PROGRAM SNAP – A TOOL FOR ELECTROMECHANICAL DRIVES ANALYSES

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The paper deals with applicability of the program SNAP on drive systems analyses. That program should be utilized especially in education of electric and mechatronic drive experts.

Keywords: *electromechanical analogy, electric drive, modelling, linear analysis*

1. Introduction

The first step to understand electric drive systems is to explore dynamic properties of their linearized models, namely transfer functions, frequency characteristics, standard time responses, etc. Common simulation programs (simulators) yield in general quantitative results of those analyses in forms of sets of numerical relationships among selected quantities, represented obviously by respective graphs. Those results are at disposal for a user immediately after he develops system model, defines analysis requirements and starts the analysis. Unfortunately those programs don't generate any intermediate results to help users to answer questions, why obtained results are such as they are.

Opposite to common simulators the programs for symbolic and semi-symbolic analyses of dynamic systems are mostly able to yield not only quantitative numerical and graphical information and results, but they generate also essential intermediate results in the form of general analytical formulae, which are very useful to understand the system in details. The program SNAP (abbr. of Symbolic Network Analysis Program) has this feature. Experts of the Faculty of Electrical Engineering and Communication, Brno University of Technology originally developed that program as a symbolic analyser of linear and linearized electric and electronic circuits under MS Windows. Application of electromechanical analogies makes it possible to utilize SNAP for symbolic and numerical analysis of electromechanical drive systems, too.

2. General features of the program SNAP

The program SNAP is based on application of symbolic algorithms [1], [4] and makes possible to generate following analyses tools:

- transfer functions (symbolic expression in terms of system-components general parameters, semi-symbolic expression, numeric values of gain, zeroes and poles). All transfer functions in symbolic and semi-symbolic forms are simplified, if it is possible;
- frequency characteristics (Bode diagrams, Nyquist diagrams, graphs of real and imaginary part characteristics);

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- step and pulse responses (semi-symbolic expression, graphs);
- sensitivity analyses (sensitivity-model transfer functions in both the symbolic and semi-symbolic terms, corresponding graphs of frequency characteristics);
- function editor (graph of root-locus diagram, etc.). Each resultant semi-symbolic transfer function can be exported to popular mathematical programs Matlab, Maple and MathCAD for advanced analyses. Any system parameter can be stepped to make easier to analyse its influence on system dynamic properties.

The user's interface of SNAP was brought in harmony with MS Windows programs and can be characterized by the following features:

- the model is represented by so called SNAP model diagram. For this purpose the program SNAP is equipped with the schematic editor. Both the models of system elements and their graphical symbols are stored in easy extensible library (up-to-date library consists of electric and electronic elements only, drive elements and others can be added);
- the model of a system together with its parameters is described symbolically and numerically in a netlist file;
- the program realizes generally the symbolic analyses. If the model parameters are given in the form of numeric values, the analyses can continue in the numeric domain.

The program SNAP is a free-ware program and its last advanced version is at disposal for download [2].

3. Physical modelling and electromechanical analogy in drive systems

Physical modelling is based on a principle, that the system to be analysed is decomposed into individual parts – *modules*. Each module represents either real constructional ensemble (gear box, driving motor, etc.) or its parts (shaft, disc, armature circuit of a dc motor, control blocks and circuits, etc.).

Dynamic behaviour of any electric drive is incorporated with transmission, accumulation and changes of different forms of energy (mechanical, electric, magnetic) among drive modules and among modules and their surroundings. It is supposed that the energy transmission runs among the modules through small number of purpose-made boundaries by means of two power quantities. The product of these quantities defines the transmitted power

$$p(t) = \sum_{k=1}^{m} u_k(t)\, i_k(t) \ , \tag{1}$$

where $u(t)$ is an effort power quantity (abbr. effort) and $i(t)$ is a flow power quantity (abbr. flow).

In order to make possible to use SNAP as a tool for electromechanical drive analyses, it is necessary to utilize analogous power quantities, abbr. *analogs*. Consequently analogous electrical and mechanical systems will have differential equations of the same forms. For the case of electromechanical systems modelling in SNAP it is convenient to prefer couples of the following analogous power quantities, collected in Tab. 1.

This system of electromechanical analogy is called *force – current analogy*. Another system of electromechanical analogy is called *force – voltage analogy* and it is preferred in modelling by bond graphs or by SimScape in Matlab/Simulink.

	analogous power quantities	
	flow	effort
electric network	current i [A]	voltage u [V]
translation	force F [N]	translational speed v [m s^{-1}]
rotation	torque τ [N m]	angular speed ω [rad s^{-1}]

Tab.1: Couples of analogous power quantities

Each electromechanical drive system consists of different physical components. Their individual physical properties are expressed in terms of *ideal elements* and described by some of the following equations

$$i(t) = \alpha\, u(t)\ , \qquad \int_0^t i(t)\,\mathrm{d}t = \alpha\, u(t) \qquad \text{and} \qquad \int_0^t u(t)\,\mathrm{d}t = \alpha\, i(t)\ , \tag{2}$$

where α is the physical parameter of an ideal element. Physical parameters of basic ideal elements of electromechanical drives are collected in Tab. 2. One can see that a conductor can model a damper (its conductance G equals to damping b), a capacitor can model a solid body or a disc (its capacitance C equals to mass m or to inertia J) and an inductance coil can model a spring (its inductance L equals to compliance d).

