EUROCODE 2 PROVISION AGAINST AMERICAN STANDARDS (ACI 209R-92 AND GARDNER&LOCKMAN MODELS) IN CREEP ANALYSIS OF COMPOSITE STEEL-CONCRETE SECTION

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The paper presents analysis of the stress-strain behaviour due to creep in statically determinate composite steel-concrete beam according to Eurocode 2, ACI209R-92 and Gardner&Lockman models. The mathematical model involves the equation of equilibrium, compatibility and constitutive relationship, i.e. an elastic law for the steel part and an integral-type creep law of Boltzmann-Volterra for the concrete part considering the above mentioned models. On the basis of the theory of the viscoelastic body of Maslov-Arutyunian-Trost-Zerna-Bažant for determining the redistribution of stresses in beam section between concrete plate and steel beam with respect to time 't', two independent Volterra integral equations of the second kind have been derived. Numerical method based on linear approximation of the singular kernel function in the integral equation is presented. Example with the model proposed is investigated.

Keywords: *steel-concrete section, integral equations, rheology, numerical method*

1. Introduction

The time-varying behaviour of composite steel-concrete members under sustained service loads drawn the attention of engineers who were dealing with the problems of their design more than 60 years [7–10]. Creep has a considerable impact upon the performance of composite beams, causing increased deflection as well as affecting stress distribution. Creep in concrete represents dimensional change in the material under the influence of sustained loading. Failure to include creep effects in the analysis of the composite steel-concrete beams may lead to excessive deformation and caused significant redistribution of stress between concrete plate and steel beam. In general, time-dependent deformation of concrete regarding creep phenomena may severely affect the serviceability, durability and stability of structures. In this paper we try to make the comparison in results, obtained from Eurocode 2, ACI209R-92 and G&L models [1] in creep analysis of composite steel concrete beams. Before that but we try to introduce the fundamentals of linear viscoelasticity of aging materials applied to concrete in the light of the great international school of hereditary mechanics of: Northwest University in USA [1], Politecnico of Turin [2] and ČVUT in Prague [3].

1.1. Fundamentals of linear viscoelasticity of aging materials applied to concrete

Concrete is considered to comply with the linear theory of viscoelasticity for aging materials [1–3]. Introducing the creep (compliance) function $J(t,\tau)$ and summing the responses

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to all uniaxial stress increments introduced at times τ, the following integral relations are obtained to model the responses at time t to sustained variable imposed stresses:

$$\varepsilon_{\mathrm{c}\sigma} = \int\limits_{t_0}^{t} \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\, J(t,\tau)\,\mathrm{d}\tau \ , \tag{1}$$

where: $\varepsilon_{\mathrm{c}\sigma} = \varepsilon_{\mathrm{c}}(t) - \varepsilon_{\mathrm{cn}}$ – stress-dependent strain, $\varepsilon_{\mathrm{c}}(t)$ – total strain at time t; $\varepsilon_{\mathrm{cn}}(t)$ – stress independent strain. Here the hereditary integrals must be intended as Stieltjes integrals in order to admit discontinuous stress histories $\sigma(t)$. If the law of variation of the imposed stress considered continuous after an initial finite step, the ordinary Rieman definition of the integral applies and Eq. (1) may be written in the form:

$$\varepsilon_{\mathrm{c}\sigma} = \sigma(t_0)\, J(t,t_0) + \int\limits_{t_0}^{t} \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\, J(t,\tau)\,\mathrm{d}\tau \ , \tag{2}$$

Equation (1) can be written in the operator form:

$$\varepsilon_{\mathrm{c}\sigma} = J\,\sigma \ ,$$

where J represents the uniaxial creep operators according Bažant [1].

The compliance function $J(t,\tau)$ is normally separated, on the basis of some convention, into an initial strain $J(\tau+\Delta,\tau)$ with $\Delta = t-\tau$ small, which is treated as instantaneous and elastic (nominal elastic strain) introducing a corresponding elastic modulus for the concrete $E_{\mathrm{c}}(\tau)$, and a creep strain $C(t,\tau)$, i.e.:

$$J(\tau+\Delta,\tau) \cong J(\tau,\tau) = \frac{1}{E_{\mathrm{c}}(\tau)} \ , \tag{3}$$

$$J(t,\tau) = \frac{1}{E_{\mathrm{c}}(\tau)} + C(t,\tau) \ . \tag{4}$$

A creep coefficient ϕ is normally introduced representing the ratio between the creep strain $C(t,\tau)$ and the initial strain at $t=\tau$ (e.g. in B3 model), or at a conventional age τ at loading (e.g. in CEB MC90 model, with $\tau = 28\,\text{days}$), i.e.:

$$J(t,\tau) = \frac{1}{E_{\mathrm{c}}(\tau)} + \frac{\phi(t,\tau)}{E_{\mathrm{c}}(\tau)} \ , \tag{5}$$

$$J(t,\tau) = \frac{1}{E_{\mathrm{c}}(\tau)} + \frac{\phi_{28}(t,\tau)}{E_{\mathrm{c28}}(\tau)} \ . \tag{6}$$

It must be noted that for structural analysis the compliance function $J(t,\tau)$ is of very importance. The conventional separation adopted in Eq. (4) and the value of Δ in Eq. (3) have no influence on the result of the analysis, except in the definition of the 'initial' (nominally elastic) state of deformation or of stress due to a sudden application of actions (respectively forces or imposed deformations). Such a state is in effect by itself a matter of convention depending on the procedures in the application of the actions at $t = t_0$ on the structure, on the initial time $t_0 + \Delta$ of observation of the effect and on the measuring procedures.

Time-dependent properties of concrete are fully characterized by $J(t,\tau)$. For realistic forms of $J(t,\tau)$ Eq. (1) is not integrable analytically, and numerical integration is mandatory.

In this respect, it must be considered that creep of concrete is a very complex phenomenon involving several interacting physical mechanisms at different scales of the microstructure, which are influenced by many factors; physical mechanisms and modelling criteria are still being debated [4]. Hence, a relatively high degree of sophistication in a realistic creep prediction model is unavoidable. A further cause of complexity of creep models to be used in structural analysis is a need to provide the average creep properties of the cross section for traditional simplified one-dimensional analysis of beams and frames. Such properties are characterized by non-uniform creep and shrinkage developing within the section, due to the drying process, and are influenced by the cross section geometry and stress distribution. As a consequence, the algebraic expressions for the prediction of the compliance function $J(t,\tau)$ for the average creep properties of the concrete cross section in drying conditions are inevitably rather complex (and less accurate than the constitutive law for a material point) in all the prevailing creep prediction models presented in recent literature [4], and/or considered by international associations [CEB (1993), ACI (2004)]. Standard numerical procedures for the solution of the integral equation of Volterra have been developed in the past by Bažant [5], and they have been incorporated in design manuals of Chiorino [2], bringing to an end a line of research, that has been investigated for more than fifty years. It has been in fact practically demonstrated that no formulation for $J(t,\tau)$ can be found that would be sufficiently accurate and would allow at the same time the precisely solution of the creep problem in the structures. The problem of the creep induced stress redistribution in the composite steel-concrete beams is dealt with within the theory of linear viscoelasticity for aging materials, which is normally considered appropriate for the creep analysis of structures of Bažant [1] and Chiorino [2]. Recent progresses of the theory of linear viscoelasticity, extended to materials like concrete showing a complex creep behavior, allow a rational interpretation of any kind of creep induced structural effects through very compact formulations on the basis of integral equations of Volterra. For this purpose, proper design aids to be inserted in manuals of bridge practice, can be offered to designers by a numerical solution of the Volterra integral equation, performed once for ever for the above mentioned creep prediction models suggested by international civil engineering associations, or of a powerful numerical solution for the automatic immediate calculation of development of internal forces in the time t from any given compliance function $J(t,t_0)$. This paper focuses on the development of these numerical solutions and theoretically consistent procedures, and of the corresponding design aids, for the evaluation of the creep induced stress redistribution in composite steel-concrete sections and on their application to steel-concrete bridges.

