TEST FACILITY DESIGN FOR EXPERIMENTAL STUDIES OF PARAMETRICALLY EXCITED VIBRATIONS

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Mechanical systems with parametric excitation have been investigated more frequently during the last years. New findings on the system behavior when excited at a parametric combination resonance frequency have triggered this increasing interest in such systems. Almost all studies, however, have been carried out analytically or numerically, so that there is a need for experimental work to prove the theoretical results. Partly the reason for this lack of experimental studies is found in several design problems that one faces when this task is tackled. In this contribution, a review of test facilities known from the literature is presented. Typical solutions suitable for experimental work and recent developments are discussed.

Key words: *parametric excitation, axially loaded beam, electromechanical and piezoelectric actuator*

1. Introduction

Systems of differential equations with periodic coefficients are the focus of scientific research since more than half a century. Such systems are also termed *parametrically excited systems*, since one or more parameters of the system are assumed to be a periodic function of time. A general case of a parametrically excited non-linear mechanical system is presented in Eq. (1)

$$\mathbf{M}(\mathbf{q},\dot{\mathbf{q}},t)\,\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}},t)\,\dot{\mathbf{q}}+\mathbf{K}(\mathbf{q},t)\,\mathbf{q}=\mathbf{0}\ , \tag{1}$$

where the mass matrix $\mathbf{M}$, the damping matrix $\mathbf{C}$ and the stiffness matrix $\mathbf{K}$ are all periodic in time $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}(t) = \mathbf{M}$, $\mathbf{C}$, $\mathbf{K}(t+T_{\mathrm{PE}})$ with period T_{PE}. Although nonlinearities will be important later, a linearized version of Eq. (1) with constant mass and damping matrix is given by

$$\ddot{\mathbf{q}}+\mathbf{M}^{-1}\,\mathbf{C}\,\dot{\mathbf{q}}+\boldsymbol{\Omega}\,\mathbf{q}+\cos(\omega_{\mathrm{PE}})\,\mathbf{M}^{-1}\,\mathbf{P}\,\mathbf{q}=\mathbf{0} \tag{2}$$

with matrix $\boldsymbol{\Omega}$ containing the natural frequencies Ω_i of the time-invariant system and the coefficient matrix $\mathbf{P}$ of the parametric stiffness excitation amplitudes p_{ij}. The frequency of the harmonic parametric excitation is denoted $\omega_{\mathrm{PE}} = 2\pi/T_{\mathrm{PE}}$.

Parametrically excited systems exhibit interesting phenomena which cannot occur in a one-dimensional system as the famous Mathieu-equation. It is well known, see e.g. [9], that in such a system instabilities may occur which are called *Principle Parametric Resonances* at frequencies $\eta^{\mathrm{pr}}_{j/n}$ and *Parametric Combination Resonances* at frequencies $\eta^{\mathrm{cr}}_{j\pm k/n}$ for the PE-frequency ω_{PE} equal to:

$$\eta^{\mathrm{pr}}_{j/n}=\frac{2\,\Omega_j}{n}\ ,\qquad \eta^{\mathrm{cr}}_{j\pm k/n}=\frac{|\Omega_j\pm\Omega_k|}{n}\ ,\qquad (j,k=1,2)\ ,\quad (n=1,2,3,\dots)\ . \tag{3}$$

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Symbols Ω_j and Ω_k denote the j-th (k-th) natural frequency of the system. The denominator n represents the order of the parametric resonance.

For a long time, almost all investigations were focused on those cases, where principle or combination resonances appeared to be resonant, i.e. become unstable and, consequently, a serious threat to a real world system. However, during the last decade it was discovered [8], that a non-resonant parametric combination resonance may exhibit interesting properties that can be used to stabilize an unstable system. The new research results also triggered a need for experimental investigations to prove the analytical and numerical findings.

In the following, an overview will be given on experimental work that has been carried out in the past. Also recent developments concerning the design of test facilities to study parametrically excited systems will be presented. Due to the limited space only a brief description of the various test stands will be given. For further details and experimental results obtained with the test facilities it is referred to the original works as listed in the references.