	physical parameter α			
	$\frac{i}{u}$	$\frac{u}{i}$	$\frac{\int_0^t i(t)\,\mathrm{d}t}{u}$	$\frac{\int_0^t u(t)\,\mathrm{d}t}{i}$
electric network	conductance G [S]	resistance R [Ω]	capacitance C [F]	inductance L [H]
translation	damping b [N s m^{-1}]	*without suitable parameter*	mass m [kg]	compliance d [m N^{-1}]
rotation	torsional damping b_{t} [N m s rad^{-1}]	*without suitable parameter*	inertia J [kg m^2]	torsional compliance d_{t} [rad N^{-1} m^{-1}]

Tab.2: Physical parameters of basic ideal elements physical parameter α

4. Analysis of electric drives using SNAP

First of all let us consider a simple electric drive with flexible coupling between a driving unit and driven machine (Fig. 1).

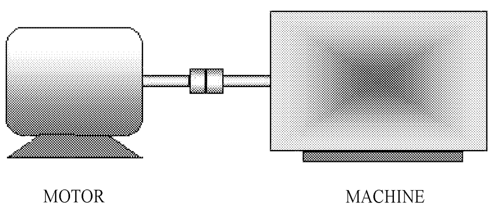

Fig.1: Simple electric drive with flexible coupling

The driving unit consists of a permanent-magnet dc motor and a dc supply-voltage source. The motor rotor is coupled with the driven machine by means of a flexible shaft through a clutch without any backlash, e.g. the coupling is considered to be linear.

The given electric drive is described by a set of the following algebraic – differential equations [3]:

– *the electric subsystem*

$$R_{\mathrm{a}}\, I_{\mathrm{a}} + L_{\mathrm{a}} \frac{\mathrm{d}I_{\mathrm{a}}}{\mathrm{d}t} + U_{\mathrm{i}} = U_{\mathrm{a}} \ , \tag{3}$$

$$U_{\mathrm{i}} = (c\Phi)\, \omega_1 \ , \qquad T_{\mathrm{i}} = (c\Phi)\, I_{\mathrm{a}} \ ; \tag{4}$$

– *the mechanical subsystem*

$$J_1 \frac{\mathrm{d}\omega_1}{\mathrm{d}t} + b_{12}\,(\omega_1 - \omega_2) + T_{12} - T_{\mathrm{i}} = 0 \ , \tag{5}$$

$$d_{12} \frac{\mathrm{d}T_{12}}{\mathrm{d}t} - (\omega_1 - \omega_2) = 0 \ , \tag{6}$$

$$J_2 \frac{\mathrm{d}\omega_2}{\mathrm{d}t} - b_{12}\,(\omega_1 - \omega_2) - T_{12} = -T_{\mathrm{L}} \ . \tag{7}$$

Friction in bearings of both discs is not taken into account.

Symbols used in above equations:

R_{a}, L_{a}	resistance and inductance of armature winding,
$c\Phi$	torque constant, resp. inverse speed constant,
J_1	inertia of the motor rotor,
d_{12}, b_{12}	compliance and proportional damping of the shaft 12,
J_2	inertia of the disc representing the machine.

Variables and inputs used:

I_{a}	armature current,
U_{i}	induced voltage,
T_{i}	electromagnetic torque,
U_{a}	supply voltage,
ω_1	motor angular speed,
ω_2	machine angular speed,
T_{12}	torque passing through the shaft 12,
T_{L}	loading torque.

Remark : Analogical symbols will be used in the following modelling equations, too.

The model circuit diagram of that drive, created by means of the SNAP schematic editor, is on Fig. 2. It is based on the electromechanical analogy according to Tab. 2.

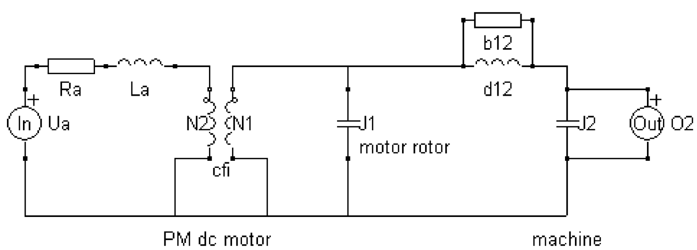

Fig.2: SNAP model diagram of a simple electric drive with flexible coupling

A component In represents the dc supply U_{a}, a component Out 'measures' an angular speed ω_2 of the machine rotor. Transfer of the input voltage U_{a} into the output angular speed ω_2 is expressed by the ratio of the output voltage across the capacitor J_2 to the voltage of the component In and it calls the voltage transfer K_{v} with the corresponding transfer function

$$K_{\mathrm{v}} = \frac{\omega_2(s)}{U_{\mathrm{a}}(s)} \equiv \frac{u_{\mathrm{Out}}(s)}{u_{\mathrm{In}}(s)} \ . \tag{8}$$

