BIFURCATION AND CHAOS IN ELECTROMENCHANICAL DRIVE SYSSTEMS WITH SMALL MPTPRS

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The purpose of this article is to provide an elementary introduction to the subject of chaos in the electromechanical drive systems with small MPTPRS. In this article, we explore chaotic solutions of maps and continuous time systems. These solutions are also bounded like equilibrium, periodic and quasiperiodic solutions.

Key words : *chaos, drive systems, bifurcation, attractor*

1. Introduction

Chaos can be defined on bounded-state behavior that isn't equilibrium solution or a periodic solution or a quasiperiodic solution. The attractor associated with chaotic motion in state space is not a simple geometrical object like a finite number of points, a closed curve or a torus. Chaotic attractor are complicated geometrical objects that posses fractal dimensions.

In contrast with the spectra of periodic and quasiperiodic attractors, which consist of a finite number of sharp spikes, the spectrum of a chaotic signal has a continuous broadband character. In addition, the spectrum of chaotic signal often contains spikes that indicate the predominant frequencies of the signal. We can also say, that chaotic motion has a very large number of unstable periodic motions. Thus, a chaotic system may dwell for a brief time on motion that is very nearly periodic and then may change to another periodic motion with a period that is k times that of the preceding motion. These constant evolutions from one periodic motion to another a long-time impression of randomness while showing short-term glimpses or order [1].

Chaotic systems are also characterized by sensitivity to initial conditions or some structural parameters of model of drive systems. That is, tiny differences in the input can be quickly amplified to create overwhelming differences in the output (this is so-called butterfly effect)

2. Deterministic chaos

Deterministic chaos is a term used to denote the irregular behavior of dynamical systems arising from a strictly deterministic time evolution without any source of noise or external

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stochasticity. This irregularity manifests itself in an extremely sensitivity dependence on initial conditions (or some structural parameters), which precludes any long-term prediction of the dynamics. Most surprisingly, it turned out that such chaotic behavior can already be found for dynamical systems with a small degree of freedom and it is, moreover, typical for a great number of mechatronic systems. A dynamical system can be described simply as a system of N first order differential equations

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = f_i(x_1, x_2, \dots, x_N, r) \ , \qquad i = 1, 2, \dots, N \ , \tag{1}$$

where the independent variable t can be read as time and the $x_i(t)$ are dynamical quantities whose time dependence is generated by (1), starting from the specific initial conditions $x_i(0)$, $i = 1, 2, \dots, N$. It should be noted that the system (1) is autonomous because it is not explicitly t-dependent. The f_i is nonlinear function of the x_i which is characterized by the parameter(s) r. The equations lead to chaotic motion, which develops and changes its characteristics with varying control parameter(s) r. The assumption of an autonomous system is not essentials, because it can be converted into an autonomous one by introducing time t as an additional variable x_{N+1}. An example of dynamical system are the Hamiltonian equations of motion in classical mechanics.

A discrete dynamical system is an iterated mapping

$$x_i(n+1) = f_i(x_1(n), \dots, x_N(n), r) \ , \qquad i = 1, 2, \dots, N \ , \tag{2}$$

starting from an initial point $x_i(0)$, $i = 1, 2, \dots, N$. Such discrete system may appear quite naturally from the setup of the problem under consideration, or it may be a reduction of the continuous system (2) in order to simplify the analysis, as for example the Poincare maps.

Basically, one can make a distinction between conservative and dissipative. In the first case, volume elements in phase space are conserved, whereas dissipative systems contract phase space element. This results in markedly different behavior [2].

3. Bifurcation behavior of real drive system

We can demonstrate the computational modeling of bifurcations and chaos on the example of DC drive with separate excitation and current controller with hysteresis. Schematic diagram of the drive and its circuits is shown on Fig. 1.