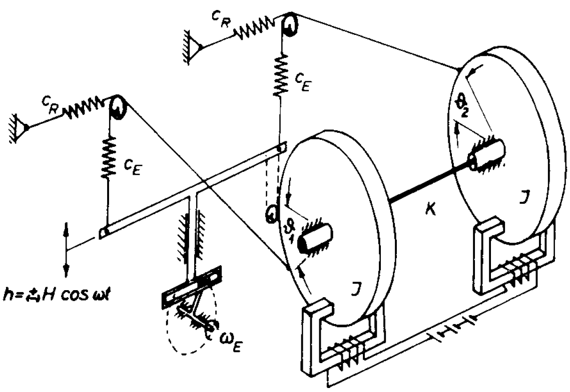

Fig.1: Experimental setup with mechanical realization of a non-linear parametric stiffness excitation as developed and used by Benz in [1]

2. Experimental work in the Sixties and Seventies

One of the very first experimental studies of a non-linear vibratory system with periodic coefficients was carried out by Benz in [1]. A sketch of the test stand developed and used is shown in Fig. 1. The test stand is primarily a mechanical torsional system where the parametric excitation is generated by a sliding crank that operates the mechanical springs c_E. Due to the nonlinearity of the geometry, stiffness parameters of the 2dof-system are time-periodic and non-linear. The only electrical elements in use are eddy current dampers for the realization of easy-to-adjust constant viscous damping.

A completely different approach was used by Yamamoto and co-authors, see [2] and [3]. They used a double-pendulum with base excitation to realize and study a parametrically excited 2dof-system. In such a system the parametric excitation is created by the periodic change of the inertia forces due to the time-dependent acceleration of the pendulum base.

Although mechanically generated parametric excitation is found in every gear system due to the time-periodic stiffness within the meshing gear wheels, it is much more complicated to realize an adjustable parametric excitation with respect to frequency and amplitude

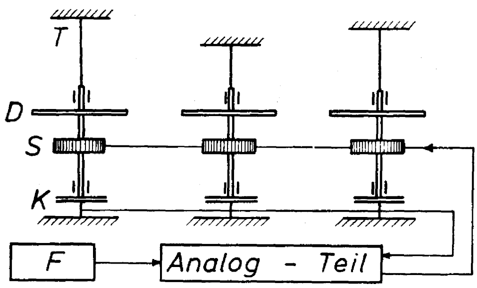

Fig.2: Early mechatronic test facility as used by Becker in [4] to study parametric excitation

by a pure mechanical system. Therefore, electromechanical systems have been employed soon to overcome this difficulty. In [4] Becker uses a current driven coil which rotates in a magnetic field. Figure 2 shows a schematic of the vertically arranged system, with torsional springs (T), eddy current dampers (D) and capacitors (K) for measurement of the system state. The electrical signals are fed into an analog control system, and control signals for the rotating coils (S) are generated. The source of the parametric excitation of the electrically coupled system is the control system in combination with the electrical actuators (S).

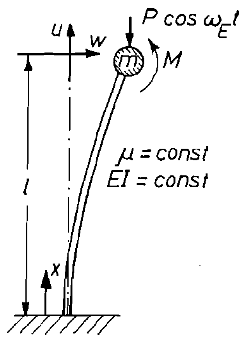

Fig.3: Axially loaded beam as used in [5]

It is known that the equations of motion of an axially loaded beam (Fig. 3) contain expressions representing parametric stiffness excitation. Based on this fact a large-scale test rig with two vertically oriented cantilever beams, connected by a rigid bar, was designed and built by Benz and Schröder in the Seventies. Results obtained by this frame structure were also documented by a very instructive movie, which is still available in various media formats [5].

In the mid-seventies, analog computers became widely available and made it possible to represent a set of differential equations by an electrical circuit. This opened the possibility to investigate parametrically excited systems comparatively easily by this new and very fast means. Although it was common to refer to such custom designed electrical circuits as *analog computers*, in a strict sense one did carry out an experiment with an electrical system.

A study of combination resonances was carried out by Schmieg in [6]. A sketch of the block diagram, which was in use, is shown in Fig. 4. Results obtained from an analog computer were also the starting point of Tondl's investigations on the nature of certain parametric combination resonances, see [8].

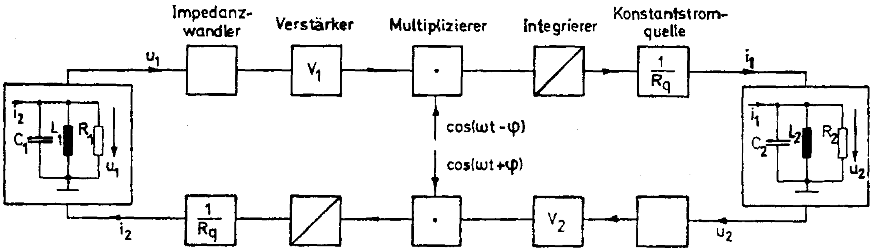

Fig.4: Block diagram of the electrical circuit (Analog Computer) as used in [6]

3. Recent test stand designs

A test stand based on more recent technology was designed and built by Knüver in [7], see Fig. 5. The sketch shows a double pendulum, which was set up horizontally and supported by air cushions for low friction. A quite smart idea was used for the follower force $F_0 \cos \Omega t$. It was generated by the thrust of two electrically driven miniature propellers attached to the tip of the second link of the double pendulum.