SNAP generates this transfer function in both the symbolic and semi-symbolic polynomial forms and moreover in the form of multiplication coefficient (gain), zeroes and poles (see Tab. 3. A dashed line is a fraction line).

```
____________symbolic________________

cfi*b12
+s*( d12*cfi )
-------------------------------------
cfi^(2)*b12
+s*( d12*cfi^(2) +J2*Ra*b12 +J1*Ra*b12 )
+s^(2)*( J2*d12*Ra +J1*La*b12 +J2*La*b12 +J2*d12*cfi^(2)*b12 +J1*d12*Ra )
+s^(3)*( J1*J2*d12*Ra*b12 +J2*d12*La +J1*d12*La )
+s^(4)*( J1*J2*d12*La*b12 )


___________semisymbolic_____________

Multip. Coefficient =  3.89759173322264E+0016

 2.33333333333333E-0004
 1.00000000000000E+0000 * s
-------------------------------------
 1.92800871070080E+0011
 8.26311167140831E+0014 * s
 9.30844206325037E+0013 * s^(2)
 6.07266548236136E+0009 * s^(3)
 1.00000000000000E+0000 * s^(4)


______________zeros_________________

-2.33333333333333E-0004


______________poles_________________

-6.07265015389386E+0009
-1.53195853407468E+0004
-8.88192157256630E+0000
-2.33333333333385E-0004
```

Tab.3: Transfer function of the input voltage into the output angular speed

On the base of the semi-symbolic transfer function SNAP can generate Bode diagrams for amplitude and phase (Fig. 3), semi-symbolic forms of step and pulse responses (Tab. 4) and their graphs, too (the step response is on Fig. 4).

Parameters of this electromechanical drive were purposely selected to avoid any oscillations in it (relatively stiff coupling between the motor and the machine). For this reason all poles and zeroes are real quantities, one pole is dominant.

Sensitivity analysis is a very useful tool of SNAP program. It consists in its capability to generate so-called *sensitivity models* for selected parameters of drive components. It is known, that on the base of sensitivity analyses it is possible to find component parameters with both the negligible and the extensive influence on drive dynamics.

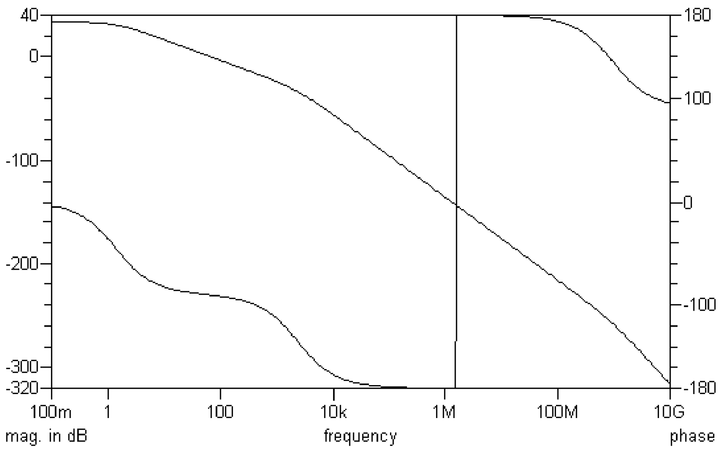

Fig.3: Bode diagrams

```
____________step response________________

 4.71698113207551E+0001
-1.74045442497533E-0013*exp(-6.07265015389386E+0009*t)
 2.73638388903929E-0002*exp(-1.53195853407468E+0004*t)
-4.71971751596558E+0001*exp(-8.88192157256630E+0000*t)
 1.04450233184838E-0011*exp(-2.33333333333385E-0004*t)


____________pulse response______________

 1.05691708316717E-0003*exp(-6.07265015389386E+0009*t)
-4.19202665131827E+0002*exp(-1.53195853407468E+0004*t)
 4.19201608226432E+0002*exp(-8.88192157256630E+0000*t)
-1.16876947564570E-0008*exp(-2.33333333333385E-0004*t)
```

Tab.4: Semi-symbolic forms of step and pulse responses

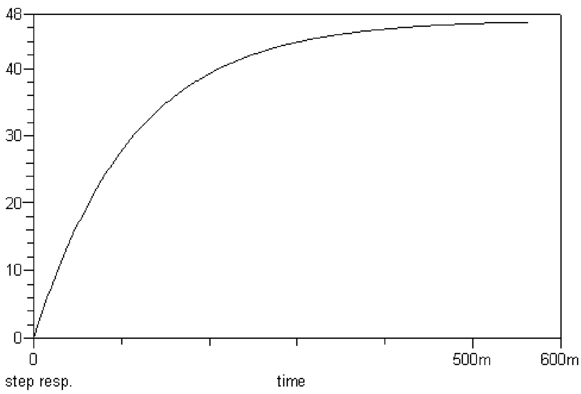

Fig.4: Step response

A sensitivity functions for the given component parameter α are defined in both the absolute and the relative forms

$$S_{\mathrm{a}}(s) = \frac{\mathrm{d}}{\mathrm{d}\alpha}\, F(s,\alpha) \qquad \text{resp.} \qquad S_{\mathrm{r}}(s) = \frac{\dfrac{\mathrm{d}}{\mathrm{d}\alpha}\, F(s,\alpha)}{\dfrac{F(s,\alpha)}{\alpha}}\ . \tag{9}$$