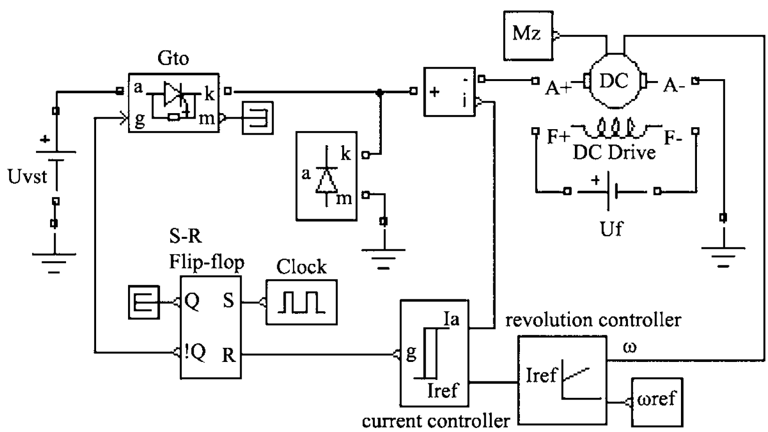

Fig.1: Schematic diagram of controlled drive

Modeled system consists of power and control parts. Power part contains power switching element and zero diode. Control part contains PI controller of angular velocity and current controller with hysteresis. Controllers output pulses are brought into R-S flip flop which is clocked by clock pulses with period T. This prevents high frequency switching of power element and defines sample period T for further signal analysis. The equations describing given system therefore contain time T on the right side, which means that it is nonautonomous system. Modeled system was modified so that during design of computational model in MATLAB we could directly use the basic mathematical description.

For on state S

$$
\begin{aligned}
\frac{\mathrm{d}i_\mathrm{a}}{\mathrm{d}t} &= \frac{1}{L_\mathrm{a}}\left(-R_\mathrm{a}\, i_\mathrm{a} - L_\mathrm{af}\, i_\mathrm{f}\, \omega_\mathrm{r} + u_\mathrm{a}\right) , \\
\frac{\mathrm{d}i_\mathrm{f}}{\mathrm{d}t} &= \frac{1}{L_\mathrm{f}}\left(-R_\mathrm{f}\, i_\mathrm{f} + u_\mathrm{f}\right) , \\
\frac{\mathrm{d}\omega_\mathrm{r}}{\mathrm{d}t} &= \frac{1}{J}\left(L_\mathrm{af}\, i_\mathrm{a}\, i_\mathrm{f} - B_\mathrm{m}\, \omega_\mathrm{r} - M_\mathrm{z}\right)
\end{aligned}
\tag{3}
$$

and for off state S

$$
\begin{aligned}
\frac{\mathrm{d}i_\mathrm{a}}{\mathrm{d}t} &= \frac{1}{L_\mathrm{a}}\left(-R_\mathrm{a}\, i_\mathrm{a} - L_\mathrm{af}\, i_\mathrm{f}\, \omega_\mathrm{r}\right) , \\
\frac{\mathrm{d}i_\mathrm{f}}{\mathrm{d}t} &= \frac{1}{L_\mathrm{f}}\left(-R_\mathrm{f}\, i_\mathrm{f} + u_\mathrm{f}\right) , \\
\frac{\mathrm{d}\omega_\mathrm{r}}{\mathrm{d}t} &= \frac{1}{J}\left(L_\mathrm{af}\, i_\mathrm{a}\, i_\mathrm{f} - B_\mathrm{m}\, \omega_\mathrm{r} - M_\mathrm{z}\right) ,
\end{aligned}
\tag{4}
$$

where i_a is current in motor armature circuit, i_f is current in excitation coil, ω_r is rotor angular speed, L_a is anchor circuit inductance, L_f is excitation coil inductance, J is motor moment of inertia, R_a is resistance in motor armature circuit, R_f is excitation coil resistance, B_m is coefficient of viscous damping, u_a is armature circuit voltage, u_f is excitation circuit voltage, M_z is load torque.

The parameters of DC drive and loads are: $U_\mathrm{vst} = 280\,\mathrm{V}$, $R_\mathrm{a} = 0.5\,\Omega$, $L_\mathrm{a} = 0.01\,\mathrm{H}$, $R_\mathrm{f} = 240\,\Omega$, $L_\mathrm{f} = 0.001\,\mathrm{H}$, $L_\mathrm{af} = 1.23\,\mathrm{H}$, $J = 0.05\,\mathrm{kg\,m^2}$, $B = 0.02\,\mathrm{N\,m/rad\,s^{-1}}$, proportional component of PI $K_\mathrm{p} = 1.6$, integration component of PI $K_\mathrm{i} = 16$, band width of controller with hysteresis is 0.8 A. Frequency of clock pulses is 200 Hz.