2. Basic equations for determining the creep coefficient according various provisions

2.1. Eurocode 2 model

The creep (compliance) function proposed by the 1990 CEB Model Code ('CEB-FIP' 1991) is given by the relationship:

$$J(t,t_0) = \frac{1}{E_\mathrm{c}(t_0)} + \frac{\phi(t,t_0)}{E_\mathrm{c}}\ ,$$

where $\phi(t,t_0)$ is the creep coefficient and $E_\mathrm{c}(t_0)$ and E_c are modulus of elasticity at the age of t_0 and 28 days, respectively [1]. The creep coefficient is evaluated with the following formula:

$$\phi(t,t_0) = \phi_{\mathrm{RH}}\,\beta(f_{\mathrm{cm}})\,\beta(t_0)\,\beta_{\mathrm{c}}(t-t_0)\ ,$$

where

$$\phi_{\mathrm{RH}} = 1 + \frac{1-\frac{RH}{100}}{0.46\left(\frac{h_0}{100}\right)^{0.33}}$$

is a factor to allow for the effect of relative humidity on the notional creep coefficient. RH is the relative humidity of the ambient environment in %,

$$\beta(f_{\mathrm{cm}}) = \frac{5.3}{\left(\frac{f_{\mathrm{cm}}}{10}\right)^{0.5}}$$

is a factor to allow for the effect of concrete strength on the notional creep coefficient,

$$\beta(t_0) = \frac{1}{0.1+(t_0)^{0.2}}$$

is a factor to allow for the effect of concrete age at loading on the notional creep coefficient (for continuous process we consider the function),

$$\beta(\tau) = \frac{1}{0.1+(\tau)^{0.2}}$$

is a function of aging, depending on the age of concrete and it characterizes the process of aging.

$$\beta_{\mathrm{c}}(t-t_0) = \left[\frac{t-t_0}{\beta_{\mathrm{H}}+(t-t_0)}\right]^{0.3}$$

is a function to describe the development of creep with time after loading,

$$\beta_{\mathrm{H}} = 150\left[1+\left(1.2\,\frac{RH}{100}\right)^{18}\right]\frac{h_0}{100} + 250 \le 1500$$

coefficient depending on the relative humidity (RH in %) and notional member size (h_0 in mm), where is the mean compressive strength of concrete at the age of 28 days (MPa) and $h_0 = 2\,A_{\mathrm{c}}/u$ is the notional size of member (mm) (A_{c} – the cross section and u – the perimeter of member in contact with the atmosphere). Constant Young's modulus is given by $E_{\mathrm{c}} = 10^4\,(f_{\mathrm{cm}})^{1/3}$. Variable Young's modulus is given by: $E_{\mathrm{c}}(t) = \beta_{\mathrm{cc}}^{0.5}\,E_{\mathrm{c}}$, where $E_{\mathrm{c}} = 10^4\,(f_{\mathrm{cm}})^{1/3}$ and $\beta_{\mathrm{cc}} = \exp[s\,(1-5.3/t^{0.5})]$, where $s = 0.25$ for normal and rapid hardening cements. So

$$E_{\mathrm{c}}(t) = 336190\,\mathrm{e}^{0.5\left[0.25\left(1-\frac{5.3}{\sqrt{t}}\right)\right]}\ ,$$

$\phi_0 = \phi_{\mathrm{RH}}\,\beta(f_{\mathrm{cm}})\,\beta(t_0)$ is a final creep.

2.2. ACI 209R-92 model

This is an empirical model developed by Branson and Christiason in 1971, with minor modification introduced in ACI 209R-92 [6]. The shape of the curve and ultimate value depend on several factors such as curing conditions, age of application of load, mixture proportioning, ambient temperature and humidity. The corrections factors are applied to ultimate values. Because creep equation for any period is a linear function of the ultimate values, however, the correction factors in this procedure may be applied to short-term creep. The creep function proposed by the ACI 209R-92 model, that presents the total stress-dependent strain by unit stress is given by the relationship :

$$J(t,t_0) = \frac{1}{E_{\mathrm{cmt0}}} + \frac{\phi(t,t_0)}{E_{\mathrm{cmt0}}} = \frac{1+\phi(t,t_0)}{E_{\mathrm{cmt0}}} \ ,$$

where $\phi(t,t_0)$ is the creep coefficient as the ratio of the creep strain to the elastic strain at the start of loading at the age t_0 (days) and E_{cm0} is the modulus of elasticity at the time of loading t_0 (MPa), respectively. The creep model proposed by ACI 209R-92 has two components that determine the ultimate asymptotic value and the time development of creep. The predicted parameter is not creep strain, but creep coefficient $\phi(t,t_0)$, defined as the ratio of the creep strain to the initial elastic strain. The creep coefficient is evaluated with the following formula :

$$\phi(t,t_0) = \phi_{\mathrm{u}}\,\beta_{\mathrm{c}}(t-t_0) \ ,$$

where $\phi(t,t_0)$ is the creep coefficient at the concrete age t due to a load applied at the age t_0, $(t-t_0)$ is the time since application of load, ϕ_{u} is the ultimate creep coefficient. For the standard conditions in the absence of specific creep date for local aggregates and conditions, the average value proposed for the ultimate creep coefficient ϕ_{u} is equal to 2.35 . For conditions other than standard conditions the value of the ultimate creep coefficient $\phi_{\mathrm{u}} = 2.35$ needs to be modified by six correction factors, depending on particular conditions, where $\phi_{\mathrm{u}} = 2.35\,\gamma_{\mathrm{c}}$ and $\gamma_{\mathrm{c}} = \gamma_{\mathrm{c,t0}}\,\gamma_{\mathrm{c,RH}}\,\gamma_{\mathrm{c,vs}}\,\gamma_{\mathrm{c,s}}\,\gamma_{\mathrm{c},\psi}\,\gamma_{\mathrm{c},\alpha}$; $\gamma_{\mathrm{c,t0}} = 1.25\,t_0^{-0.118}$ corresponds to $\beta(t_0)$ in CEB MC90, is a function of aging, depending on the age of concrete and it characterizes the process of aging; $\gamma_{\mathrm{c,RH}} = 1.27 - 0.67\,h$ for $h \geq 0.40$ is the ambient humidity factor, where the relative humidity h is in decimal (corresponds to ϕ_{RH} CEB MC90); $\gamma_{\mathrm{c,vs}} = 2\,(1 + 1.13\,\mathrm{e}^{-0.0213\,(V/S)})/3$ (corresponds to β_{H} in CEB MC90), where V is the specimen volume in mm^3 and S the specimen surface area in mm^2, allows to consider the size of member in terms of the volume-surface ratio; $\gamma_{\mathrm{c,s}} = 0.82 + 0.00624\,s$ is a slump factor, where $s = 75$ mm is the slump of fresh concrete; $\gamma_{\mathrm{c},\psi} = 0.88 + 0.0024\,\psi$ is fine aggregate factor, where $\psi = 40$ is the ratio of fine aggregate to total aggregate by weight expressed as percentage; $\gamma_{\mathrm{c},\alpha} = 0.46 + 0.009\,\alpha \geq 1$ is an air content factor, where $\alpha = 2$ is the air content in percentage.

$$\beta_{\mathrm{c}}(t-t_0) = \frac{(t-t_0)^{0.6}}{10 + (t-t_0)^{0.6}}$$

is a function to describe the development of creep with time after loading. The secant modulus of elasticity of concrete E_{cmt0} at any time t_0 of loading is given by $E_{\mathrm{cmt0}} = 0.043\,\varrho_{\mathrm{c}}^{1.5}\,\sqrt{f_{\mathrm{cmt0}}}$ MPa, where ϱ_{c} is the unit weight of concrete (kg/m^3) and f_{cmt0} is the mean concrete compressive strength at the time of loading (MPa).The general equation for predicting compressive strength at an time t is given by $f_{\mathrm{cmt0}} = f_{\mathrm{cm28}}\,t/(a + b\,t)$, where f_{cm28} is the concrete mean compressive strength of 28 days in MPa; a (in days) and b are constants and t is the age of the concrete.