SNAP makes possible to generate sensitivity functions in both the symbolic and semi-symbolic forms. Moreover it can generate corresponding frequency characteristic of that function to make visible the component parameter influence. In Tab. 5 one can see, as an example, the resultant sensitivity function of the drive to changes of the motor-rotor inertia J_1 and its corresponding amplitude frequency characteristic on Fig. 5.

```
____________symbolic________________

-s*( J1*Ra*b12^(2) )
-s^(2)*( 2*J1*d12*Ra*b12 +J1*La*b12^(2) )
-s^(3)*( 2*J1*d12*La*b12 +J1*d12^(2)*Ra +J1*J2*d12*Ra*b12^(2) )
-s^(4)*( J1*J2*d12^(2)*Ra*b12 +J1*d12^(2)*La +J1*J2*d12*La*b12^(2) )
-s^(5)*( J1*J2*d12^(2)*La*b12 )
----------------------------------------
cfi^(2)*b12^(2)
+s*( 2*d12*cfi^(2)*b12 +J2*Ra*b12^(2) +J1*Ra*b12^(2) )
+s^(2)*( d12^(2)*cfi^(2) +2*J2*d12*Ra*b12 +2*J1*d12*Ra*b12 +J2*d12*cfi^(2)*b12^(2)
+J1*La*b12^(2) +J2*La*b12^(2) )
+s^(3)*( 2*J1*d12*La*b12 +2*J2*d12*La*b12 +J2*d12^(2)*Ra +J2*d12^(2)*cfi^(2)*b12 +J1*d12^(2)
*Ra +J1*J2*d12*Ra*b12^(2) )
+s^(4)*( J1*J2*d12^(2)*Ra*b12 +J2*d12^(2)*La +J1*d12^(2)*La +J1*J2*d12*La*b12^(2) )
+s^(5)*( J1*J2*d12^(2)*La*b12 )


___________semisymbolic_____________

Multip. Coefficient = -1.00000000000000E+0000

 8.63923269508798E+0005 * s
 7.40505665215040E+0009 * s^(2)
 1.58679789026447E+0013 * s^(3)
 1.03521201583798E+0009 * s^(4)
 1.00000000000000E+0000 * s^(5)
----------------------------------------
 4.49868699163520E+0007
 3.85606810069607E+0011 * s
 8.26332886838978E+0014 * s^(2)
 9.30844220494590E+0013 * s^(3)
 6.07266548236159E+0009 * s^(4)
 1.00000000000000E+0000 * s^(5)
```

Tab.5: Sensitivity function

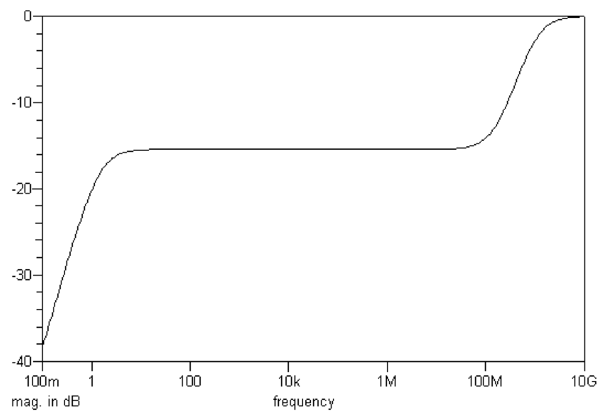

Fig.5: Amplitude frequency characteristic of a sensitive function

Due to the limited capability of SNAP and its graphical tools it is convenient in some cases to utilize the SNAP feature to export get results into the program MATLAB in the form of user's function, containing a guide how to process exported data (see Tab. 6).

```
function Z = name(s)
% Voltage gain (open output) of circuit c:\progra~1\snap\examples\name.snn
%
% usage: name (s) - return complex value of circuit function for given s
%        name ('numer')  - return vector of numerator coefficients
%        name ('denom')  - return vector of denominator coefficients
%        name ('export') - export network parameters to global workspace
%        name ('show')   - show network parameters
%        name ('clear')  - clear global workspace
%
% polynomials are in the Matlab style, ie. C(1)*s^N + ... + C(N)*s + (N+1)
.........................................................................
.........................................................................
```

Tab.6: Heading of the exported user's function

The program Matlab can generate the respective transfer function by the following command

$$\mathrm{Tfl} = \mathrm{tf(name('number'), name('denom'))} \ . \tag{10}$$

Then it can accomplish more detailed analyses and simulations of the drive by means of LTI viewer.

In the following part it is shown how to create SNAP model diagrams of another configurations of electromechanical drives. In order to make analyses easier, it is desirable to refer all parameters of mechanical components to the motor shaft. Moreover other possible transfer functions will be introduced.

a) a single-motor drive with a gear box

A single-motor drive with a gear box is seen on Fig. 6. It consists of 3 basic parts, e.g. the driving permanent-magnet dc motor, the gear box and the driven machine. Therefore the model of its mechanical part is the three-mass model.