We can determine the behavior of the system on the base of time courses of selected variables, characters of attractors in both stable and unstable states and spectral analysis of selected variables. Such data are shown on figures 2–5.

Figure 2 shows courses of motor current and speed in steady state with single period, corresponding to simple attractor – limit cycle. The same parameters are shown in Figure 3 at steady state with double period. One can see that attractor has more complex character and corresponds to more complex motion. State close to chaos is shown in Fig. 4. Mainly the attractor (Fig. 4c) is represented by more complex motion with open course, corresponding to unstable state of the system.

These facts are illustrated also by spectra of motor speed courses for both stable and chaotic state, see Fig. 5. The modulation of base frequency component in state close to chaos can not be overlooked and documents the complex character of motion.

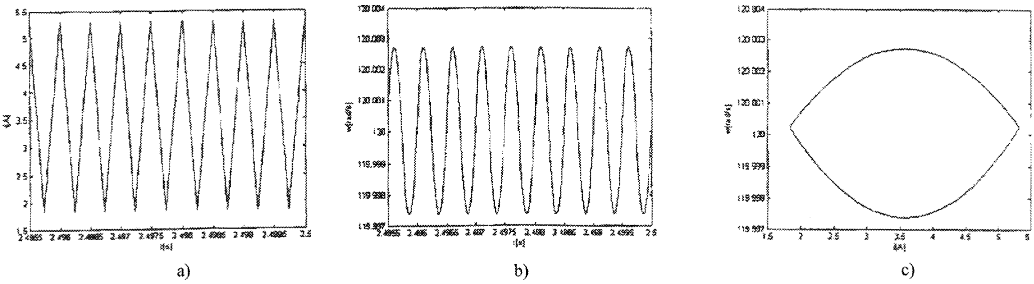

Fig.2: Steady state of the system with single period – a) time course of current, b) time course of speed, c) attractor

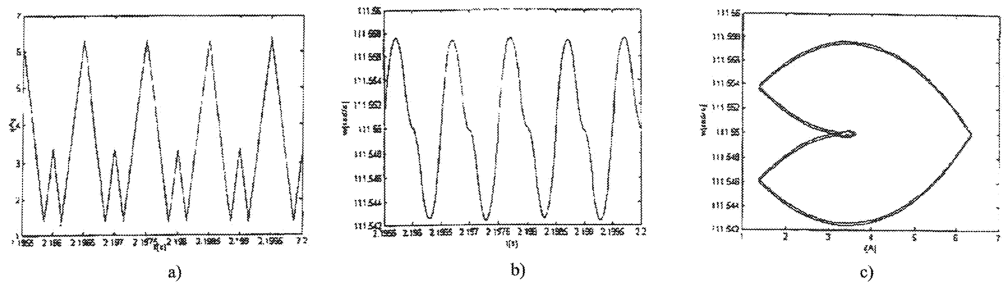

Fig.3: Steady state of the system with two periods – a) time course of current, b) time course of speed, c) attractor

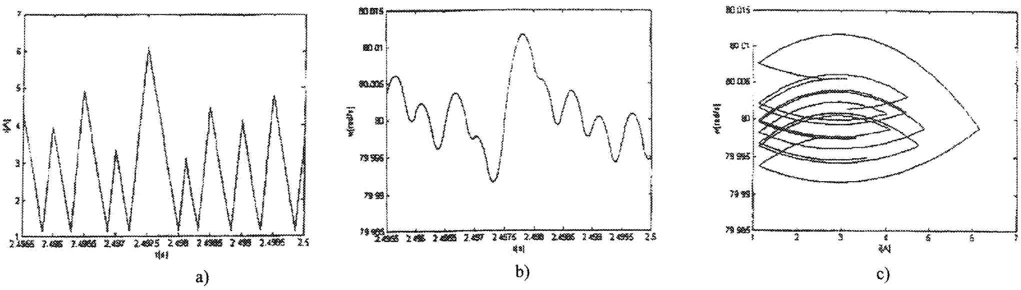

Fig.4: Chaotic state of the system – a) time course of current, b) time course of speed, c) attractor

4. Bifurcation and chaos

Bifurcation diagram is often used for analysis of how the change of certain parameter influences the behavior of the system in question. To create bifurcation diagram we have to build a circuit to generate required signal for oscilloscope.

