EXPERIMENTAL RESEARCH OF CHAOS IN DRIVE SYSTEMS

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Chaos can be defined on bounded-state behaviour that is not equilibrium solution or a periodic solution or a quasiperiodic solution. The article is focused on analysis of dynamic properties of controlled drive systems and also on bifurcation of steady states and possible occurrence of chaotic behaviour. The purpose of this article is to provide an elementary introduction to the subject of chaos in the electromechanical drive systems. In this article, we explore chaotic solutions of maps and continuous time systems. The attractor associated with chaotic motion in state space is not a simple geometrical object like a finite number of points, a closed curve or a torus. Chaotic attractor is complex geometrical object that posses fractal dimensions.

Keywords: *chaos, dynamical system, drive systems, nonlinear systems*

1. Introduction

Dissipative dynamic systems can be characterized as systems whose behaviour with increasing time asymptotically approaches steady states if there is no energy added from the outside. Such system description is in many cases possible with relatively simple nonlinear equations of motion. For certain values of parameters of those equations the solution does not converge towards expected values, but chaotically oscillates. Strong dependency on small changes of initial conditions occurs as well. When analyzing such phenomena its mathematical essence can be connected with existence of 'strange attractor' in phase plane. Possible origination of chaos can be seen in repeated bifurcation of solution, with so called cumulation point behind which the strange attractor is generated. Phase diagram of system solution then transfers from stable set of trajectories towards new, unstable and chaotic set. Creating the global trajectory diagrams is of essential importance. When succesful, the asymptotic behaviour of systems model is described [1], [3].

2. Deterministic chaos

Deterministic chaos is a term used to denote the irregular behaviour of dynamical systems arising from a strictly deterministic time evolution without any source of noise or external stochasticity. This irregularity manifest itself in an extremely sensitivity dependence on initial conditions or some structural and control parameters, which preclude any long-term prediction of the dynamics. Most surprisingly, it turned out that such chaotic behaviour can already be found for dynamical systems with a small degree of freedom and it is, moreover, typical for great number of mechatronic systems.

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A dynamical systems can be described simply as a systems of n first order differential equations

$$x_i'(t) = f_i(x_1, x_2, \ldots, x_n, r) \ , \qquad i = 1, 2, \ldots, n \ , \tag{1}$$

where the independent variable t can be read as time and $x_i(t)$ are dynamical quantities whose time dependence is generated by (1), starting from the specific initial conditions $x_i(0)$, $i = 1, 2, \ldots, n$. It should be noted that the system (1) is autonomous because it is not explicitly t-dependent. The $f_i(\ldots)$ is nonlinear functions of the x_i which is characterized by the parameters r. The equations lead to chaotic motion, which develops and changes its characteristics with varying control parameter(s) r. The assumption of an autonomous system is not essential, because it can be converted into an autonomous one by introducing time t as an additional variable x_{n+1}. An example of dynamical system are the Hamiltonian equations of motion in classical mechanics.

A discrete dynamical system is an iterated mapping

$$x_i(k+1) = f_i(x_1(k), \ldots, x_n(k), r) \ , \qquad i = 1, 2, \ldots, n \ , \tag{2}$$

where starting from the initial points $x_i(0)$, $i = 1, 2, \ldots, n$. This discrete system may appear quite naturally from the setup of the problem under consideration, or it may be a reduction of the continuous system (2) in order to simplify the analysis, as for example the Poincare maps.

Basically, one can make a distinction between conservative and dissipative. In the first case, volume elements in phase space are conserved, whereas dissipative systems contract phase space element. It results in markedly different behaviour [2].

3. Bifurcation behaviour of real drive system (drive with DC motor with permanent magnets)

Direct current drive with permanent magnets is often used in mechatronic drive systems together with semiconductor transducers. Good properties of such drives are given by the fact that vector of exciting magnetic flux is perpendicular to current direction in armature circuit and motor therefore always produces maximal torque. For evaluation of drive system with DC motor, possibilities of origination of bifurcation of state variables depending on small changes of control parameter, we have used *chaos module* [4], which generated chaotic input signal for small DC motor Maxon. Block diagram of this system is shown on Fig. 1.

