EXACT LINEARIZATION OF NONHOLONOMIC SYSTEM DYNAMICS APPLIED TO CONTROL OF DIFFERENTIALLY DRIVEN SOCCER ROBOT

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This paper describes the dynamic modeling of nonholonomic system for control purposes. The equations of motion together with nonholonomic constraint are reduced into system of five 1st order equations. Further, the exact linearization is performed and finally we obtain a decoupled system of two independent integrator chains. Next we describe the controller design, and at the end, the simulation results are presented.

Key words: *nonholonomic system dynamics, exact linearization, soccer robot*

1. Introduction

This paper deals with modeling of dynamics of two-wheeled differentially-driven soccer robot of FIRA MIROSOT category. An overview of the system is depicted in Fig. 1. The game runs autonomously controlled by a computer. Two teams (of up to 11 players) play on a playground surrounded by a 50 mm high side walls. A camera is located above the centre of the playground and it is connected to the computer. The image of the playground is analyzed to gain positions of the robots and the ball (orange golf ball). Based on this information, the robots are controlled remotely.

The rules for this competition limit the size of the robots to a cube with 75 mm long edges and its mass to 650 g. Therefore, the speed and acceleration can be rather high (up to 4 m/s and 10 m/s^2 respectively) and thus the dynamics of the robot plays an important role. The goal is to replace the former control based on kinematics only by the control based on dynamics of the system, as the kinematics-based control has its clear drawbacks.

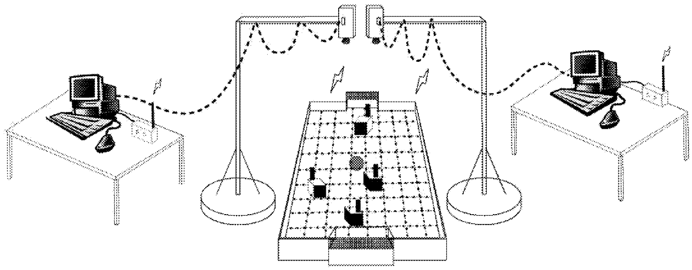

Fig.1: Robot soccer system (FIRA MIROSOT category)

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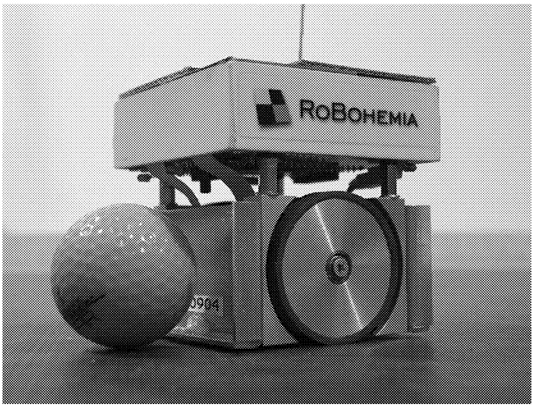

Fig.2: Soccer robot of FIRA MIROSOT category

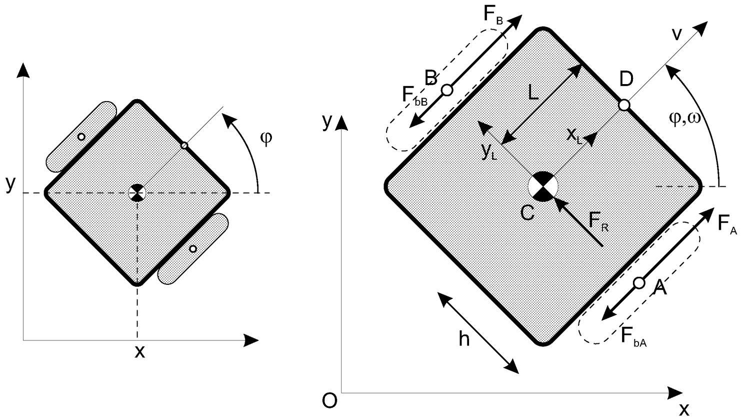

Fig.3: Schema of differentially driven soccer robot

In this paper, we neglect the dynamics of the wheels (considering their mass and power of the drives) and consider the robot as one rigid body. The described procedure can be generalized for more complex systems. Fig. 3 shows the schema of two-wheeled robot.

The dynamics of electrical drives is also neglected and drives are modeled as force inputs.