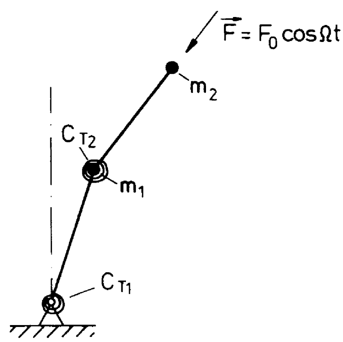

Fig.5: Experimental setup as used in [7]

Tondl's findings in [8] were based on analog computer results. The numerous subsequent publications to further investigate the nature of non-resonant parametric combination resonances and the increase of damping in the system were all carried out analytically and/or numerically. Therefore, there was a need for a well designed experiment to demonstrate the validity of the results. A review of previous experimental results by other authors did not reveal any related results.

The first test stand designed by the author and his cooperators was a test setup for the linear motion of two masses, see Fig. 6 (left). To allow for an almost frictionless linear motion, an air-track was used to support the two moving masses. Conventional coil springs were used to connect masses to each other and to the inertia reference frame. The parametric stiffness excitation (PSE) was created by an elastic string with time-periodic tension. Like for a guitar string the tension determines the lateral stiffness of a string. To change the tension in the string periodically, a crank was driven by an electric motor, resembling the first mechanical parametric stiffness excitation as designed by Benz.

In Fig. 6 (right) the principle of the PSE-mechanism is depicted. Numbers (1) refer to the pulleys which are used to guide a comparably inextensible string (3), which is attached to the crank (4). The crank is driven at constant rotational speed by an electrical motor (5).

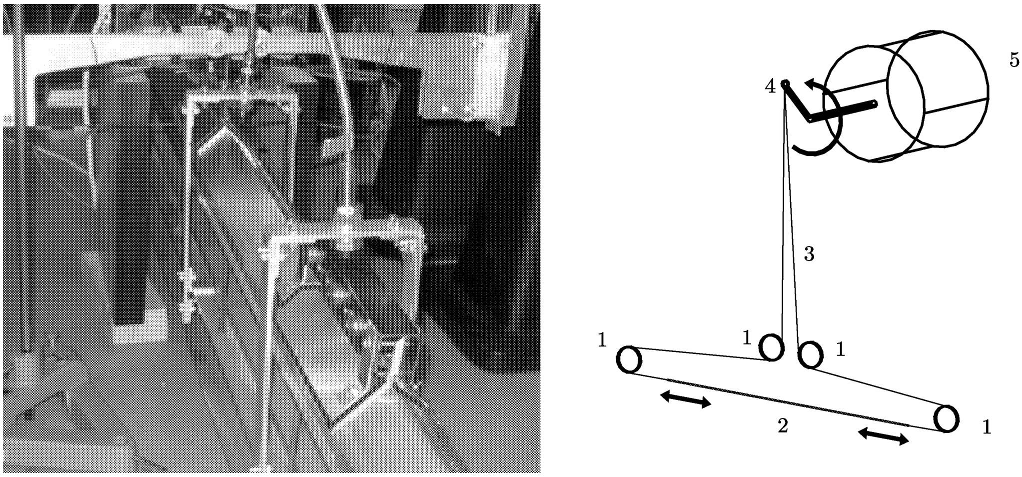

Fig.6: Schematic of test stand and parametric stiffness excitation device as used in [11]

The flexible part in the system is a rubber string (2), which is stretched according to the position of the crank and the rotational speed of the motor.

This PSE-mechanism did work quite well and in first results the stabilization of an otherwise unstable system could be proved, see [10]. However, there were some drawbacks with this test rig concerning the damping of the masses when moving on the air track. Due to limited equipment it was not possible to supply a sufficiently constant air pressure. Also geometric track errors led to somewhat unreliable damping parameters.