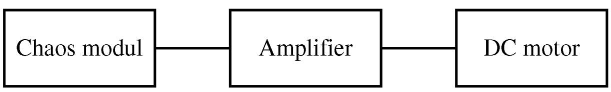

Fig.1: Block diagram of real drive system

Output signal x_n from chaos circuit is used for feeding small DC motor. Amplifier must be used enabling variable current and voltage (power) amplification without limitation or deformation of amplified signal. For such a purpose a simple amplifier was built, consisting of tracker enabling impedance isolation, voltage amplifier and two-way emitter tracker.

Emitter tracker is basically a transistor arranged with common collector, which has the voltage gain basically equal to one and output signal taken from the emitter follows the input signal. High input impedance and low output impedance are main advantages. The drawback of two-way arrangement is dead zone from −0.6 V to +0.6 V, given by B-E transition threshold voltage. The connection with operational amplifier reduces this drawback by feedback connected to the output of the tracker.

Amplifier gain is set by the voltage part resistor ratio to approximately $K_u = 4$ due to the limitation of operational amplifiers by feeding voltage (see Fig. 2).

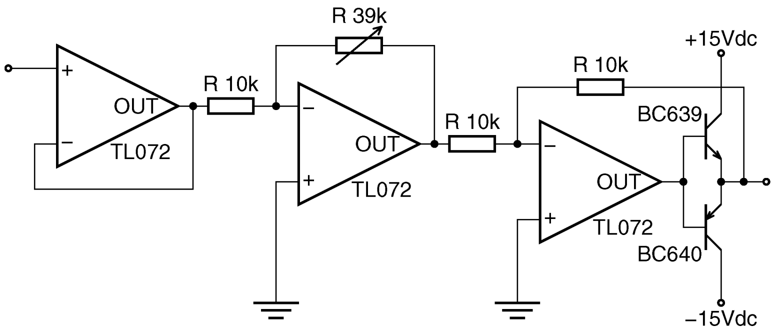

Fig.2: Amplifier circuit diagram

DC motor Maxon Re-16 fed by above described amplifier is connected through shaft with second DC motor Maxon with permanent magnets. This motor represents tachogenerator, scanning the angular velocity of powered motor shaft. Induced voltage of tachogenerator which is directly proportional to angular velocity is displayed on oscilloscope, see Fig. 3.

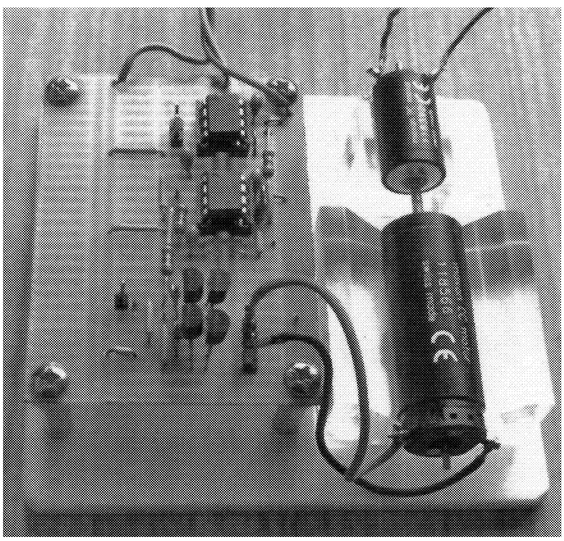

Fig.3: Overall view on real system

For limit states existence determination the shape of the signals dependent on time is important, therefore we can directly observe the shape of induced voltage which corresponds to the course of shaft angular velocity. The armature circuit current $i(t)$ is observed by oscilloscope using shunt resistance. With respect to bifurcation analysis the magnitude of such current (the current value) is not important in contrast to its shape-time course. Diagram of observed variables record we can see in Fig. 4.