2. Simplified dynamics of nonholonomic robot

We use a vector $\mathbf{q}$ of three generalized coordinates $\mathbf{q} = [x, y, \varphi]^{\mathrm{T}}$ for description of robot state, where x and y denote position of the reference point and φ orientation of the robot w.r.t. positive direction of the x-axis (please refer to Fig. 3) in global Cartesian coordinate frame O.

There are following external forces:
– action forces F_{A} a F_{B} from drives,
– viscous friction against the wheel movement

$$F_{\mathrm{bA}} = b\, v_{\mathrm{A}} = b\, (v_{\mathrm{C}} + h\, \dot{\varphi}) \tag{1}$$

where v_{C} is the CG velocity, $\dot{\varphi}$ is angular velocity and b is viscous friction coefficient and
– lateral force F_{R} which guarantees no movement in y_{L} direction.

Free body diagram leads to these three equations of force and torque balance

$$F_{\rm A}\,\cos\varphi + F_{\rm B}\,\cos\varphi - F_{\rm R}\,\sin\varphi - \underbrace{b\,(\dot{x} + h\,\dot{\varphi}\,\cos\varphi)}_{F_{\rm bAx}} - \underbrace{b\,(\dot{x} - h\,\dot{\varphi}\,\cos\varphi)}_{F_{\rm bBx}} = m\,\ddot{x}\ , \tag{2}$$

$$F_{\rm A}\,\sin\varphi + F_{\rm B}\,\sin\varphi + F_{\rm R}\,\cos\varphi - b\,(\dot{x} + h\,\dot{\varphi}\,\sin\varphi) - b\,(\dot{x} - h\,\dot{\varphi}\,\sin\varphi) = m\,\ddot{y}\ , \tag{3}$$

$$F_{\rm A}\,h - F_{\rm B}\,h - 2\,b\,h^2\,\dot{\varphi} = I\,\ddot{\varphi}\ , \tag{4}$$

which can be easily expressed in matrix form

$$\begin{bmatrix} m & & \\ & m & \\ & & I \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} -2\,b & & \\ & -2\,b & \\ & & -2\,b\,h^2 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi & \cos\varphi \\ \sin\varphi & \sin\varphi \\ h & -h \end{bmatrix}\begin{bmatrix} F_A \\ F_B \end{bmatrix} - \begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix} F_{\rm R}\ ,$$

$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{V}\,\dot{\mathbf{q}} = \mathbf{B}\,\mathbf{f} - \mathbf{A}^{\rm T}\,\boldsymbol{\lambda}\ . \tag{5}$$

In these three equations ($n=3$) and there are the following unknown variables: $\{\ddot{x}, \ddot{y}, \ddot{\varphi}, F_{\rm R}\}$.

3. Model reduction

If we consider zero lateral movement (rolling without (lateral) skid of the wheel), the nonholonomic constraint is formulated as

$$\dot{y}_{\rm L} = 0\ , \tag{6}$$

which can be transformed into fixed coordinate system O as follows:

$$\frac{\dot{y}}{\dot{x}} = \tan\varphi\ . \tag{7}$$

Introduced one nonholonomic constraint ($k = 1$) can be expressed in matrix form:

$$\dot{x}\,\tan\varphi - \dot{y} = 0\ , \tag{8}$$

$$\begin{bmatrix} \tan\varphi & -1 & 0 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = 0\ , \tag{9}$$

$$\mathbf{A}(\mathbf{q})\,\dot{\mathbf{q}} = \mathbf{0}\ . \tag{10}$$

Note here, that our system has no holonomic constraint and therefore the total number of constraints is $m = k + 0 = 1$. We define matrix $\mathbf{S}(\mathbf{q}) \in \mathbb{R}^{n\times m-n}$ such as

$$\mathbf{A}(\mathbf{q})\,\mathbf{S}(\mathbf{q}) = \mathbf{0}\ . \tag{11}$$

For our case, the fitting definition is [4]:

$$\mathbf{S} = \begin{bmatrix} \cos\varphi & 0 \\ \sin\varphi & 0 \\ 0 & 1 \end{bmatrix}\ . \tag{12}$$