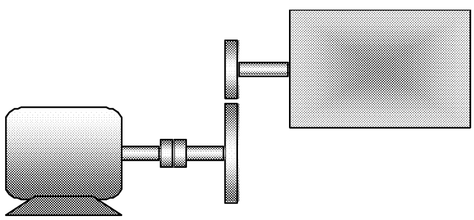

Fig.6: Single-motor drive with a gear box

This electric drive is described by a set of the following algebraic – differential equations:

– *the electric subsystem*

$$R_\mathrm{a}\, I_\mathrm{a} + L_\mathrm{a}\, \frac{\mathrm{d}I_\mathrm{a}}{\mathrm{d}t} + U_\mathrm{i} = U_\mathrm{a} \ , \tag{11}$$

$$U_\mathrm{i} = (c\Phi)\, \omega_1 \ , \qquad T_\mathrm{i} = (c\Phi)\, I_\mathrm{a} \ ; \tag{12}$$

– *the mechanical subsystem*

$$J_1\, \frac{\mathrm{d}\omega_1}{\mathrm{d}t} + b_{12}\, (\omega_1 - \omega_2) + T_{12} - T_\mathrm{i} = 0 \ , \tag{13}$$

$$d_{12}\,\frac{\mathrm{d}T_{12}}{\mathrm{d}t} - (\omega_1 - \omega_2) = 0\;, \tag{14}$$

$$J_2\,\frac{\mathrm{d}\omega_2}{\mathrm{d}t} + b_{23}\,(\omega_2 - \omega_3) + T_{23} - b_{12}\,(\omega_1 - \omega_2) - T_{12} = 0\;, \tag{15}$$

$$d_{23}\,\frac{\mathrm{d}T_{23}}{\mathrm{d}t} - (\omega_2 - \omega_3) = 0\;, \tag{16}$$

$$J_3\,\frac{\mathrm{d}\omega_3}{\mathrm{d}t} - b_{23}\,(\omega_2 - \omega_3) - T_{23} = -T_\mathrm{L}\;. \tag{17}$$

The respective SNAP model diagram of that model is on Fig. 7.

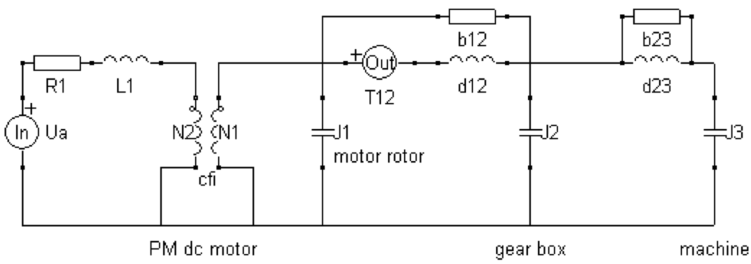

Fig.7: SNAP model diagram of the single-motor drive with the gear box

The component In represents the dc supply U_a, the component Out 'measures' the torque T_{12} in the shaft 12. Transfer of the input voltage U_a into the torque T_{12} is expressed by the ratio of the current passing through the coil 12 to the voltage of the component In and it calls the transconductance Y_T with the corresponding transfer function

$$Y_\mathrm{T} = \frac{T_{12}(s)}{U_\mathrm{a}(s)} \equiv \frac{i_\mathrm{Out}(s)}{u_\mathrm{In}(s)}\;. \tag{18}$$

It is generated by SNAP in corresponding forms.

b) a single-motor with a branching gear box

A single-motor drive with a branching gear box is seen on Fig. 8. It consists of 4 basic parts, e.g. the driving permanent-magnet dc motor, the branching gear box and 2 driven machines 1 and 2. Therefore the model of its mechanical part is the four-mass model.

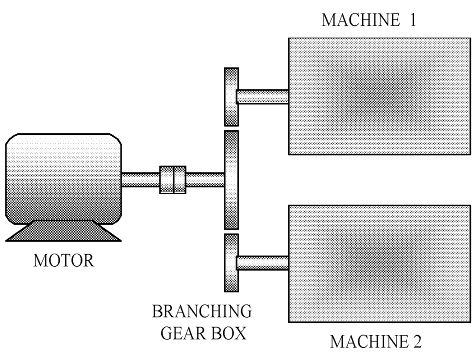

Fig.8: Single-motor drive with a dividing gear box

This electric drive is described by a set of the following algebraic – differential equations:
– *the electric subsystem*

$$R_\mathrm{a}\, I_\mathrm{a} + L_\mathrm{a}\, \frac{\mathrm{d}I_\mathrm{a}}{\mathrm{d}t} + U_\mathrm{i} = U_\mathrm{a} \;, \tag{19}$$

$$U_\mathrm{i} = (c\Phi)\, \omega_1 \;, \qquad T_\mathrm{i} = (c\Phi)\, I_\mathrm{a} \;; \tag{20}$$