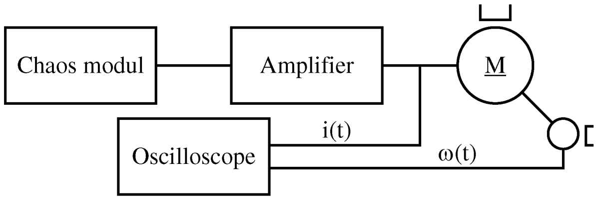

Fig.4: Diagram of observed variables record

To verify properties and behaviour of drive with DC motor in various steady states we used :

- theoretical model of DC motor and model of chaos module [5],
- real systems of motor chaos (for input signal generation), amplifier and small DC motor Maxon with revolutions recording and armature current measurement [6].

Mathematical model of DC motor can be expressed via state variables of current i and angular velocity ω by state equation :

$$\begin{bmatrix} \frac{\mathrm{d}i}{\mathrm{d}t} \\ \frac{\mathrm{d}\omega}{\mathrm{d}t} \end{bmatrix} = \begin{bmatrix} -\frac{R_\mathrm{a}}{L_\mathrm{a}} & -\frac{C_\Phi}{L_\mathrm{a}} \\ \frac{C_\Phi}{J} & 0 \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} = \begin{bmatrix} -\frac{1}{L_\mathrm{a}} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} u_\mathrm{a} \\ m_\mathrm{z} \end{bmatrix} , \tag{3}$$

where particular symbols represent : i – armature circuit current; ω – motor angular velocity; u_a – motor feeding voltage; R_a, L_a – resistance and inductance of rotor coil; Φ – magnetic flux; J – rotor moment of inertia.

Inner structure of chaos chip is showed in Fig. 5.

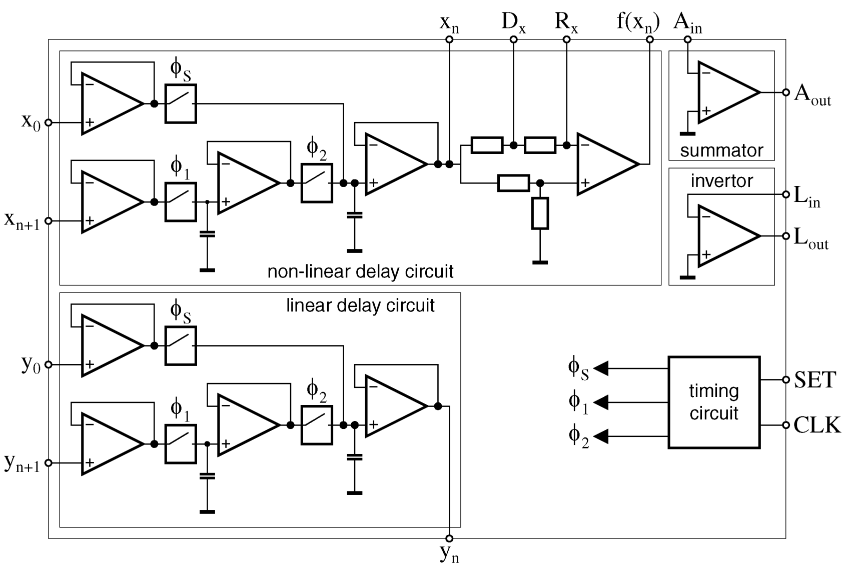

Fig.5: Inner structure of chaos chip

4. Dynamical behaviour and attributes of drive model with chaos-modul [7]

For evaluation of DC drive behaviour exhibiting possibilities of origination of bifurcation of state variables depending on small change of control parameter we have used chaos module which generated chaotic input signal for DC motor module corresponding to MAXON Re 16. Block diagram of drive model made in Matlab/Simulink is shown on Fig. 6.