2.3. Gardner&Lockman 2000 model

It is modified Atlanta 97 model 1993, which itself was influenced by CEB MC90-99 [6]. The compliance expression is based on the modulus of elasticity at 28 days, instead of the modulus of elasticity at age of loading. This model includes a term for drying before loading, which applies to both basic and drying creep. Required parameters: Age of concrete when drying starts, usually taken as the age at the end of moist curing (days); Age of concrete at loading (days); Concrete mean compressive strength at 28 days (MPa or psi); Concrete mean compressive strength at loading (MPa or psi); Modulus of elasticity of concrete at 28 days (MPa or psi); Modulus of elasticity of concrete at loading (MPa or psi); Relative humidity expressed as a decimal; and Volume-surface ratio (mm or in.); The creep (compliance) function proposed by the (Gardner and Lockman 2001), is composed of the elastic and creep strains. The elastic strain is reciprocal of the modulus of elasticity at the age of loading E_{cmt0} and the creep strain is the 28 day creep coefficient divided by the modulus of elasticity at 28 days E_{cm28}. The creep coefficient $\phi_{28}(t,t_0)$ is the ratio of the creep strain to the elastic strain due to the load applied at the age of 28 days. So,

$$J(t,t_0)=\frac{1}{E_{\mathrm{cmt0}}(t_0)}+\frac{\phi_{28}(t,t_0)}{E_{\mathrm{cmt0}}}\ ,$$

where E_{cmt0} is the modulus of elasticity of concrete at the time of loading t_0, E_{cm28} is the mean modulus of elasticity concrete at 28 days (MPa), $1/E_{\mathrm{cmt0}}$ represents the initial strain per unit stress at loading. $\phi(t,t_0)$ gives the ratio of the creep strain since the start of loading at the age t_0 to the elastic strain due to a constant stress applied at a concrete age of 28 days. The 28-day creep coefficient $\phi_{28}(t,t_0)$ is calculated using the next formulae:

$$\phi_{28}(t,t_0)=\Phi(t_{\mathrm{c}})\left[2\,\frac{(t-t_0)^{0.3}}{(t-t_0)^{0.3}+14}+\left(\frac{7}{t_0}\right)^{0.5}\left(\frac{(t-t_0)}{(t-t_0)+7}\right)^{0.5}+\right.$$
$$\left.+\,2.5\,(1-1.086\,h^2)\left(\frac{(t-t_0)}{(t-t_0)+0.12\left(\frac{V}{S}\right)^2}\right)^{0.5}\right].$$

The creep coefficient includes three terms. The first two terms are required to calculate the basic creep and the third term is for the drying creep. At a relative humidity of 0.96 there is only basic creep. There is no drying creep. $\Phi(t_{\mathrm{c}})$ is the correction term for the effect of drying before loading. If $t_0=t_{\mathrm{c}}$; $\Phi(t_{\mathrm{c}})=1$. When $t_0>t_{\mathrm{c}}$,

$$\Phi(t_{\mathrm{c}})=\left[1-\left(\frac{(t_0-t_{\mathrm{c}})}{(t_0-t_{\mathrm{c}})+0.12\left(\frac{V}{S}\right)^2}\right)^{0.5}\right]^{0.5}\ ,$$

t_0 is an age of concrete at loading, (days), t_{c} is an age of concrete when drying starts at the end of moist curing, (days). To calculate relaxation, remains constant at the initial value throughout the relaxation period. For creep recovery calculation, $\Phi(t_{\mathrm{c}})$ remains constant at the value at the age of loading. If experimental values are not available the modulus of elasticity E_{cmt} at any time t is given by $E_{\mathrm{cmt}}=3500+4500\sqrt{f_{\mathrm{cmt}}}$, where strength development with time can be calculated from the compressive strength using equation $f_{\mathrm{cmt}}=\beta_{\mathrm{e}}^2\,f_{\mathrm{cm28}}$. This equation is a modification of the CEB strength-development relationship. So,

$$\beta_{\mathrm{e}}=\exp\left[\frac{s}{2}\left(1-\sqrt{\frac{28}{t}}\right)\right]\ ,$$

where $s = 0.4$ is CEB (1993) style strength-development parameter and β_e relates strength development to cement type. Relationship between specified and mean compressive strength of concrete can be estimated from the equation: $f_{cm28} = 1.1\, f'_c + 5.0$. According [6] many basic features of Model GL are questionable on the basis of the current understanding of the mechanics and physics of concrete shrinkage and creep, and violate the guidelines published by a RILEM Committee. Some of them are as follows: disagreement with diffusion theory, the effect of age on creep according this model is far too weak and too short-lived, the creep coefficient for the additional creep due to drying is given in this model by a curve that does not have a bounded final value, the creep recovery curve calculated according to the principle of superposition is violated by the GL model (Bažant&Baweja 2000).

3. Basic equations of equilibrium

Let us denote both normal forces and bending moments in the cross-section of the plate and the girder after the loading in the time $t = 0$ with $N_{c,0}$, $M_{c,0}$, $N_{a,0}$, $M_{a,0}$ and with $N_{c,r}(t)$, $M_{c,r}(t)$, $N_{a,r}(t)$, $M_{a,r}(t)$ a new group of normal forces and bending moments, arising due to creep and shrinkage of concrete. For a composite bridge girder with $J_c = A_c(n\, I_c)\, n/(A_s\, I_s) \leq 0.2$ according to the suggestion of Sonntag [2] we can write the equilibrium conditions in time t as follows:

$$N(t) = 0\ , \qquad N_{c,r}(t) = N_{a,r}(t)\ , \tag{7a}$$

$$\sum M(t) = 0\ , \qquad M_{c,r}(t) + N_{c,r}(t) = M_{a,r}(t)\ . \tag{7b}$$

Due to the fact that the problem is a twice internally statically indeterminate system, the equilibrium equations (7) are not sufficient to solve it. It is necessary to produce two additional equations in the sense of compatibility of deformations of both steel girder and concrete slab in time t (Fig. 1).

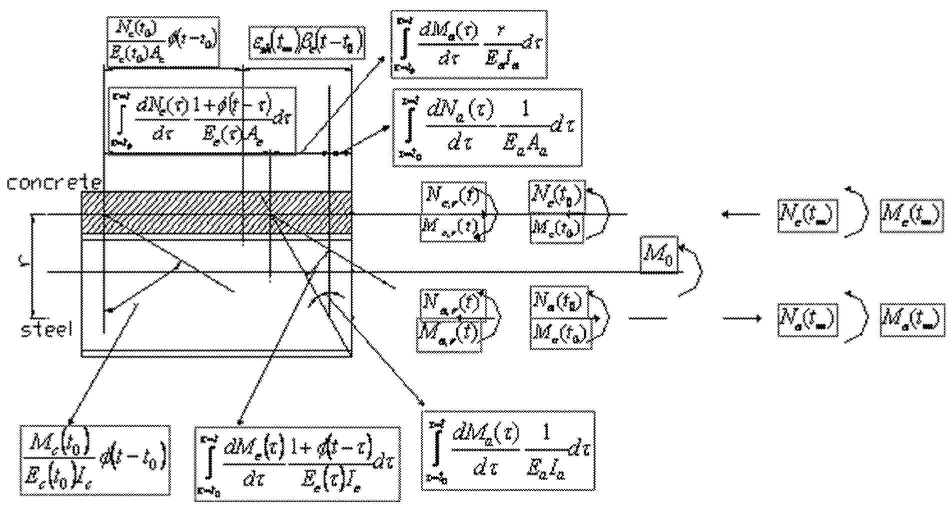

Fig.1: Mechanic-mathematical model for deformations in cross-section, regarding creep of the concrete