– *the mechanical subsystem*

$$J_1\, \frac{\mathrm{d}\omega_1}{\mathrm{d}t} + b_{12}\,(\omega_1 - \omega_2) + T_{12} - T_\mathrm{i} = 0 \;, \tag{21}$$

$$d_{12}\, \frac{\mathrm{d}T_{12}}{\mathrm{d}t} - (\omega_1 - \omega_2) = 0 \;, \tag{22}$$

$$J_2\, \frac{\mathrm{d}\omega_2}{\mathrm{d}t} + b_{23}\,(\omega_2 - \omega_3) + T_{23} + b_{24}\,(\omega_2 - \omega_4) + T_{24} - b_{12}\,(\omega_1 - \omega_2) - T_{12} = 0, \tag{23}$$

$$d_{23}\, \frac{\mathrm{d}T_{23}}{\mathrm{d}t} - (\omega_2 - \omega_3) = 0 \;, \tag{24}$$

$$d_{24}\, \frac{\mathrm{d}T_{24}}{\mathrm{d}t} - (\omega_2 - \omega_4) = 0 \;, \tag{25}$$

$$J_3\, \frac{\mathrm{d}\omega_3}{\mathrm{d}t} - b_{23}\,(\omega_2 - \omega_3) - T_{23} = -T_\mathrm{L1} \;, \tag{26}$$

$$J_4\, \frac{\mathrm{d}\omega_4}{\mathrm{d}t} - b_{24}\,(\omega_2 - \omega_4) - T_{24} = -T_\mathrm{L2} \;. \tag{27}$$

The respective SNAP model diagram of that model is on Fig. 9.

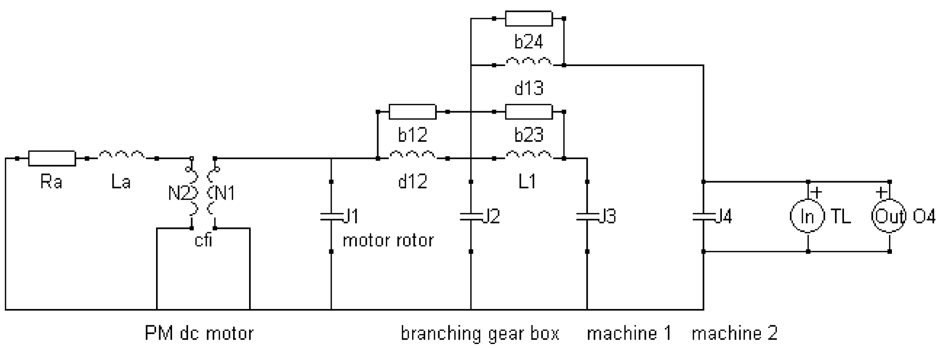

Fig.9: SNAP model diagram of the single-motor drive with the branching gear box

The component In represents the loading torque T_L, the component Out 'measures' the angular speed ω_4 of the machine 2. Transfer of the loading torque T_L into the angular speed ω_4 is expressed by the ratio of the output voltage across the capacitor J_4 to the current of the component In and it calls the transimpedance Z_T with the corresponding transfer function

$$Z_\mathrm{T} = \frac{\omega_4(s)}{T_\mathrm{L}(s)} \equiv \frac{u_\mathrm{Out}(s)}{i_\mathrm{In}(s)} \;. \tag{28}$$

It is generated by SNAP in corresponding forms. In order to generate this transfer function the input of the motor must be short-circuited.

c) a side-by-side double-motor drive with a linking gear box

A side-by-side double-motor drive with a linking gear box is seen on Fig. 10. It consists of 4 basic parts, e.g. two driving permanent-magnet dc motors, a linking gear box and a driven machine. Therefore the model of its mechanical part is the four-mass model.

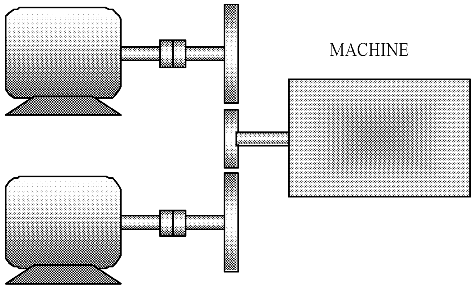

Fig.10: Side-by-side double-motor drive with a linking gear box

This electric drive is described by a set of the following algebraic – differential equations:
– *the electric subsystem (parallel supply)*

$$R_{a1}\, I_{a1} + L_{a1}\, \frac{\mathrm{d}I_{a1}}{\mathrm{d}t} + U_{i1} = U_{a1}\ , \tag{29}$$

$$U_{i1} = (c\Phi)_1\, \omega_1\ , \qquad T_{i1} = (c\Phi)_1\, I_{a1}\ , \tag{30}$$

$$R_{a2}\, I_{a2} + L_{a2}\, \frac{\mathrm{d}I_{a2}}{\mathrm{d}t} + U_{i2} = U_{a2}\ , \tag{31}$$

$$U_{i2} = (c\Phi)_2\, \omega_2\ , \qquad T_{i2} = (c\Phi)_2\, I_{a2}\ ; \tag{32}$$