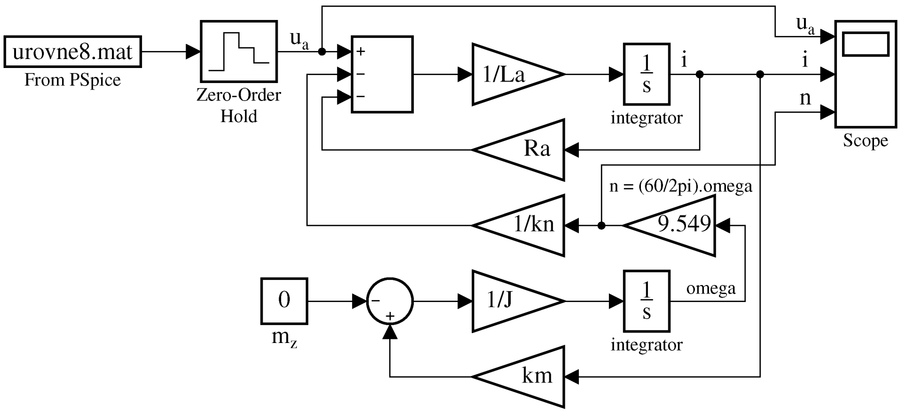

Fig.6: Block diagram of drive model

Chaos module is based on NJH 1101 chip, developed in Yamakowa's Lab & FLSI for modeling and analysis of chaotic states in discrete nonlinear systems [4]. Chaotic signal generated by chaos module in PSpice environment was sampled, saved as value matrix and transferred to Matlab/Simulink environment. We observed the courses of armature voltage $u_a(t)$, angular velocity $\omega(t)$ and armature current $i(t)$, which corresponded to steady states of the system for given values of control current – value of extremely stabilized resistor R_1 in nonlinear circuit, thus the chaos module.

Fig.7: Time signal of feeding voltage $u_a(t)$, induced voltage $u_i(t)$ and armature current $i(t)$, one period

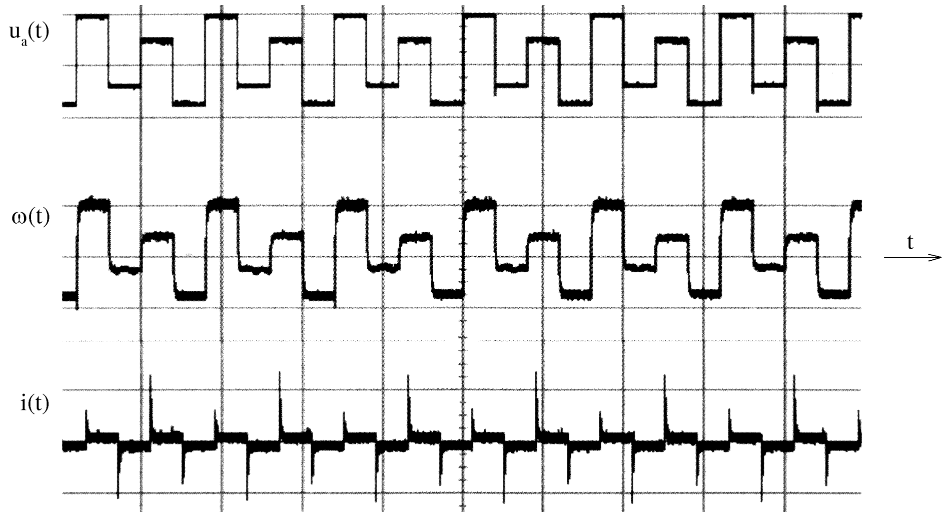

Fig.8: Time signal of feeding voltage $u_\mathrm{a}(t)$, induced voltage $u_\mathrm{i}(t)$ and armature current $i(t)$, two periods