4. Deriving of the generalized mechanic-mathematical model according EC 2

4.1. Strain compatibility on the contact surfaces between the concrete and steel members of composite girder

For constant elasticity module of concrete strain compatibility on the contact surfaces between the concrete and steel members of composite girder is:

$$\begin{aligned}
&\varepsilon_{\mathrm{sh}}(t_0)\, f(t-t_0) + \frac{N_{\mathrm{c},0}}{E_{\mathrm{c}}(t_0)\, A_{\mathrm{c}}}\left[1+\phi_{\mathrm{RH}}\,\beta(f_{\mathrm{cm}})\,\beta(t_0)\,\beta_{\mathrm{c}}(t-t_0)\right] - \\
&\quad - \frac{1}{E_{\mathrm{c}}(t_0)\, A_{\mathrm{c}}}\int_{t_0}^{t}\frac{\mathrm{d}N_{\mathrm{c,r}}(\tau)}{\mathrm{d}\tau}\left[1+\phi_{\mathrm{RH}}\,\beta(f_{\mathrm{cm}})\,\beta(\tau)\,\beta_{\mathrm{c}}(t-\tau)\right]\mathrm{d}\tau + \\
&\qquad + \frac{N_{\mathrm{a},0}}{E_{\mathrm{a}}\, A_{\mathrm{a}}} - \frac{1}{E_{\mathrm{a}}\, A_{\mathrm{a}}}\int_{t_0}^{t}\frac{\mathrm{d}N_{\mathrm{a,r}}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau = \frac{M_{\mathrm{a},0}}{E_{\mathrm{a}}\, I_{\mathrm{a}}}\, r + r\,\frac{1}{E_{\mathrm{a}}\, I_{\mathrm{a}}}\int_{t_0}^{t}\frac{\mathrm{d}M_{\mathrm{a,r}}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau\ .
\end{aligned}$$

4.2. Compatibility of curvatures

Compatibility of curvatures when $\tau = t$ is

$$\begin{aligned}
&\frac{M_{\mathrm{c},0}}{E_{\mathrm{c}}(t_0)\, I_{\mathrm{c}}}\left[1+\phi_{\mathrm{RH}}\,\beta(f_{\mathrm{cm}})\,\beta(t_0)\,\beta_{\mathrm{c}}(t-t_0)\right] - \\
&\quad - \frac{1}{E_{\mathrm{c}}(t_0)\, I_{\mathrm{c}}}\int_{t_0}^{t}\frac{\mathrm{d}M_{\mathrm{c,r}}(\tau)}{\mathrm{d}\tau}\left[1+\phi_{\mathrm{RH}}\,\beta(f_{\mathrm{cm}})\,\beta(\tau)\,\beta_{\mathrm{c}}(t-\tau)\right]\mathrm{d}\tau = \\
&\qquad = \frac{M_{\mathrm{a},0}}{E_{\mathrm{a}}\, I_{\mathrm{a}}} + \frac{1}{E_{\mathrm{a}}\, I_{\mathrm{a}}}\int_{t_0}^{t}\frac{\mathrm{d}M_{\mathrm{a,r}}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau\ .
\end{aligned}$$

Here is the place to explain the meaning of the second term of the above mentioned integral relations. This type of integral, known as a hereditary integral of Stieltjes expresses time history of loading. (If in 1854 Georg F. B. Rieman gave a set of necessary and sufficient conditions under which a bounded function is said to be integrable and if Rieman dominated the field of integration until 1894, then a Dutch mathematician named Thomas Jan Stieltjes (1856–1894) developed the Rieman-Stieltjes integral while investigating a very specific problem concerning a thin rod of nonuniformly distributed mass). Since in our case the stress history $\sigma_{\mathrm{c,r}}(\tau)$, which represents the distribution of stress between concrete plate and steel beam, is continuous summing strain histories due to all small stress increments before time t yields to perfect satisfying the strain compatibilities.

After integrating the two equations by parts and using (7a) and (7b) for assessment of normal forces $N_{\mathrm{c,r}}(t)$ and bending moment $M_{\mathrm{c,r}}(t)$ two linear integral Volterra equations of the second kind are derived.

$$\begin{aligned}
N_{\mathrm{c,r}}(t) = \lambda_{\mathrm{N}}\int_{t_0}^{t} N_{\mathrm{c,r}}(\tau)\,\frac{\mathrm{d}}{\mathrm{d}\tau}\left[1+\phi_{\mathrm{RH}}\,\beta(f_{\mathrm{cm}})\,\beta(\tau)\,\beta_{\mathrm{c}}(t-\tau)\right]\mathrm{d}\tau + \\
+ \lambda_{\mathrm{N}}\, N_{\mathrm{c},0}\,\phi_{\mathrm{RH}}\,\beta(f_{\mathrm{cm}})\,\beta(t_0)\,\beta_{\mathrm{c}}(t-t_0) + \lambda_{\mathrm{N}}\, N_{\mathrm{sh}}\,\beta_{\mathrm{c}}(t-t_0)\ ,
\end{aligned} \tag{8}$$

$$M_{\mathrm{c,r}}(t) = \lambda_{\mathrm{M}} \int\limits_{t_0}^{t} M_{\mathrm{c,r}}(\tau)\, \frac{\mathrm{d}}{\mathrm{d}\tau}[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(\tau)\, \beta_{\mathrm{c}}(t-\tau)]\, \mathrm{d}\tau + \\ + \lambda_{\mathrm{M}}\, M_{\mathrm{c},0}\, \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta_{\mathrm{c}}(t-t_0) - \lambda_{\mathrm{M}}\, \frac{E_{\mathrm{c}}\, I_{\mathrm{c}}}{E_{\mathrm{a}}\, I_{\mathrm{a}}}\, N_{\mathrm{c,r}}(t)\, r \ , \tag{9}$$

where

$$\lambda_{\mathrm{N}} = \left[1 + \frac{E_{\mathrm{c}}\, A_{\mathrm{c}}}{E_{\mathrm{a}}\, A_{\mathrm{a}}} \left(1 + \frac{A_{\mathrm{a}}\, r^2}{I_{\mathrm{a}}}\right)\right]^{-1} \ , \qquad \lambda_{\mathrm{M}} = \left[1 + \frac{E_{\mathrm{c}}\, I_{\mathrm{c}}}{E_{\mathrm{a}}\, I_{\mathrm{a}}}\right]^{-1} \ .$$

5. Deriving of the mechanic-mathematical model according ACI 209R-92

Using above mentioned approach, for constant elasticity module of concrete for assessment of normal forces $N_{\mathrm{c,r}}(t)$ and bending moment $M_{\mathrm{c,r}}(t)$ two linear integral Volterra equations of the second kind are derived.

$$N_{\mathrm{c,r}}(t) = \lambda_{\mathrm{N}} \int\limits_{t_0}^{t} N_{\mathrm{c,r}}(\tau)\, \frac{\mathrm{d}}{\mathrm{d}\tau}[1 + 2.35\, \gamma_{\mathrm{c}5}\, \beta(\tau)\, \beta_{\mathrm{c}}(t-\tau)]\, \mathrm{d}\tau + \\ + \lambda_{\mathrm{N}}\, N_{\mathrm{c},0}\, 2.35\, \gamma_{\mathrm{c}5}\, \beta(t_0)\, \beta_{\mathrm{c}}(t-t_0) \ , \tag{10}$$

$$M_{\mathrm{c,r}}(t) = \lambda_{\mathrm{M}} \int\limits_{t_0}^{t} M_{\mathrm{c,r}}(\tau)\, \frac{\mathrm{d}}{\mathrm{d}\tau}[1 + 2.35\, \gamma_{\mathrm{c}5}\, \beta(\tau)\, \beta_{\mathrm{c}}(t-\tau)]\, \mathrm{d}\tau + \\ + \lambda_{\mathrm{M}}\, M_{\mathrm{c},0}\, 2.35\, \gamma_{\mathrm{c}5}\, \beta(t_0)\, \beta_{\mathrm{c}}(t-t_0) - \lambda_{\mathrm{M}}\, \frac{E_{\mathrm{c}}\, I_{\mathrm{c}}}{E_{\mathrm{a}}\, I_{\mathrm{a}}}\, N_{\mathrm{c,r}}(t)\, r \ , \tag{11}$$

6. Deriving of the generalised mechanic-mathematical model according Gardner&Lockman model

Using above mentioned approach for constant elasticity module of concrete for assessment of normal forces $N_{\mathrm{c,r}}(t)$ and bending moment $M_{\mathrm{c,r}}(t)$ two linear integral Volterra equations of the second kind are derived.