Typical bifurcation diagram has horizontal axis corresponding to the parameter change and vertical axis corresponding with sampled steady values of the variable of tested system. To draw bifurcation diagram we have to bring the required signals for X and Y inputs of the oscilloscope. To obtain those signals we must perform two actions:

1. change of selected parameter of examined system corresponding to the small change of voltage sawtooth brought to X input of the oscilloscope
2. sampling of selected signal of examined system and sending samples to Y input of the oscilloscope.

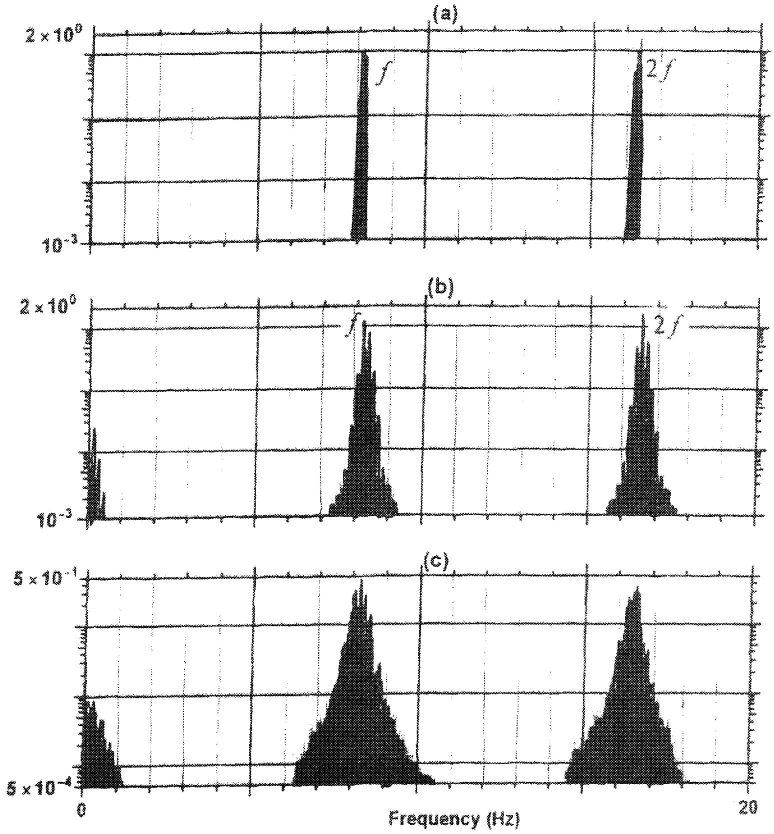

Fig.5: Motor speed spectra – a) steady state with single period, b) steady state with two periods, c) chaotic state

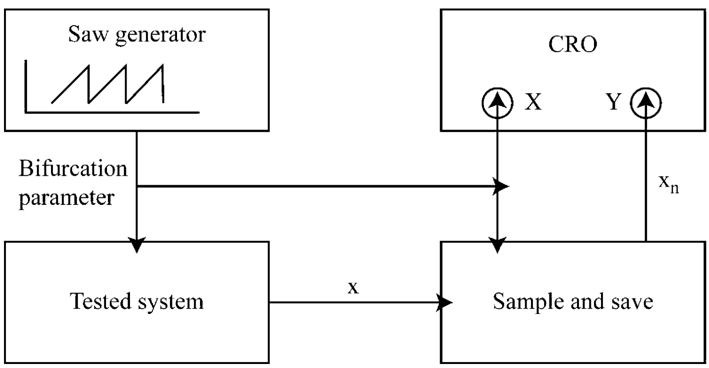

Fig.6: Block diagram of bifurcation diagram

Those two actions must be coordinated and saw increment must be relatively small. Then for each value of bifurcation diagram the sampled data are brought to Y input of the oscilloscope. The system implementing this way of bifurcation diagram construction is shown on Fig. 6.