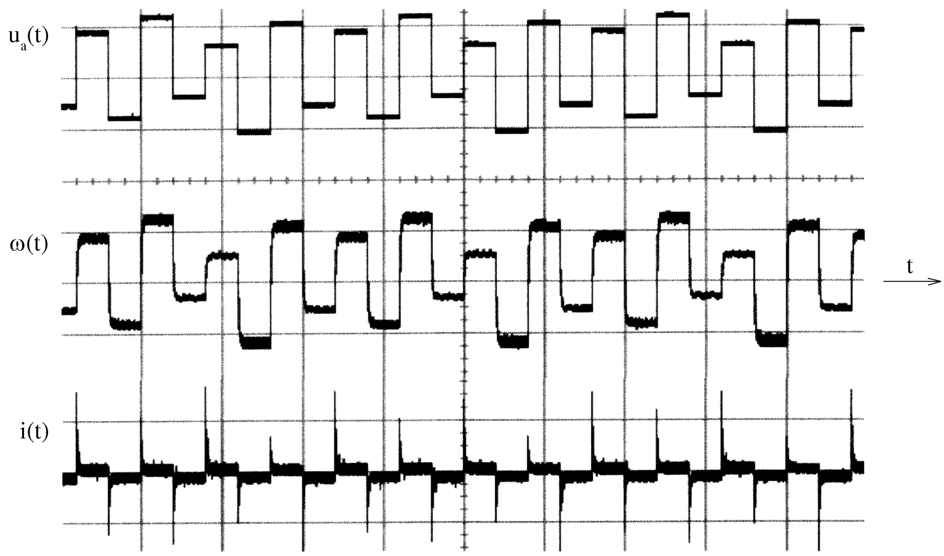

Fig.9: Time signal of feeding voltage $u_\mathrm{a}(t)$, induced voltage $u_\mathrm{i}(t)$ and armature current $i(t)$, four periods

Figures 7–10 show the course of feeding voltage $u_\mathrm{a}(t)$, induced voltage $u_\mathrm{i}(t)$ of tachogenerator corresponding to angular velocity of the motor $\omega(t)$ and armature current $i(t)$ when changing the control parameter R_1 in circuit scheme of chaos module. The feeding voltage $u_\mathrm{a}(t)$ is shown in top portion on figures 7–10. Induced voltage $u_\mathrm{i}(t)$ is shown in midlle part of those figures. Armature current $i(t)$ is shown in bottom portion of those figures.

Apart from time dependencies of selected variables we also generated given attractors corresponding to basic stable period, its double, quadruple and following chaotic state when periodicity of the motion can not be ensured – see Fig. 11.

Fig.10: Time signal of feeding voltage $u_a(t)$, induced voltage $u_i(t)$ and armature current $i(t)$, chaos

