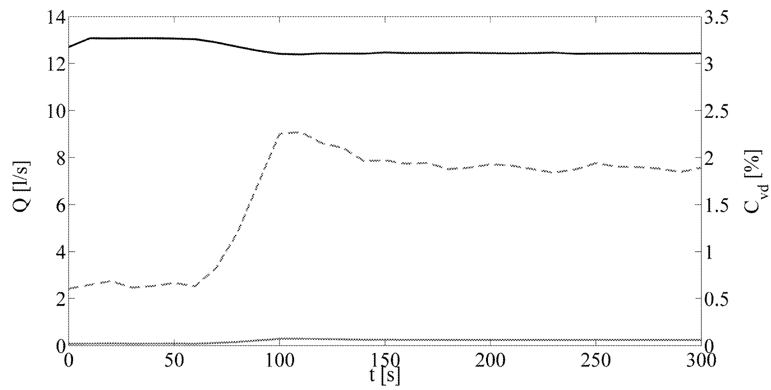


Fig.7: Boundary condition at the flume inlet: black solid line – discharge of water, grey solid line – discharge of sediment, grey dash line – delivered concentration

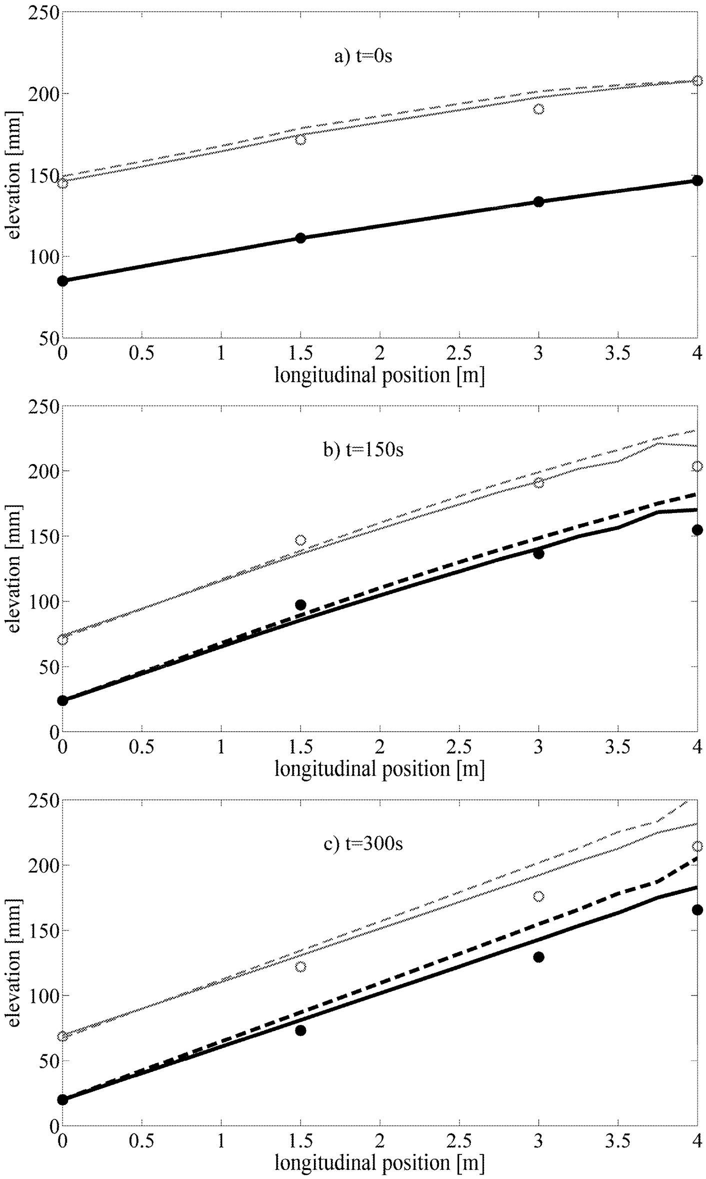


Fig.8: Development of longitudinal profiles of top of bed and water surface measured and simulated for flow with gradually varied transport of sediment in tilting flume at 4 locations (Cross sections I, II, III, IV) in 3 time steps ($t = 0$, 150, and 300 second); legend: black circles – measured position of top of bed, blank circles – measured position of water surface, black line – simulated top of bed, grey line – simulated water surface (solid lines – using Eqs. 3–6, dash lines – using Eqs. 1a–2)