$$N_{\mathrm{c,r}}(t) = \lambda_{\mathrm{N}} \int\limits_{t_0}^{t} N_{\mathrm{c,r}}(\tau)\, \frac{\mathrm{d}}{\mathrm{d}\tau}[1 + \Phi(t_{\mathrm{c}})\, \beta_{\mathrm{c}}(t-\tau)]\, \mathrm{d}\tau + \\ + \lambda_{\mathrm{N}}\, N_{\mathrm{c},0}\, \Phi(t_{\mathrm{c}})\, \beta_{\mathrm{c}}(t-t_0) \ , \tag{12}$$

$$M_{\mathrm{c,r}}(t) = \lambda_{\mathrm{M}} \int\limits_{t_0}^{t} M_{\mathrm{c,r}}(\tau)\, \frac{\mathrm{d}}{\mathrm{d}\tau}[1 + \Phi(t_{\mathrm{c}})\, \beta_{\mathrm{c}}(t-\tau)]\, \mathrm{d}\tau + \\ + \lambda_{\mathrm{M}}\, M_{\mathrm{c},0}\, \Phi(t_{\mathrm{c}})\, \beta_{\mathrm{c}}(t-t_0) - \lambda_{\mathrm{M}}\, \frac{E_{\mathrm{c}}\, I_{\mathrm{c}}}{E_{\mathrm{a}}\, I_{\mathrm{a}}}\, N_{\mathrm{c,r}}(t)\, r \ , \tag{13}$$

7. Numerical method for solution of integral equation of Volterra

Integral equations (9–13) are weakly singular Volterra integral equation of the second kind:

$$y(t) = g(t) + \lambda \int_{t_0}^{t} K(t,\tau)\, y(\tau)\, \mathrm{d}\tau \ , \quad t \in [t_0, T] \ , \quad 0 < t_0 < T < \infty \ . \tag{14}$$

So in our case discontinuous kernel functions $K(t,\tau)$ have an infinite singularity of type $(t-\tau)^{\gamma-1}$, $\gamma > 0$.

In order to solve (14), we use the idea of product integration by considering the special case of

$$y(t) = g(t) + \lambda \int_{t_0}^{t} L(t,\tau)\, (t-\tau)^{\gamma-1}\, y(\tau)\, \mathrm{d}\tau \ , \qquad t \in [t_0, T] \ , \quad 0 < t_0 < T < \infty \ , \quad 0 < \gamma < 1 \ . \tag{15}$$

where given functions $g(t)$ and $L(t,\tau)$ are sufficiently smooth which guarantee the existence and uniqueness of the solution (see Yosida (1960), Miller&Feldstein (1971) [7, 8, 9, 10]).

To solve (15) we use the method called product trapezoidal rule.

Let $n \geq 1$ be an integer and points $\{t_j = t_0 + j\,h\}_{j=0}^{n} \in [t_0, T]$. Then for general $y(t) \in C_{[t_0,T]}$ we define

$$(L(t,\tau)\, y(\tau))_n = \frac{1}{h}\left[(t_j - \tau)\, L(t, t_{j-1}) + (\tau - t_{j-1})\, L(t, t_j)\, y(t_j)\right] \ , \tag{16}$$

for $f_{j-1} \leq \tau \leq t_j$, $t \in [t_0, T]$.

This is piecewise linear in τ and it interpolates $L(t,\tau)\, y(\tau)$ at $\tau = t_0, \ldots, t_n$. Using numerical approximation (16) we obtain the following method for solving the integral equation (15):

$$\tilde{y}(t_i) = g(t_i) + \lambda \sum_{j=0}^{i} \omega_{n,j}(t_i)\, [L(t_i, t_j)\, \tilde{y}_n(t_j)] \qquad \text{for} \quad i = 0, 1, \ldots, n \ , \tag{17}$$

with weights

$$\omega_{n,0}(t_i) = \frac{1}{h} \int_{t_0}^{t_1} (t_1 - \tau)\, (t_i - \tau)^{\gamma-1}\, \mathrm{d}\tau \ ,$$

$$\omega_{n,n}(t_n) = \frac{1}{h} \int_{t_{n-1}}^{t_n} (\tau - t_{n-1})\, (t_n - \tau)^{\gamma-1}\, \mathrm{d}\tau \ ,$$

$$\omega_{n,j}(t_i) = \frac{1}{h} \int_{t_{j-1}}^{t_j} (\tau - t_{j-1})\, (t_i - \tau)^{\gamma-1}\, \mathrm{d}\tau + \frac{1}{h} \int_{t_j}^{t_{j+1}} (t_{j+1} - \tau)\, (t_i - \tau)^{\gamma-1}\, \mathrm{d}\tau \ ,$$

for $i = 0, 1, \ldots, n$.

Calculating analytically the weights, we compute the approximate solution values $y_n(t_i)$ from the system (17).

Theorem 1. Consider the numerical approximation defined with piecewise linear interpolation (17). Then for all sufficiently large n, the equation (15) is uniquely solvable and moreover if $y(t) \in C^2_{[t_0,T]}$, then we have

$$||y - y_n|| \leq \frac{c\,h^2}{8} \max_{t_0 \leq t,\tau \leq T} \left| \frac{\partial^2 L(t,\tau)\, y(\tau)}{\partial \tau^2} \right| . \tag{18}$$

Since $L(t,.) \in C^2_{[t_0,T]}$, $t_0 \leq t \leq T$ the estimate (18) is immediate consequence of theorem 4.2.1 in Atkinson [10].

8. Numerical example

The method presented in the previous paragraph is now applied to a simply supported beam, subjected to a uniform load, whose cross section is shown in Fig. 2.

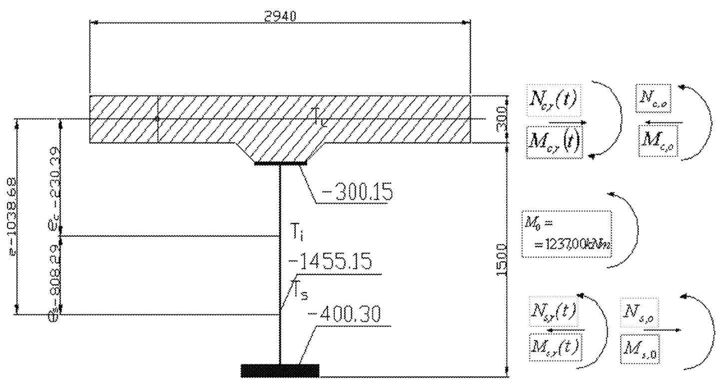

Fig.2: Composite beam with cross-section characteristic

8.1. The following parameters are according EUROCODE 2 model

$$E_{\mathrm{c}} = 3.2{\times}10^4\,\mathrm{MPa}\ , \quad E_{\mathrm{a}} = 2.1{\times}10^5\,\mathrm{MPa}\ , \quad A_{\mathrm{c}} = 8820\,\mathrm{cm}^2\ , \quad A_{\mathrm{a}} = 383.25\,\mathrm{cm}^2\ ,$$

$$n = \frac{E_{\mathrm{a}}}{E_{\mathrm{c}}} = 6.56\ , \quad I_{\mathrm{c}} = 661500\,\mathrm{cm}^4\ , \quad I_{\mathrm{a}} = 1217963.7\,\mathrm{cm}^4\ , \quad r_{\mathrm{c}} = 23.039\,\mathrm{cm}\ ,$$

$$r_{\mathrm{a}} = 80.829\,\mathrm{cm}\ , \quad r = 103.868\,\mathrm{cm}\ , \quad A_{\mathrm{i}} = 2453.05\,\mathrm{cm}^2\ , \quad I_{\mathrm{i}} = 4536360.758\,\mathrm{cm}^4\ ,$$

$$M_0 = 1237\,\mathrm{kNm}\ , \quad N_{\mathrm{c},0} = 846.60\,\mathrm{kN}\ , \quad M_{\mathrm{c},0} = 27.56\,\mathrm{kNm}\ , \quad M_{\mathrm{a},0} = 330.13\,\mathrm{kNm}\ ,$$