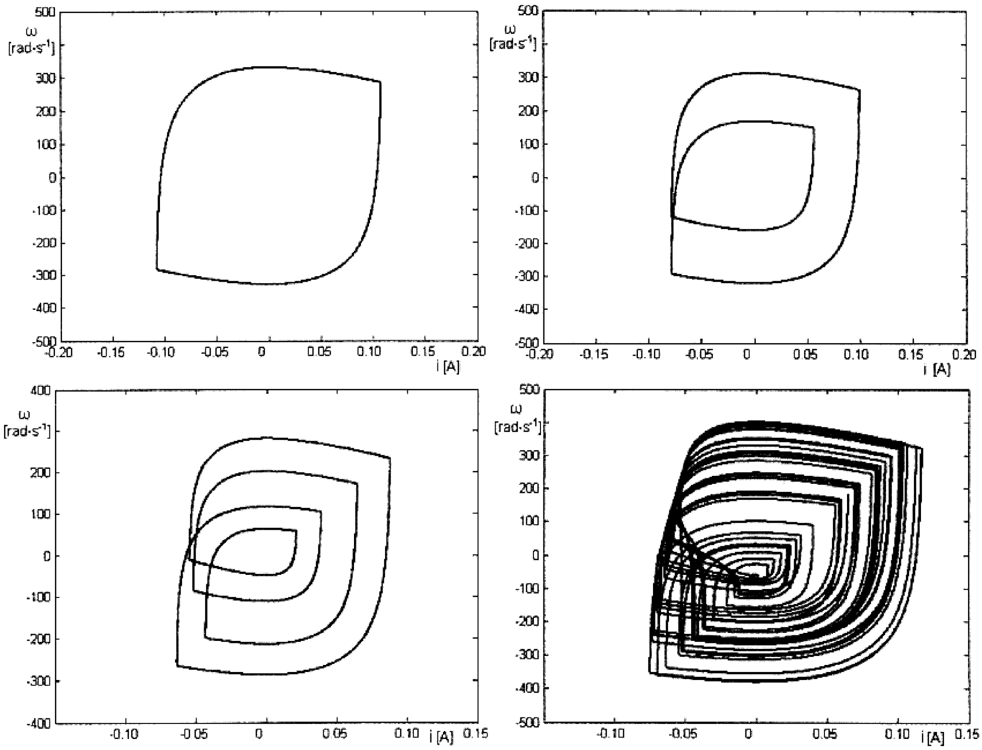

Fig.11: Attractors of model system

These attractors we can see direct on recordings of oscilloscope – see Fig. 12. We can see the existence of limit cycles with primitive period (Fig. 12a) and when changing the control parameter with double period (Fig. 12b), quadruple period (Fig. 12c) and subsequently with chaotic attractor (Fig. 12d).

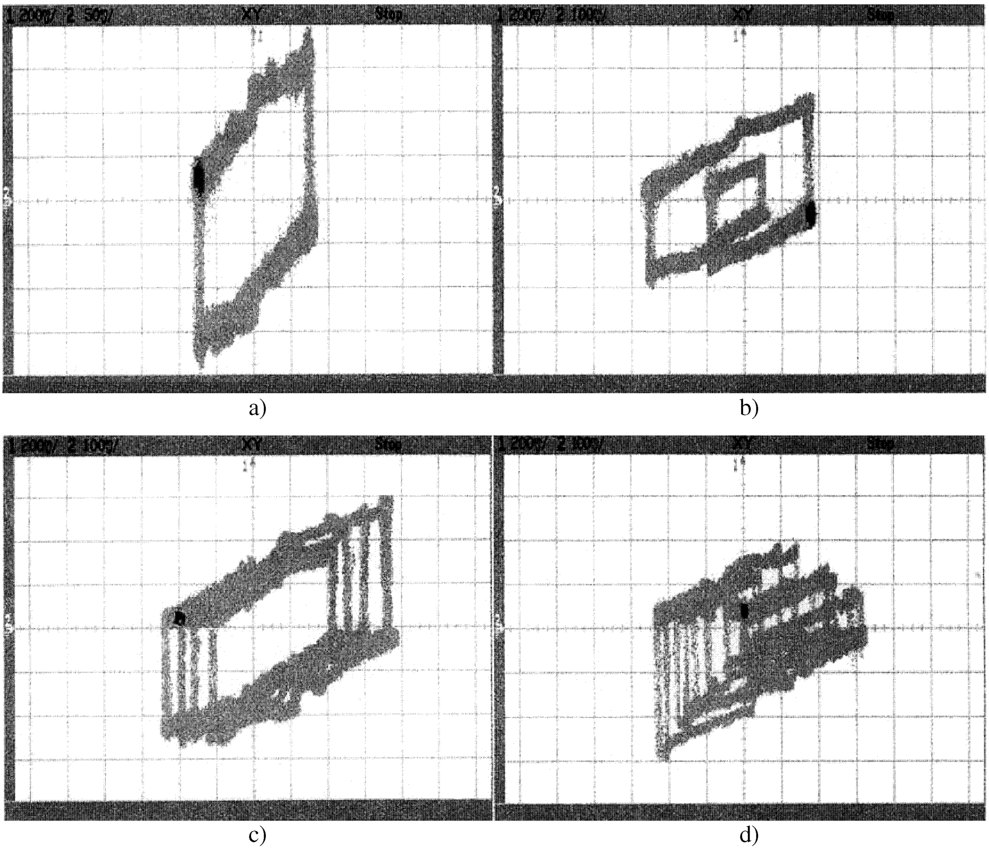

Fig.12: Attractors of real system : primitive period (a), double period (b), quadruple period (c) and chaotic attractor (d)