Next, we define the vector $\mathbf{v}$ with respect to the following equation:

$$\dot{\mathbf{q}}(\mathbf{q}) = \mathbf{S}(\mathbf{q})\,\mathbf{v}(t)\ , \tag{13}$$

which is in our case (remarked for clearer understanding):

$$\mathbf{v} = \begin{bmatrix} \dot{x}_\mathrm{L} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} v \\ \omega \end{bmatrix} . \tag{14}$$

Let's differentiate eq. 13 with respect to time

$$\ddot{\mathbf{q}} = \dot{\mathbf{S}}\,\mathbf{v} + \mathbf{S}\,\dot{\mathbf{v}} , \tag{15}$$

$$\dot{\mathbf{S}} = \begin{bmatrix} -\sin\varphi & 0 \\ \cos\varphi & 0 \\ 0 & 0 \end{bmatrix} \dot{\varphi} . \tag{16}$$

Let's substitute resulting formulas into eq. 5 and the whole equation multiply by $\mathbf{S}^\mathrm{T}$.

$$\mathbf{M}\,\dot{\mathbf{S}}\,\mathbf{v} + \mathbf{M}\,\mathbf{S}\,\dot{\mathbf{v}} + \mathbf{V}\,\mathbf{S}\,\mathbf{v} = \mathbf{B}\,\mathbf{f} - \mathbf{A}^\mathrm{T}\,\boldsymbol{\lambda} , \tag{17}$$

$$\mathbf{S}^\mathrm{T}\,\mathbf{M}\,\mathbf{S}\,\dot{\mathbf{v}} + \mathbf{S}^\mathrm{T}\,(\mathbf{V}\,\mathbf{S}\,\mathbf{M}\,\dot{\mathbf{S}})\,\mathbf{v} = \mathbf{S}^\mathrm{T}\,\mathbf{B}\,\mathbf{f} - \underbrace{\mathbf{S}^\mathrm{T}\,\mathbf{A}^\mathrm{T}}_{=(\mathbf{A}\,\mathbf{S})^\mathrm{T}=\mathbf{0}}\,\boldsymbol{\lambda} . \tag{18}$$

As we can see, the described procedure leads to elimination of vector $\boldsymbol{\lambda}$, which is a key success. Final resulting equation together with eq. 13 constitutes the new reduced model of the system

$$\mathbf{S}^\mathrm{T}\,\mathbf{M}\,\mathbf{S}\,\dot{\mathbf{v}} + \mathbf{S}^\mathrm{T}\,(\mathbf{V}\,\mathbf{S} + \mathbf{M}\,\dot{\mathbf{S}})\,\mathbf{v} = \mathbf{S}^\mathrm{T}\,\mathbf{B}\,\mathbf{f} , \tag{19}$$

$$\dot{\mathbf{q}} = \mathbf{S}\,\mathbf{v} . \tag{20}$$

Resulting model consists of five ODEs of the first order. The state vector is $[v, \omega, x, y, \varphi]^\mathrm{T}$.

4. Exact linearization

We reformulate eq. 19 into simpler form

$$\overline{\mathbf{M}}\,\dot{\mathbf{v}} + \overline{\mathbf{K}}\,\mathbf{v} = \overline{\mathbf{B}}\,\mathbf{f} , \tag{21}$$

$$\dot{\mathbf{q}} = \mathbf{S}\,\mathbf{v} . \tag{22}$$

Further, we define the input vector of action forces $\mathbf{f}$ as follows [2]:

$$\mathbf{f} = \overline{\mathbf{B}}^{-1} \left(\overline{\mathbf{M}}\,\mathbf{u} + \overline{\mathbf{K}}\,\mathbf{v}\right) \tag{23}$$

where $\mathbf{u}$ is the new system input. Thus, we obtain new partially linearized system

$$\dot{\mathbf{v}} = \mathbf{u} \tag{24}$$

$$\dot{\mathbf{q}} = \mathbf{S}\,\mathbf{v} \tag{25}$$

This result is interesting, but the use of x_L coordinate as a control input is rather inconvenient.

Equation 24 represents simple decoupled linear time invariant system, but together eq. 25 the system is still nonlinear and we want to control $\mathbf{q}$ not $\mathbf{v}$ only. The main target is to control the position (i.e., the x and y coordinates) of the robot.