Therefore, a second test stand was designed from scratch to avoid the previous disadvantages. The solution for the air track friction was to hang the masses on long and very thin but almost inextensible wires, which reduces damping almost to nil. Figure 7 shows the entire test stand, but due to limited resolution of the graphic the wires are invisible. Although the pendulum motion is in principle non-linear, for limited amplitudes this is not a problem. In Fig. 8 a schematic of the test stand shows the two masses that are connected by conventional coil springs. The motion of the masses is measured by a laser-distance measurement equipment.

The second crucial component in the system is the PE-stiffness generation. A related idea as in [4] was used and a mechatronic actuator was designed, see [12]. The principle of the actuator is shown in Fig. 8. A current driven circular coil can perform a linear motion between two permanent magnets. If the polarity of the permanent magnet facing one side of the coil is the same as for the magnetic field generated by the coil, then repelling forces will be generated. The same will happen on the opposite side. The magnitude of the forces will depend on the distance and on the strength of the magnetic fields. The field of the current driven coil depends on the current provided and can therefore be changed in an arbitrary manner with time. Therefore, the actuator can be driven by a time-periodic current and then represents a parametric stiffness. In the present design the stiffness generated by the actuator is not only a function of time but depends also quadratically on distance, leading to a non-linear parametric stiffness excitation. With regard to the phenomenon of vibration suppression at combination resonance frequencies this is not a problem. The system works very well and the measurement results are in perfectly good agreement with the numerical results.

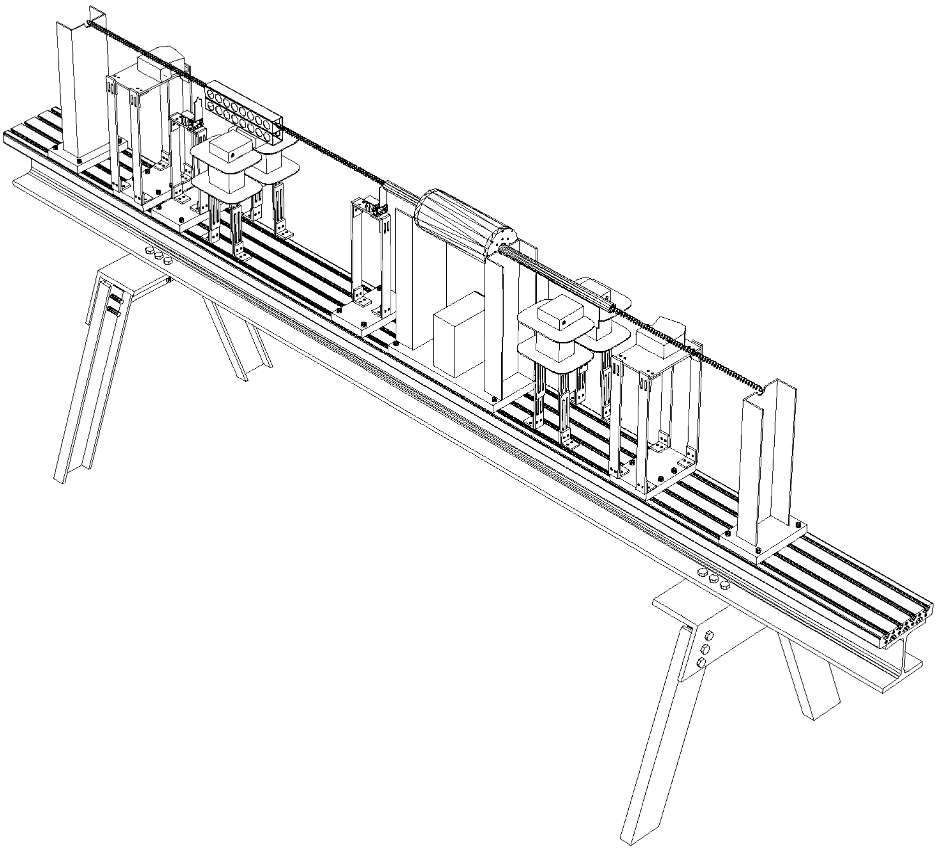

Fig.7: Schematic of a 2dof test facility to investigate parametrically excited vibrations at the Institute of Mechanics and Mechatronics at TU-Vienna