5.2. Unsteady gradually-varied transport – comparison of model and experiment

In the model for the gradually varied transport of sediment, the length of the modelled flume region was 4 m and it was divided into 0.25 m long computational sections. The time step of 0.5 second was used in numerical simulations. The steady state at the beginning of the experiment served as the initial condition for the numerical simulation. Boundary conditions for the model were: the measured position of the top of the bed in the outlet cross section (Cross section I in Figure 3) of the flume, the measured depth of flow in the uppermost cross section (Cross section IV) in the flume (the flow was supercritical), and the discharges of water and sediment at the inlet to the flume (see Figure 7).

There is a section at the flume inlet, where the flow distributor is installed to make the flow calm and homogeneous. Solids accumulation is possible in this section and in parts of connecting pipes of the recirculating system. As a result, the solids discharge at the beginning of modelled region can differ from the discharge of solids measured in the vertical pipe. To take the effect of solids accumulation into account, a 3 m long section of the flume was considered at the beginning of flume, where the position of the bed was assumed to vary in the same way as the bed position calculated in the inlet cross-section of the modelled region. This enabled to simulate a transformation of the solids discharge hydrogram between the flow meter and the inlet cross sections of the modelled region.

Figure 8 shows results of two numerical simulations in the form of longitudinal profiles of the top of the bed and the water surface. The transport formula (Equations 1a, 1b) and friction formula (Equation 2) were used in one simulation (dash line) whereas our formulae (Equations 3, 4, 6) were used to produce the solid lines in Figure 8. The results are plotted in three time steps. For the first time step at $t = 0$ s (with the bed surface given by the initial condition), both sets of formulae provide good predictions of the water surface profile, although Equations 3–6 match the experiment slightly better.

At the next two time steps (one in the middle of the experiment, $t = 150$ s, and the other at the end of the experiment, $t = 300$ s), the position of the top of the bed is slightly overestimated by both simulations and Equations 3–6 get closer to the observed situation. The observed slight discrepancy can be partially attributed to uncertainty in sediment accumulation in front of the modelled part of the flume as discussed above.

6. Conclusions

A simple model is proposed for flow with gradually varied transport of bed load in the upper plane bed regime. The model contains alternative formulae for sediment transport and bed friction suitable for the condition of high bed shear and intense transport of bed load in open channel flow.

Experiments were carried out in the tilting flume to validate the formulae and the model for the required flow conditions. The results of the physical and mathematical modelling revealed that

- the transport formula (Equation 3 and 4) calibrated in pressurized pipes performs well also in open-channel flows at high bed shear;
- the roughness of the eroded bed with a developed shear layer is related to Shields parameter and neglecting this effect leads to considerable underestimation of the bed roughness;
- the slope of the energy line predicted using alternative roughness formulae (with and without Shields parameter) is not significantly different, i.e. a big underestimation of the roughness leads to a relatively small underestimation of the slope;
- the computation of the complete flow-transport model containing the roughness formula with the Shields parameter often crashes for open channel flows;

– the model for gradually varied transport performs quite well in predicting a development of longitudinal profiles of both the top of the bed and the water surface; further refinement of the model is work currently in progress.

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List of symbols

B – width of flume
d – diameter of grain
d_{50} – median diameter of grain
g – gravitational acceleration
h – depth of flow
i – index of cross-section
I_e – slope of energy grade line (hydraulic gradient)
j – index of time step
k_s – equivalent roughness of bed
y_b – position of top of bed
q – specific discharge
Q – volumetric discharge
Re_p – particle Reynolds number
R_w – hydraulic radius associated with wall
R_b – hydraulic radius associated with bed
V_m – mean velocity of mixture flow
V_t – terminal settling velocity of grain
V_t^* – dimensionless terminal settling velocity of grain
α – coefficient in Equation 3
β – coefficient in Equation 3
λ – Darcy-Weisbach friction coefficient
ϱ – density
ν – kinematic viscosity
θ – Shields parameter
Φ – Einstein parameter

Indices s, f, m, c – sediment, fluid, mixture, critical

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