Let's denote $\mathbf{z} = [\,x, y\,]^{\mathrm{T}}$. Then

$$\dot{\mathbf{z}} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \cos\varphi & 0 \\ \sin\varphi & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} , \tag{26}$$

$$\dot{\mathbf{z}} = \mathbf{P}\,\mathbf{v} \tag{27}$$

and

$$\ddot{\mathbf{z}} = \dot{\mathbf{P}}\,\mathbf{v} + \mathbf{P}\,\dot{\mathbf{v}} \ . \tag{28}$$

Now we can reformulate eq. 24 with help of eq. 27 and eq. 28 as

$$\mathbf{P}\,\dot{\mathbf{v}} = \mathbf{P}\,\mathbf{u} \ , \tag{29}$$

$$\ddot{\mathbf{z}} - \dot{\mathbf{P}}\,\mathbf{v} = \mathbf{P}\,\mathbf{u} \ , \tag{30}$$

$$\ddot{\mathbf{z}} = \mathbf{P}\,\mathbf{u} + \dot{\mathbf{P}}\,\mathbf{v} = \mathbf{r} \tag{31}$$

where $\mathbf{r}$ can be considered a new control input related to $\mathbf{u}$ according

$$\mathbf{u} = \mathbf{P}^{-1}\big(\mathbf{r} - \dot{\mathbf{P}}\,\mathbf{v}\big) \ . \tag{32}$$

Equation 31 represents decoupled LTI system. Unfortunately, $\mathbf{P}$ is singular and thus we cannot obtain $\mathbf{u}$. However, there exists the following solution.

The very interesting result can be obtained by simple modification (see [2]). Remember that our goal is to control the position of robot in the plane. Normally, the position is given by coordinates of COG point C. If we use the point D instead of C as a control point, the new control vector can be defined as

$$\mathbf{z} = \begin{bmatrix} x + L\,\cos\varphi \\ y + L\,\sin\varphi \end{bmatrix} \ . \tag{33}$$

Please note that if we use the reference point C, the second control input (ω) would have no direct effect on its position. The steering affects the position of the reference point only indirectly in this case, through the change of orientation of the robot. This case of dynamic feedback linearization (exact linearization) is described in [3]. Also, more on controllability and stabilizability if the wheeled mobile robots can be found in [5].

Let's differentiate it with respect to time and we obtain

$$\dot{\mathbf{z}} = \begin{bmatrix} \dot{x} - L\,\dot{\varphi}\,\sin\varphi \\ \dot{y} + L\,\dot{\varphi}\,\cos\varphi \end{bmatrix} = \begin{bmatrix} \cos\varphi & -L\,\sin\varphi \\ \sin\varphi & L\,\cos\varphi \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} , \tag{34}$$

$$\dot{\mathbf{z}} = \mathbf{P}\,\mathbf{v} \ . \tag{35}$$

After second differentiation, we have

$$\ddot{\mathbf{z}} = \dot{\mathbf{P}}\,\mathbf{v} + \mathbf{P}\dot{\mathbf{v}} \ , \tag{36}$$

$$\dot{\mathbf{P}} = \begin{bmatrix} -\sin\varphi & -L\,\cos\varphi \\ \cos\varphi & -L\,\sin\varphi \end{bmatrix} \omega \ . \tag{37}$$

Let's multiply the eq. 24 by matrix $\mathbf{P}$. Further, we reformulate equations

$$\mathbf{P}\,\dot{\mathbf{v}} = \mathbf{P}\,\mathbf{u}\ , \tag{38}$$

$$\ddot{\mathbf{z}} - \dot{\mathbf{P}}\,\mathbf{v} = \mathbf{P}\,\mathbf{u}\ , \tag{39}$$

$$\ddot{\mathbf{z}} = \mathbf{P}\,\mathbf{u} + \dot{\mathbf{P}}\,\mathbf{v} \tag{40}$$

and define the input $\mathbf{u}$ as follows:

$$\mathbf{u} = \mathbf{P}^{-1}\big(\mathbf{r} - \dot{\mathbf{P}}\,\mathbf{v}\big) \tag{41}$$

where $\mathbf{r}$ is again the new redefined system input.