$$\lambda_{\mathrm{N}} = \left[1 + \frac{E_{\mathrm{c}}\,A_{\mathrm{c}}}{E_{\mathrm{a}}\,A_{\mathrm{a}}}\left(1 + \frac{A_{\mathrm{a}}\,r^2}{I_{\mathrm{a}}}\right)\right]^{-1} = 0.060545358\ , \quad \lambda_{\mathrm{M}} = \left[1 + \frac{E_{\mathrm{c}}\,I_{\mathrm{c}}}{E_{\mathrm{a}}\,I_{\mathrm{a}}}\right]^{-1} = 0.922950026\ ,$$

$$h_0 = \frac{2\,A\,C}{u} = 300\,\mathrm{mm}\ , \quad \beta_{\mathrm{H}} = 150\left[1 + \left(1.2\,\frac{80}{100}\right)^{18}\right]\frac{h_0}{100} + 250 = 915.82 < 1500\ ,$$

$$\beta(f_{\mathrm{cm}}) = \left.\frac{5.3}{(f_{\mathrm{cm}}/10)^{0.5}}\right|_{f_{\mathrm{cm}}=30} = 3.06\ , \quad \beta(t_0) = \left.\frac{1}{0.1 + t_0^{0.2}}\right|_{t_0=60} = 0.4223\ ,$$

$$\phi_{\mathrm{RH}} = 1 + \left.\frac{1 - RH/100}{0.46\sqrt[3]{h_0/100}}\right|_{RH=80,h_0=300} = 1.3014\ , \quad \phi_0 = \phi_{\mathrm{RH}}\,\beta(f_{\mathrm{cm}})\,\beta(t_0) = \mathbf{1.6817}\ ,$$

$$\beta_{\mathrm{c}}(36500-60) = 0.9925811\ , \quad \phi_{t=36500} = \phi_0\,\beta_{\mathrm{c}}(36500-60) = 1.669242\ .$$

8.2. The following parameters are according ACE 209R-92 model

$$E_{\mathrm{c}} = 2.8178{\times}10^4\,\mathrm{MPa}\ , \quad E_{\mathrm{a}} = 2.1{\times}10^5\,\mathrm{MPa}\ , \quad A_{\mathrm{c}} = 8820\,\mathrm{cm}^2\ , \quad A_{\mathrm{a}} = 383.25\,\mathrm{cm}^2\ ,$$

$$n = \frac{E_{\mathrm{a}}}{E_{\mathrm{c}}} = 7.452\ , \quad I_{\mathrm{c}} = 661500\,\mathrm{cm}^4\ , \quad I_{\mathrm{a}} = 1207963.7\,\mathrm{cm}^4\ , \quad r_{\mathrm{c}} = 25.407\,\mathrm{cm}\ ,$$

$$r_{\mathrm{a}} = 78.463\,\mathrm{cm}\ , \quad r = 103.870\,\mathrm{cm}\ , \quad A_{\mathrm{i}} = 1566.8248\,\mathrm{cm}^2\ , \quad I_{\mathrm{i}} = 4420140.76\,\mathrm{cm}^4\ ,$$

$$M_0 = 1237\,\mathrm{kNm}\ , \quad N_{\mathrm{c},0} = 837.286\,\mathrm{kN}\ , \quad M_{\mathrm{c},0} = 24.716\,\mathrm{kNm}\ , \quad M_{\mathrm{a},0} = 338.05\,\mathrm{kNm}\ ,$$

$$\lambda_{\mathrm{N}} = \left[1 + \frac{E_{\mathrm{c}}\,A_{\mathrm{c}}}{E_{\mathrm{a}}\,A_{\mathrm{a}}}\left(1 + \frac{A_{\mathrm{a}}\,r^2}{I_{\mathrm{a}}}\right)\right]^{-1} = 0.068220902\ , \quad \lambda_{\mathrm{M}} = \left[1 + \frac{E_{\mathrm{c}}\,I_{\mathrm{c}}}{E_{\mathrm{a}}\,I_{\mathrm{a}}}\right]^{-1} = 0.93155\ .$$

Mean 28-day strength: $f_{\mathrm{cm28}} = 33.3\,\mathrm{MPa}$ ($f_{\mathrm{cm28}} = 33.0\,\mathrm{MPa}$ according to CEB MC90-99). Mean 28-day elastic modulus $E_{\mathrm{cm28}} = 28178\,\mathrm{MPa}$ ($E_{\mathrm{cm28}} = 32009\,\mathrm{MPa}$ according to CEB MC90-99).
$\varrho_{\mathrm{c}} = 2345\,\mathrm{kg/m}^3$, $E_{\mathrm{c}}(\tau) = E_{\mathrm{c}}(t_0) = E_{\mathrm{const}} = E_{\mathrm{cmt0}} = 0.43 \times 2345^{1.5}\sqrt{33.30} = 28178\,\mathrm{MPa}$, according to ACI 209R-92. $\gamma_{\mathrm{c,t0}} = 1.25\,t_0^{-0.118}$ corresponds to $\beta(t_0) = 0.61684$ for $t = 60$ days, $\gamma_{\mathrm{c,RH}} = 1.27 - 0.67\,h = 0.734$ for $h = 0.80$, $\gamma_{\mathrm{c,vs}} = 2\,(1 + 1.13\,\mathrm{e}^{-0.0213(V/S)})/3 = 0.6975$, where $V/S = 150$, $\gamma_{\mathrm{c,s}} = 0.82 + 0.00624\,s = 1.018$, where $s = 75\,\mathrm{mm}$, $\gamma_{\mathrm{c},\psi} = 0.88 + 0.0024\,\psi = 0.976$, where $\psi = 40$, $\gamma_{\mathrm{c},\alpha} = 0.46 + 0.09\,\alpha \geq 1$ is air content factor, where $\alpha = 2$, $\gamma_{\mathrm{c},\alpha} = 1$, $\beta_{\mathrm{c}}(36500-60) = 0.982004$.

8.3. The following parameters are according Gardner&Lockman model

$$E_{\mathrm{c}} = 2.8014{\times}10^4\,\mathrm{MPa}\ , \quad E_{\mathrm{a}} = 2.1{\times}10^5\,\mathrm{MPa}\ , \quad A_{\mathrm{c}} = 8820\,\mathrm{cm}^2\ , \quad A_{\mathrm{a}} = 383.25\,\mathrm{cm}^2\ ,$$

$$n = \frac{E_{\mathrm{a}}}{E_{\mathrm{c}}} = 7.496\ , \quad I_{\mathrm{c}} = 661500\,\mathrm{cm}^4\ , \quad I_{\mathrm{a}} = 1207963.7\,\mathrm{cm}^4\ , \quad r_{\mathrm{c}} = 25.50\,\mathrm{cm}\ ,$$

$$r_{\mathrm{a}} = 78.37\,\mathrm{cm}\ , \quad r = 103.870\,\mathrm{cm}\ , \quad A_{\mathrm{i}} = 1560.82\,\mathrm{cm}^2\ , \quad I_{\mathrm{i}} = 4415813.859\,\mathrm{cm}^4\ ,$$

$$M_0 = 1237\,\mathrm{kNm}\ , \quad N_{\mathrm{c},0} = 840.50\,\mathrm{kN}\ , \quad M_{\mathrm{c},0} = 24.7206\,\mathrm{kNm}\ , \quad M_{\mathrm{a},0} = 338.386\,\mathrm{kNm}\ ,$$

$$\lambda_{\mathrm{N}} = \left[1 + \frac{E_{\mathrm{c}}\,A_{\mathrm{c}}}{E_{\mathrm{a}}\,A_{\mathrm{a}}}\left(1 + \frac{A_{\mathrm{a}}\,r^2}{I_{\mathrm{a}}}\right)\right]^{-1} = 0.068593645\ , \quad \lambda_{\mathrm{M}} = \left[1 + \frac{E_{\mathrm{c}}\,I_{\mathrm{c}}}{E_{\mathrm{a}}\,I_{\mathrm{a}}}\right]^{-1} = 0.931921295\ .$$

9. Conclusion

In Figs. 3–16 there are shown values of normal forces and bending moments in time t. A numerical method for time-dependent analysis of composite steel-concrete sections according EC2, ACI 209R-92 and G&L models are presented. Using MATLAB code a numerical algorithm was developed and subsequently applied to a simple supported beam. These numerical procedures, suited to a PC, are employed to better understand the influence of the creep of the concrete in time-dependent behaviour of composite section. For a good accuracy of the time values, the numerical results are presented on logarithmic time scales.