To obtain the information about the behavior of modeled system during the change of selected parameters following examples were chosen:

– First we evaluated bifurcation diagram of motor current with parameter u_{vst} (input voltage) for $\omega_{\mathrm{r}} = 80\,\mathrm{rad\,s^{-1}}$. The diagram is shown on Fig. 7.

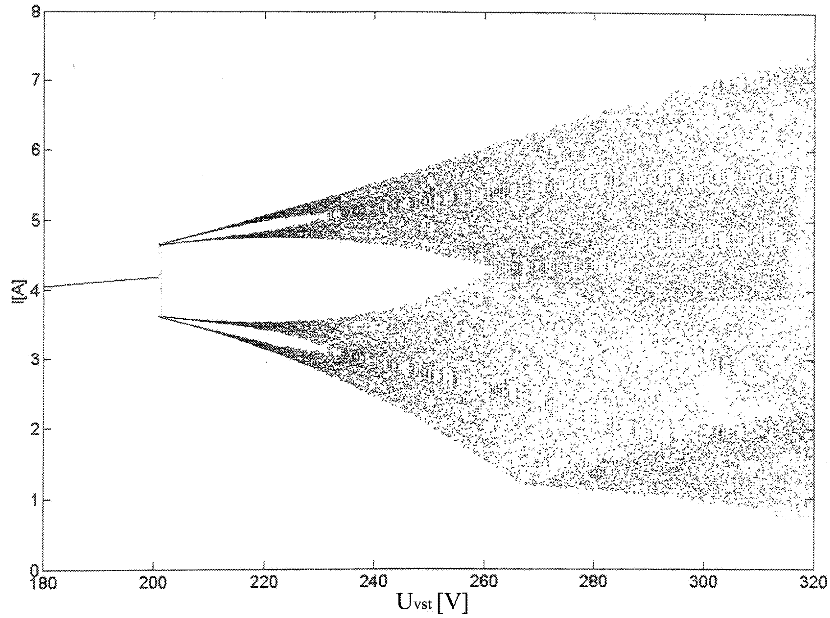

Fig.7: Bifurcation diagram of motor current with u_{vst} parameter for $\omega_{\mathrm{r}} = 80\,\mathrm{rad\,s^{-1}}$

– Another example is bifurcation diagram of motor current with ω_{r} parameter for $u_{\mathrm{vst}} = 280\,\mathrm{V}$ which is shown on Fig. 8.

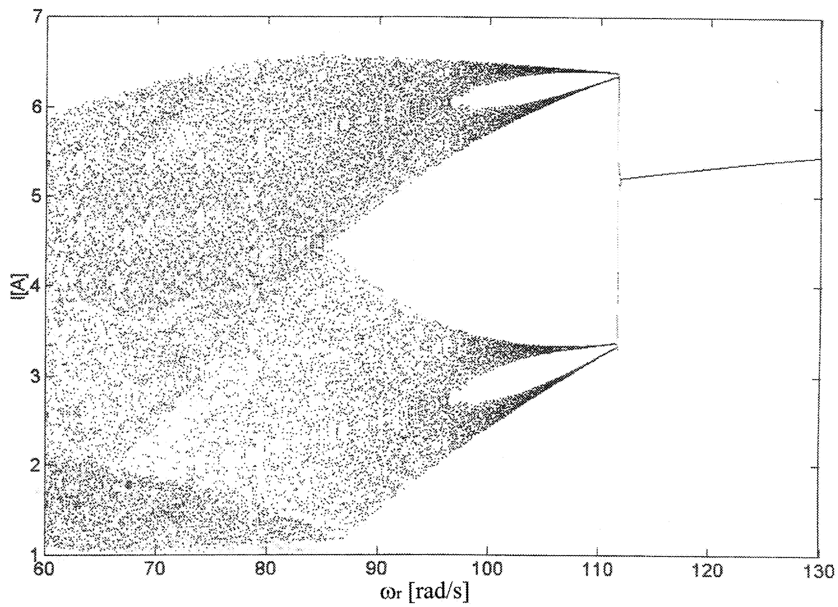

Fig.8: Bifurcation diagram of motor current with ω_{r} parameter for constant value of $u_{\mathrm{vst}} = 280$ V

– The last example is the most interesting regarding the mechanism of transition from stable state to unstable state of the system.