Finally, we have (globally) linearized system with usable control variable. The system is decoupled, which allows to adjust the dynamic properties of both subsystems independently.

$$\ddot{\mathbf{z}} = \mathbf{r}\ . \tag{42}$$

5. Control design

After the exact linearization, we control the decoupled system of second order represented by eq. 42. Using the theory of second order system behaviour, we can form the controller as follows (please refer to Fig. 5):

$$\mathbf{r} = \ddot{\mathbf{z}}^{\mathrm{d}} - \mathbf{K}_0\,\tilde{\mathbf{z}} - \mathbf{K}_1\,\dot{\tilde{\mathbf{z}}}\ , \tag{43}$$

$$\tilde{\mathbf{z}} = \mathbf{z} - \mathbf{z}^{\mathrm{d}} \tag{44}$$

where $\mathbf{z}^{\mathrm{d}}$ is required and $\mathbf{z}$ is real position of reference point D. After simple modification, we get

$$\ddot{\tilde{\mathbf{z}}} + \mathbf{K}_1\,\dot{\tilde{\mathbf{z}}} + \mathbf{K}_0\,\tilde{\mathbf{z}} = 0\ . \tag{45}$$

Remember, that matrixes $\mathbf{K}_1$ and $\mathbf{K}_0$ are diagonal ones and eq. 45 is thus a system of $n - m = 2$ independent second order systems, whose properties we can arbitrarily chosen.

If we set the $\mathbf{K}_0$, the critical damping (the fastest stabilization without overshooting) is guaranteed by

$$\mathbf{K}_1 = \begin{bmatrix} 2\sqrt{k_{01}} & 0 \\ 0 & 2\sqrt{k_{02}} \end{bmatrix} \tag{46}$$

where k_{0i} is arbitrary value which defines the response dynamics for i-th component.

The controlled system is schematically shown in Fig. 4.

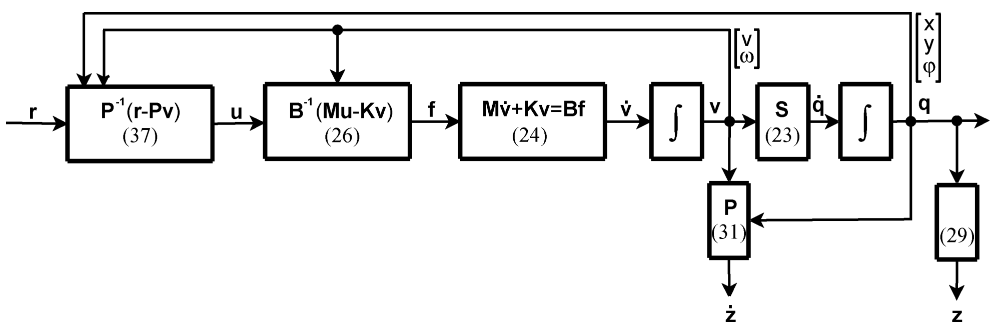

Fig.4: Schema of system control

Let us complete the control algorithm description with the analytically expressed control input vector $\mathbf{f}$. Using eqns. 23 and 41 we get:

$$\mathbf{f} = \overline{\mathbf{B}}^{-1}\,\overline{\mathbf{M}}\,\mathbf{P}^{-1}\,\mathbf{r} - \overline{\mathbf{B}}^{-1}\left(\overline{\mathbf{M}}\,\mathbf{P}^{-1}\,\dot{\mathbf{P}} + \overline{\mathbf{K}}\right)\mathbf{v}\ , \tag{47}$$

which can be rewritten as

$$\mathbf{f} = \mathbf{U}(\varphi)\,\mathbf{r} + \mathbf{W}(\omega)\,\mathbf{v} \tag{48}$$

$$\mathbf{U}(\varphi) = \frac{1}{2\,h\,L}\begin{bmatrix} m\,h\,L\,\cos\varphi - I\,\sin\varphi & m\,h\,L\,\sin\varphi + I\,\cos\varphi \\ m\,h\,L\,\cos\varphi + I\,\sin\varphi & m\,h\,L\,\sin\varphi - I\,\cos\varphi \end{bmatrix}\ . \tag{49}$$

Resulting matrix can be expressed as follows:

$$\mathbf{W}(\omega) = \begin{bmatrix} \dfrac{2\,b\,h\,L + I\,\omega}{2\,h\,L} & \dfrac{\omega\,m\,L}{2} + h\,b \\ \dfrac{2\,b\,h\,L + I\,\omega}{2\,h\,L} & \dfrac{\omega\,m\,L}{2} - h\,b \end{bmatrix} \tag{50}$$

We can see, that we can compute control vector for all cases $h, L \neq 0$. As can be seen from Fig. 3, these values represent physical dimensions of the robot and x-coordinate of the reference point in robot local coordinates frame, respectively. Therefore, the singularity should be avoided.