The choice of the length of time step of the proposed numerical algorithm is based on numerous numerical experiments with different steps (seven, three and one days). So we conclude that good results can be achieved from practical point of view with one day step. For our purpose we consider a period of about 33–35 years. We derive our mathematical model using a Stieltjes hereditary integral, which represents time loading history. For the service load analysis, the proposed numerical method makes it possible to follow with great precision the migration of the stresses from the concrete slab to the steel beam, which occurs gradually during the time as a result of the creep of the concrete.

The parametric analysis results are characterized by the following effects:

- the state of stress in the concrete slab depends on the age of the concrete at loading time ;
- the stress in the top flange of the steel section increases strongly with time;
- the stress in the bottom flange undergoes small variations;
- the stress increases more for young concrete and little for old one.

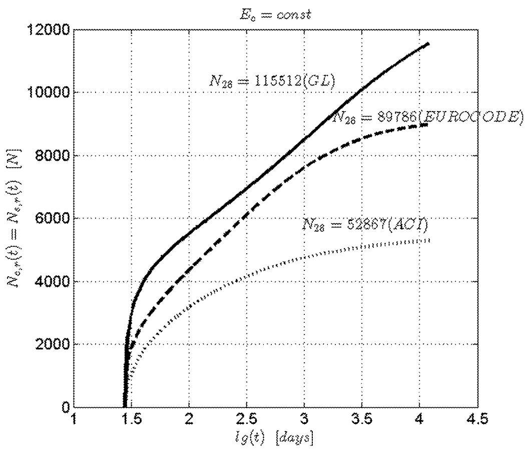

Fig.3: Normal forces $N_{\mathrm{c,r}}(t) = N_{\mathrm{a,r}}(t)$, $t = 12028$ days, $t_0 = 28$ days, (humidity 80 %)

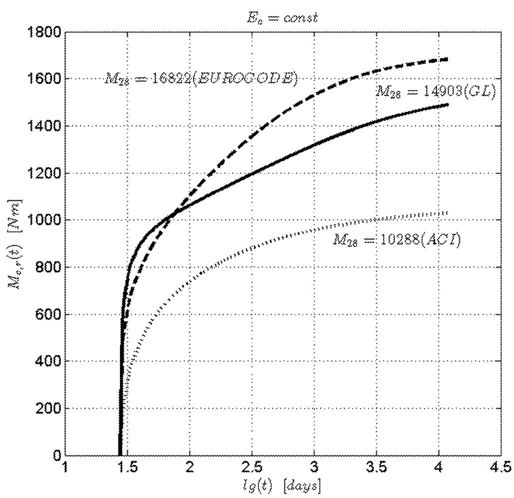

Fig.4: Bending moments $M_{\mathrm{c,r}}(t)$, $t = 12028$ days, $t_0 = 28$ days, (humidity 80 %)

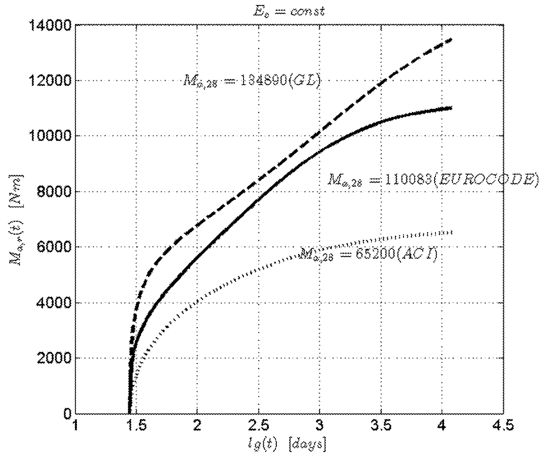

Fig.5: Bending moments $M_{\mathrm{a,r}}(t)$, $t = 12028$ days, $t_0 = 28$ days, (humidity 80 %)

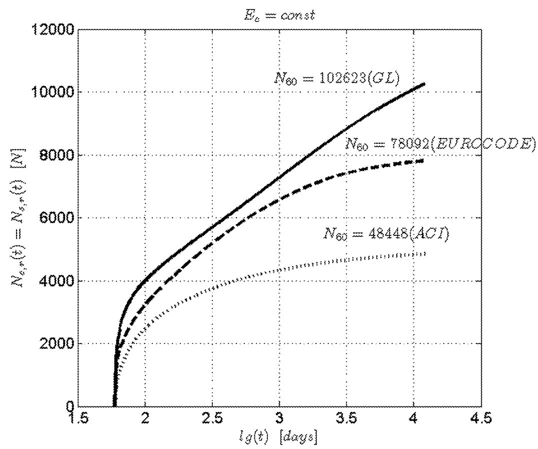

Fig.6: Normal forces $N_{\mathrm{c,r}}(t) = N_{\mathrm{a,r}}(t)$, $t = 12060$ days, $t_0 = 60$ days, (humidity 80 %)

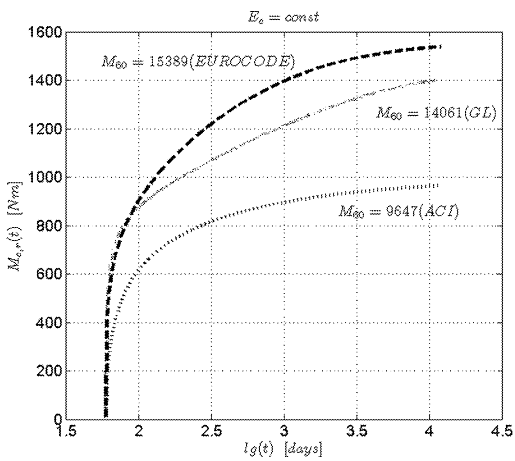

Fig.7: Bending moments $M_{\mathrm{c,r}}(t)$, $t = 12060$ days, $t_0 = 60$ days, (humidity 80 %)

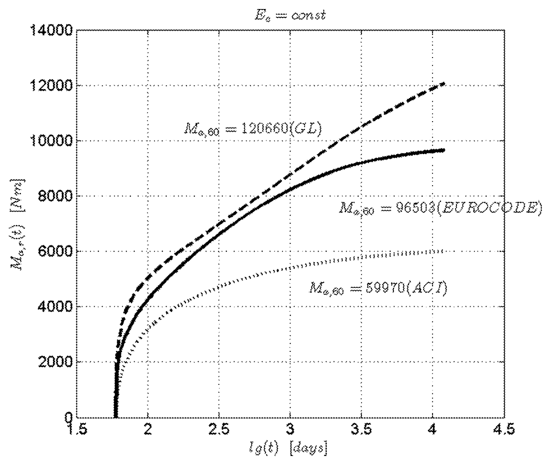

Fig.8: Bending moments $M_{\mathrm{a,r}}(t)$, $t = 12060$ days, $t_0 = 60$ days, (humidity 80 %)

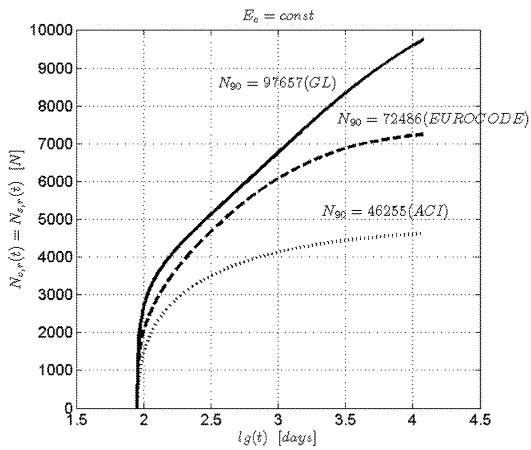

Fig.9: Normal forces $N_{\mathrm{c,r}}(t) = N_{\mathrm{a,r}}(t)$, $t = 12090$ days, $t_0 = 90$ days, (humidity 80 %)