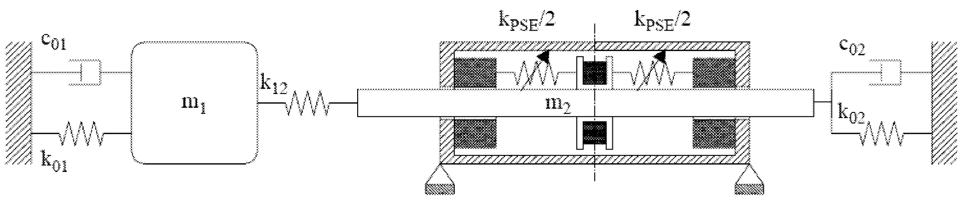

Fig.8: Principal sketch of the test rig (Fig. 7) and the actuator as discussed in [12]

Finally, the most recent test rig developed at TU-Vienna is briefly introduced. The core of this testing facility is a single cantilever beam in an upright position, see Fig. 9. The beam is slotted and a string with a high Youngs modulus runs within the slot from the tip of the beam to the root. There, a hole allows the string to pass the base and continue beyond. At that end the string is connected to a piezoelectric actuator. This actuator exerts a constant prestress to the string, which results in an axial force at the upper end of the cantilever beam. Moreover, the actuator can be controlled such that a periodic force can be generated and a system related to the previous works [5] and [7] is obtained. The direction of the force applied at the tip of the beam is neither constant in direction nor is it a follower force, since the string will always have the direction of the secant of the curved beam.

This test rig is used to demonstrate new concepts of active vibration control. By measuring the vibration at the tip of the beam, this signal can be used by a control system to control the force of the actuator for vibration suppression. In the near future, however, it will be also used in experiments to study passive vibration control methods of parametrically excited systems, as investigated already in theoretical studies, see [13].

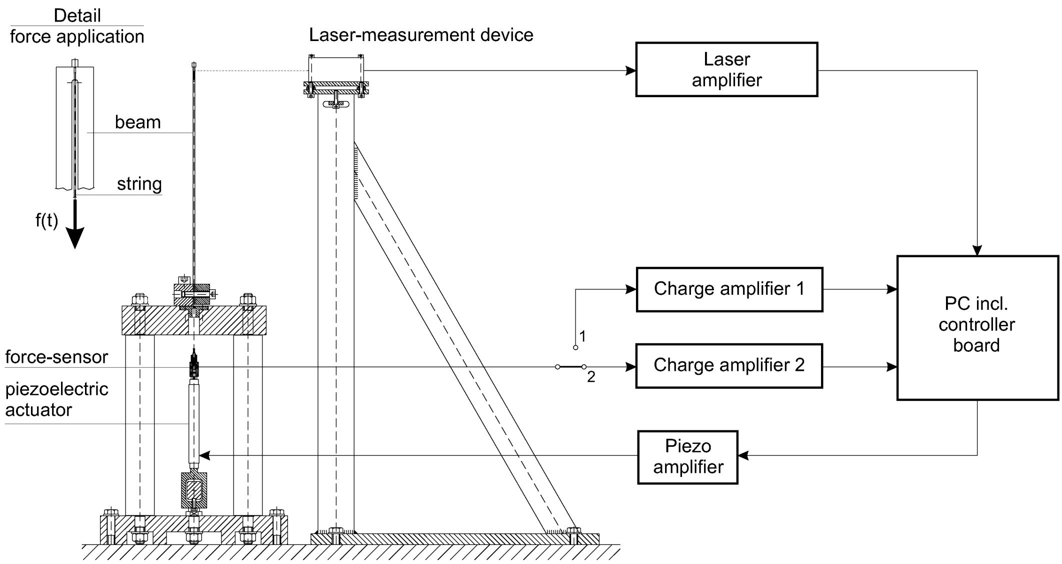

Fig.9: Test stand at TU-Vienna for active and passive vibration control of a cantilever beam by axial force control, see [13]

4. Conclusions

This review of experimental work on parametrically excited multi-degree of freedom systems reaches back to the Sixties of the last century. It turns out that a few basic concepts have been employed several times and have proven to be suitable for experiments. The axially loaded beam is a classical setup to demonstrate effects of parametric stiffness excitation. It is somewhat difficult to create a pure follower force or a force with constant direction, therefore it is more convenient to generate a combination of both types. Recent experiments show that this can be also very successful.

There are possibilities to mechanically generate a time-periodic stiffness, but for experimental studies most investigators rely on electromechanical devices, since they are much more versatile and easier to use. A drawback of such actuators is that they may be inherently non-linear, leading to a nonlinear system that is more challenging in analytical and numerical studies.

Finally it is noted that no report on experimental work was found, that deals with time-periodic mass parameters as another type of parametrically excited systems, although such systems are very common in various types of machines and drive trains. It might be worth to explore the chances for experimental studies with such systems in the future.

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