Fig. 9a shows the bifurcation diagram of current with parameter K_p for $u_\mathrm{vst} = 332\,\mathrm{V}$ and $\omega_\mathrm{r} = 135\,\mathrm{rad\,s^{-1}}$. Further changed parameters of the system are: integration component of PI controller $K_\mathrm{i} = 0$ and band width of controller with hysteresis which is set to 2 A. It is clear that with increasing PI controller gain we are moving towards unstable area.

The complex mechanism of transition between stable and unstable states of the system is shown in detail on Fig. 9b.

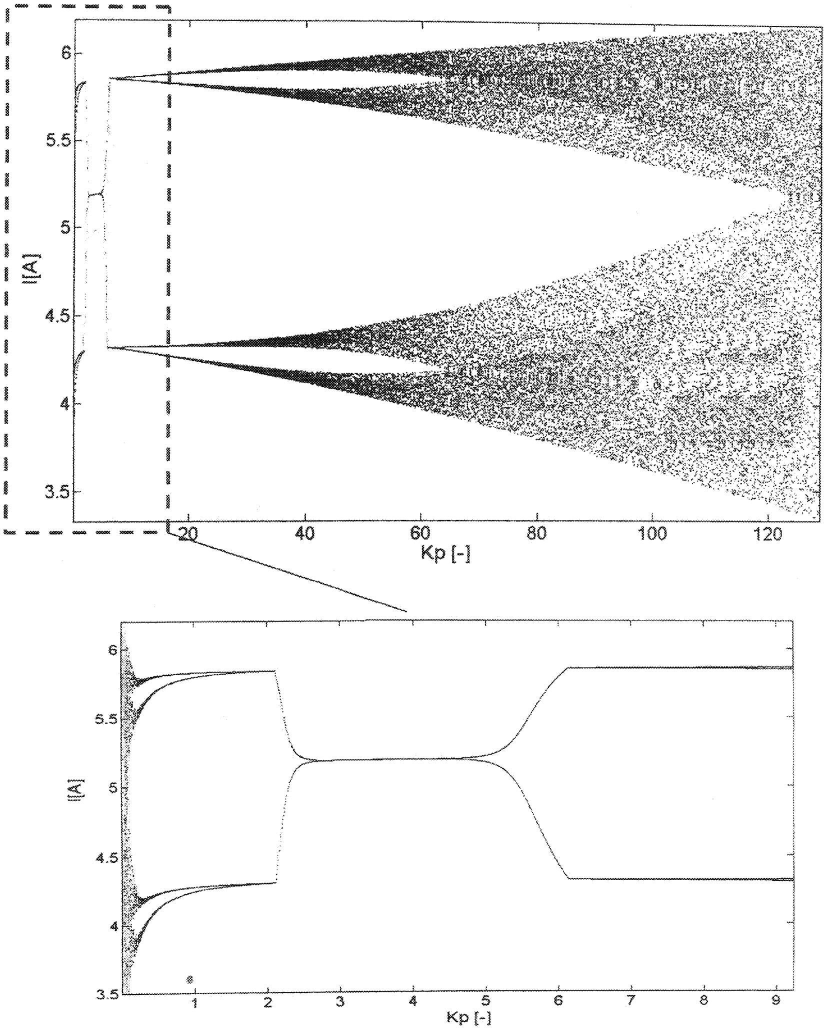

Fig.9: Bifurcation diagram of motor current with K_p parameter for constant values of $u_\mathrm{vst} = 332$ V and $\omega_\mathrm{r} = 135\,rad\,s^{-1}$

5. Conclusion

Bifurcation analysis of dynamic systems responses (or real systems models) can be used for relatively simple description of steady states of nonlinear systems and for evaluation of changes in their behavior depending on changes of certain control parameter or initial conditions.

In the case of electric drives the bifurcation analysis can be used as a mean for limit parameter value identification or for diagnostics of the drive as whole. Bifurcation analysis is in such case based on observation and analysis of angular velocity and armature current. Those variables are commonly observed in industrial drive systems for the feedback control loops purposes.

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