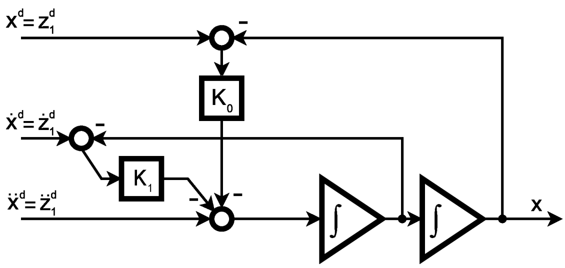

Fig.5: Control of integrator chain

6. Simulation experiments

The algorithm described above has been implemented in Matlab environment. We introduce the vector of disturbance into the system (eq. 5)

$$\mathbf{f}_\mathrm{P} = [\,F_\mathrm{Px},\ F_\mathrm{Py},\ M_\mathrm{Pz}\,]^\mathrm{T}\ , \tag{51}$$

$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{V}\,\dot{\mathbf{q}} = \mathbf{B}\,\mathbf{f} - \mathbf{A}^\mathrm{T}\,\boldsymbol{\lambda} + \mathbf{f}_\mathrm{P} \tag{52}$$

which affects the eq. 21 as follows:

$$\overline{\mathbf{M}}\,\dot{\mathbf{v}} + \overline{\mathbf{K}}\,\mathbf{v} = \overline{\mathbf{B}}\,\mathbf{f} + \mathbf{S}^\mathrm{T}\,\mathbf{f}_\mathrm{P}\ . \tag{53}$$

For the trajectory planning we use function `jtraj` from Robotic Toolbox for Matlab. The function define the position, velocity and acceleration $\mathbf{z}^\mathrm{d}$, $\dot{\mathbf{z}}^\mathrm{d}$ and $\ddot{\mathbf{z}}^\mathrm{d}$ using the polynomial of second order. The required and simulated trajectories of two examples are shown in Fig. 6. The matrix $\mathbf{K}_0$ was chosen differently, the $\mathbf{K}_1$ always respect critical damping.

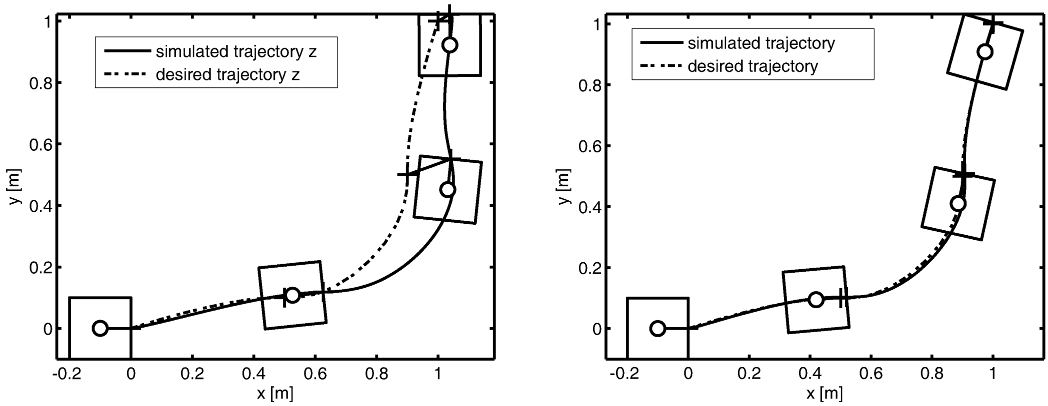

Fig.6: Simulated result for $k_{01} = k_{02} = 5$ *(left) and* $k_{01} = k_{02} = 50$ *(right)*

7. Conclusion

Resulting control algorithm is computationally efficient and can potentially be used in a real device. It should be noted that it is usual to control the position of the reference point C located on the wheels' axis (it's the center of rotation of the robot). In our case, it's necessary to generate appropriate reference trajectory for the point D (moved forward).

Acknowledge

The presented results were acquired using the subsidization of projects AV0Z20760514 and GAČR 101/06/0063.

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Received in editor's office: March 13, 2008
Approved for publishing: April 14, 2008

Note: The paper is an enlarged version of the contribution presented at conference *Dynamics of Machines 2008, Prague.*