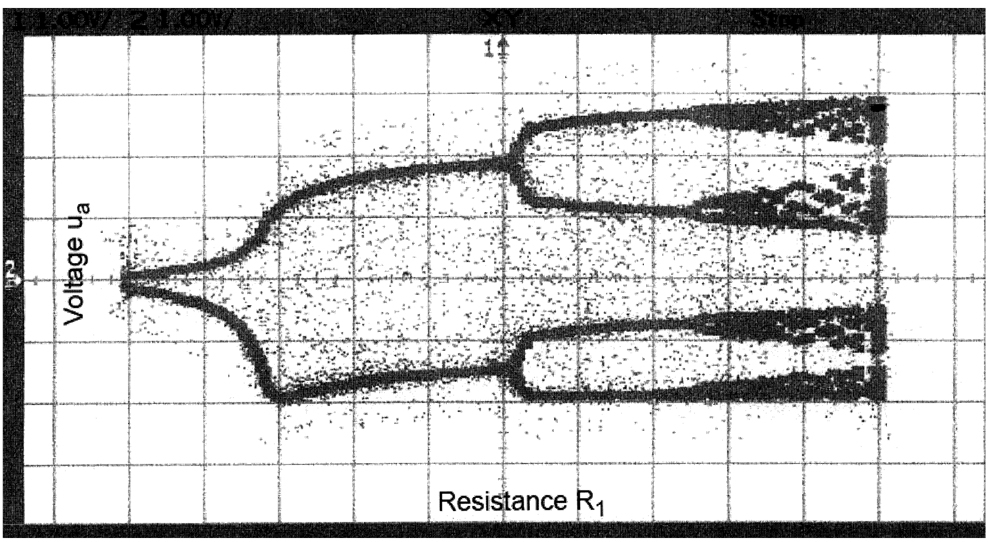

Fig.13: Bifurcation diagram

Time curses of feeding voltage $u_a(t)$, angular velocity $\omega(t)$ and armature current $i(t)$ in chaotic state are recorded on Fig. 7–10. For completeness' sake let us show the 'classical' bifurcation diagram as was shown on oscilloscope screen [6] – Fig. 13.

5. Valuation of current results and possibilities of its utilization

If we compare results obtained on model system chaos module + DC motor model and real system chaos module + amplifier + DC motor, we can state that results are very similar. From performed simulations analysis it is clear that in both cases the existence of steady limit cycles precede the occurrence of chaos. Those cycles can be uniquely determined in phase space using typical attractors (cycles with primitive period, double period and quadruple period). Chaotic behaviour in both cases is uniquely described by open state trajectories, which can not be predicted precisely. Those trajectories are situated in certain limiting areas. This has particular technical product :

Both angular velocity of the motor ω and armature current i will not exceed certain limits and observed DC motor is not in direct danger by exceeding the through current even in chaotic state.

Bifurcation analysis can be used as mean for identification of limit values of parameters, or as a mean for drive diagnostic. Bifurcation analysis could extend algorithms used of adaptive controllers design. We can not exclude the use of chaotic signals as control signals for special industrial mechanisms requiring irregular mechanical motion depending on control parameter.

6. Conclusion

Chaos became phenomenon in variety of engineering problems in last years. Therefore we focused on it also in analysis of drive systems. Based on performed analysis we can state following recommendations :

- when evaluating the properties and behaviour of dynamic system it is useful to define such parameters of models, which can influence the occurrence of parasitic motion including chaotic one (fluctuation of initial conditions, links gaps, control parameters),
- to observe the evolution of responses in phase planes based on changes of selected parameters and to identify typical chaos effects,
- if such effect occurs then evaluate Fourier spectrum of responses. Chaotic motion corresponds to broadband spectra, even when exciting spectra is narrowband.

With respect to recommendations it is not difficult to identify the areas of possible occurrence of chaos in technical systems using mathematical modelling. However, we do not want to disvalue the analytical approaches with above described alternative approach.

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