– *the mechanical subsystem*

$$J_1\, \frac{\mathrm{d}\omega_1}{\mathrm{d}t} + b_{13}\,(\omega_1 - \omega_3) + T_{13} - T_{i1} = 0\ , \tag{33}$$

$$d_{13}\, \frac{\mathrm{d}T_{13}}{\mathrm{d}t} - (\omega_1 - \omega_3) = 0\ , \tag{34}$$

$$J_2\, \frac{\mathrm{d}\omega_2}{\mathrm{d}t} + b_{23}\,(\omega_2 - \omega_3) + T_{23} - T_{i2} = 0\ , \tag{35}$$

$$d_{23}\, \frac{\mathrm{d}T_{23}}{\mathrm{d}t} - (\omega_2 - \omega_3) = 0\ , \tag{36}$$

$$J_3\, \frac{\mathrm{d}\omega_3}{\mathrm{d}t} + b_{34}\,(\omega_3 - \omega_4) + T_{34} - b_{13}\,(\omega_1 - \omega_3) - T_{13} - b_{23}\,(\omega_2 - \omega_3) - T_{23} = 0, \tag{37}$$

$$d_{34}\, \frac{\mathrm{d}T_{34}}{\mathrm{d}t} - (\omega_3 - \omega_4) = 0\ , \tag{38}$$

$$J_4\, \frac{\mathrm{d}\omega_4}{\mathrm{d}t} - b_{34}\,(\omega_3 - \omega_4) - T_{34} = -T_{\mathrm{L}}\ . \tag{39}$$

The respective SNAP model diagram of that model is on Fig. 11.

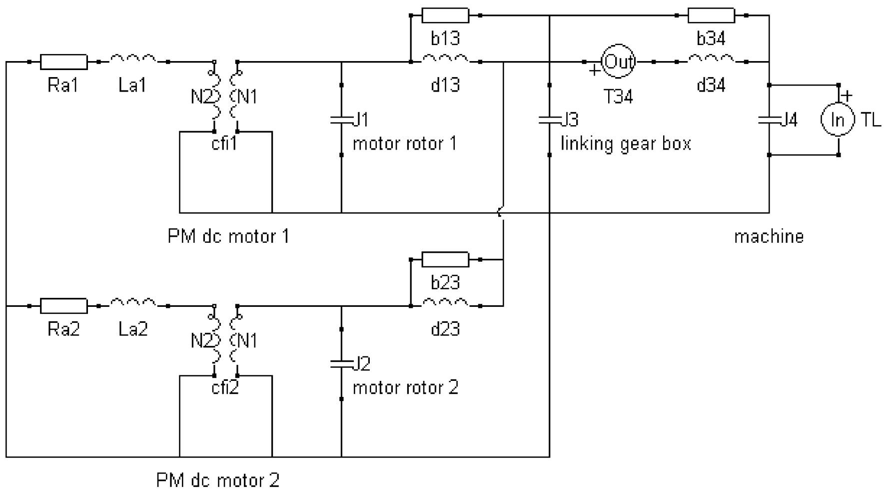

Fig.11: SNAP model diagram of the side-by-side double-motor drive with a linking gear box

The component In represents the loading torque T_L, the component Out 'measures' the torque T_{34} in the shaft 34. Transfer of the loading torque T_L into the torque T_{34} is expressed by the ratio of the current passing through the coil 34 to the current of the component In and it calls the current transfer K_I with the corresponding transfer function

$$K_\mathrm{I} = \frac{T_{34}(s)}{T_\mathrm{L}(s)} \equiv \frac{i_\mathrm{Out}(s)}{i_\mathrm{In}(s)} \; . \tag{40}$$

It is generated by SNAP in corresponding forms. In order to generate this transfer function the input of both motors must be short- circuited.

d) a cascade double-motor drive with a gear box

A cascade double-motor drive with a gear box is seen on Fig. 12. It consists of 4 basic parts, e.g. two driving permanent-magnet dc motors, a gear box and a driven machine. Therefore the model of its mechanical part is the four-mass model.

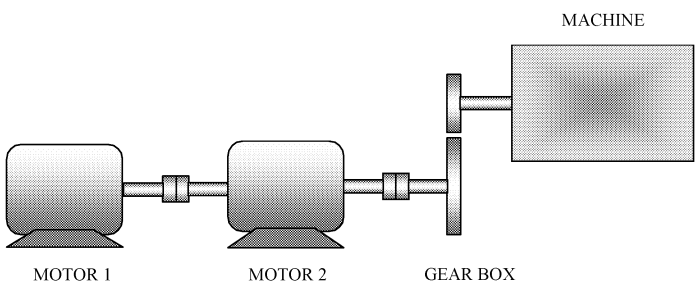

Fig.12: Cascade double-motor drive with a gear box

This electric drive is described by a set of the following algebraic – differential equations:

– *the electric subsystem (parallel supply)*

$$R_{a1}\, I_{a1} + L_{a1}\,\frac{\mathrm{d}I_{a1}}{\mathrm{d}t} + U_{i1} = U_a\ , \tag{41}$$

$$U_{i1} = (c\Phi)_1\,\omega_1\ , \qquad T_{i1} = (c\Phi)_1\, I_{a1}\ , \tag{42}$$

$$R_{a2}\, I_{a2} + L_{a2}\,\frac{\mathrm{d}I_{a2}}{\mathrm{d}t} + U_{i2} = U_a\ , \tag{43}$$

$$U_{i2} = (c\Phi)_2\,\omega_2\ , \qquad T_{i2} = (c\Phi)_2\, I_{a2}\ ; \tag{44}$$