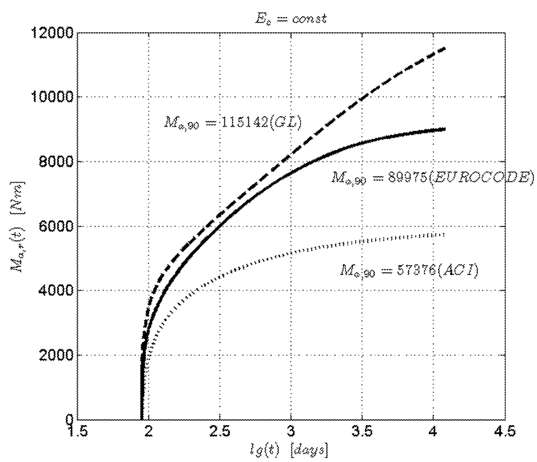

Fig.10: Bending moments $M_{\mathrm{a,r}}(t)$, $t = 12090$ days, $t_0 = 90$ days, (humidity 80 %)

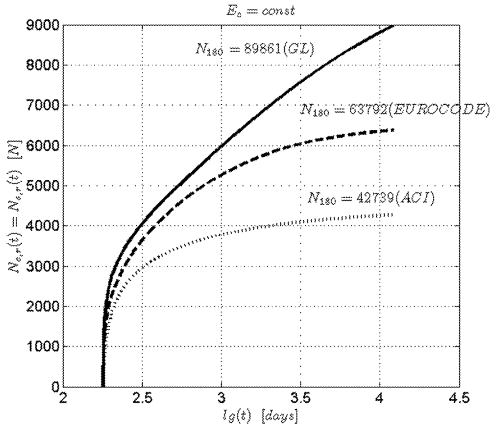

Fig.11: Normal forces $N_{\mathrm{c,r}}(t) = N_{\mathrm{a,r}}(t)$, $t = 12180$ days, $t_0 = 180$ days, (humidity 80 %)

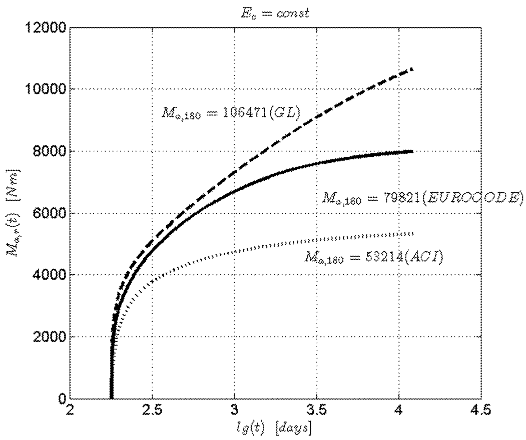

Fig.12: Bending moments $M_{\mathrm{a,r}}(t)$, $t = 12180$ days, $t_0 = 180$ days, (humidity 80 %)

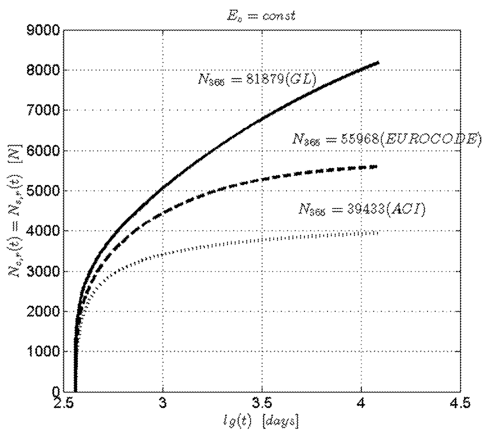

Fig.13: Normal forces $N_{\mathrm{c,r}}(t) = N_{\mathrm{a,r}}(t)$, $t = 12365$ days, $t_0 = 365$ days, (humidity 80 %)

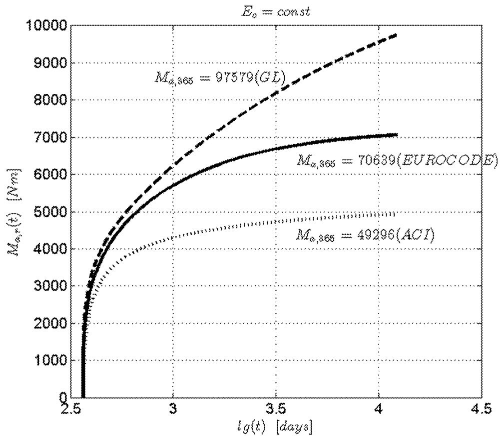

Fig.14: Bending moments $M_{\mathrm{a,r}}(t)$, $t = 12365$ days, $t_0 = 365$ days, (humidity 80 %)

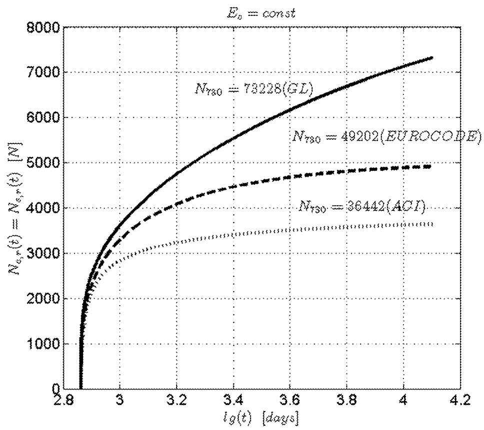

Fig.15: Normal forces $N_{\mathrm{c,r}}(t) = N_{\mathrm{a,r}}(t)$, $t = 12730$ days, $t_0 = 730$ days, (humidity 80 %)

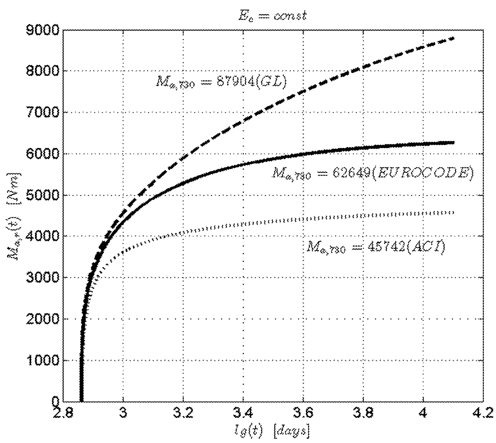

Fig.16: Bending moments $M_{\mathrm{a,r}}(t)$, $t = 12730$ days, $t_0 = 730$ days, (humidity 80 %)

From Figs. 3–16 it's clearly seen, that according to the proposed method, according GL2000, forces $N_{\mathrm{c,r}}$, $N_{\mathrm{a,r}}$ and $M_{\mathrm{a,r}}$ are much greater (between 25–30 %) than the same ones in the CEB MC90-99 method. It reminds the differences obtained from the results, when solving the same task using the methods based on the theory of viscoelastic body and the theory of aging of Dischinger and modified theory of aging of Rüsch-Jungwirth. Then we explain these facts through the assumptions of the viscoelastic body theory. According to this theory, which takes into account the delayed elastic strain, developing in constrained conditions, it leads to appearance of recovery of the stresses. They themselves decrease the relaxation of stresses in concrete of composite beams. That's why this fact leads to lower $N_{\mathrm{c,r}}$ and respectively $M_{\mathrm{a,r}}$. So as a result, we have had according the theory of viscoelastic body less stresses in the steel beam, which lead to the more economic designing of composite beam. The neglecting of the reversal of the creep recovery curves obtained from the GL2000 model according to the principle of superposition denoted from Bažant and Baweja (2000) can be a reason for the significance differences between the results obtained with the GL2000 and CEB MC90-99 methods. It means, that in the light of theory of viscoelastic body, the relaxation process in the concrete plate will be essentially greater compared with the results when the creep recovery is taken correctly into account according to the CEB FIP model (CEN 2004a).

In our opinion the influence of creep on time dependent behavior of composite steel-concrete beams according to ACI 209R-92 code provisions, in comparison with CEB FIB model code-1990 is underestimated. It is observed from the numerical results shown on Figs. 3–16.

According to our results based on numerous practical examples we can state that about 90–92 % of the maximum values of the stressed in concrete or steel in time are reached after about three years. Besides that 98 % are reached after about twenty years in comparison with the period of hundred years obtained by the EM Method [7–10].

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