– *the mechanical subsystem*

$$J_1\,\frac{\mathrm{d}\omega_1}{\mathrm{d}t} + b_{12}\,(\omega_1-\omega_2) + T_{12} - T_{i1} = 0\ , \tag{45}$$

$$d_{12}\,\frac{\mathrm{d}T_{12}}{\mathrm{d}t} - (\omega_1-\omega_2) = 0\ , \tag{46}$$

$$J_2\,\frac{\mathrm{d}\omega_2}{\mathrm{d}t} + b_{23}\,(\omega_2-\omega_3) + T_{23} - b_{12}\,(\omega_1-\omega_2) - T_{12} - T_{i2} = 0\ , \tag{47}$$

$$d_{23}\,\frac{\mathrm{d}T_{23}}{\mathrm{d}t} - (\omega_2-\omega_3) = 0\ , \tag{48}$$

$$J_3\,\frac{\mathrm{d}\omega_3}{\mathrm{d}t} + b_{34}\,(\omega_3-\omega_4) + T_{34} - T_{23} = 0\ . \tag{49}$$

$$d_{34}\,\frac{\mathrm{d}T_{34}}{\mathrm{d}t} - (\omega_3-\omega_4) = 0\ , \tag{50}$$

$$J_4\,\frac{\mathrm{d}\omega_4}{\mathrm{d}t} - b_{34}\,(\omega_3-\omega_4) - T_{34} = -T_L\ . \tag{51}$$

The respective SNAP model diagram of that model is on Fig. 13.

The component In represents the input voltage U_a, the component Out 'measures' the angular speed ω_2 of the motor 2. Transfer of the input voltage U_a into the angular speed ω_2 is expressed by the ratio of the voltage across the capacitor J_2 to the voltage of the component In and it calls the voltage transfer K_v with the corresponding transfer function

$$K_v = \frac{\omega_2(s)}{U_a(s)} \equiv \frac{u_{Out}(s)}{u_{In}(s)}\ . \tag{52}$$

It is generated by SNAP in corresponding forms.

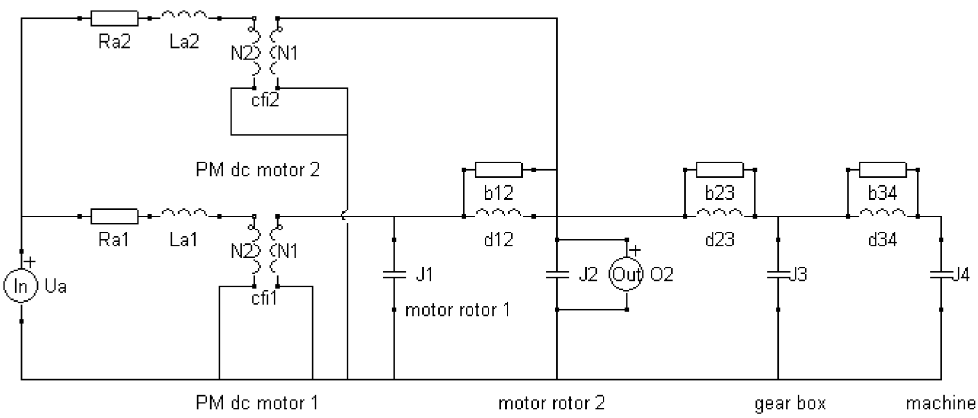

Fig.13: SNAP model diagram of a cascade double-motor drive with a gear box

5. Conclusion

Up-to-date experience confirms, that SNAP was found to be very useful for both the electric, electronic and mechatronic experts education as well as for basic system analyses because it reduces amount of time-consuming manual calculations [3]. In comparison with the program Matlab/Simulink and its special block sets (SimScape, SimPowerSystems, SimMechanics, SimElectronics, etc.) mastering of the program SNAP occupies 1 up to 1.5 hours at maximum. The user need not compile any mathematical model of the drive to be analysed, because SNAP generates the model netlist file automatically on the base of the SNAP model diagram. Using SNAP schematic editor the creation of the model diagram is simpler and quicker, because all electric and electronic component models and their graphical symbols are concentrated in one library only (in near future it is intended to add models of drive mechanical and control components into the SNAP library, too) and this library is an integral part of the SNAP schematic editor window. From the viewpoint of users, the main goal of the program SNAP is that it is the free-ware program, e.g. it saves user's money. In contradistinction to the program Matlab/Simulink the program SNAP is not a simulation program, it can serve as a tool of analyses of linear system dynamics only.

In order to make SNAP friendly for mechatronic and non-electrician experts new library elements could be developed and new names of analysis functions could be defined, because voltage and current gains, impedance, conductance, transimpedance, transconductance and others, used in current version of SNAP, are worse intelligible for non-electricians.

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