THE STRIP DAMAGE INFLUENCE ON THE STRESS AND SAFETY OF THE WOUND HIGH PRESSURE VESSEL

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The influence of the steel strip cracking at the wound part of the high pressure compound vessel on the stress, strain and vessel safety has been investigated. An equivalent orthotropic material model with the zero tangential elasticity modulus E_t has been utilised to describe this phenomenon. The stress and strain analysis was carried out by means of the problem oriented finite element method (FEM) code PROKOP [2]. The damaged strip packet of thickness 10 mm at any radial position influences negatively the vessel safety. The steel strip cracking near the inner winding radius is more dangerous than the cracking near the outer radius.

Keywords: *wound vessel, strip cracking, equivalent orthotropy, FEM*

1. Introduction

The high pressure compound vessel of mixed construction with the outer strip wound part posses several advantages compared to the usual construction with supporting steel rings. For example the controlled strip winding prestress makes possible to optimize the stress distribution with respect to the vessel safety, the strip steel exhibits higher strength than ring steel and also the manufacturing and assembly technology can be simpler in some aspects. A pressure vessel designed for the production or sintering of synthetic diamond or cubic boron nitride under pressure 5–8 GPa at temperature about 1600 °C [1] is illustrated in Fig. 1.

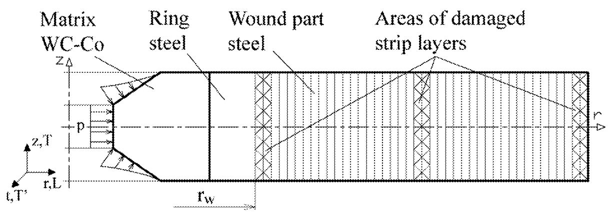

Fig.1: High pressure compound vessel of mixed construction with outer wound part

During the vessel service the strip damage may lead to the radial stiffness decrease of the supporting wound part. It causes the higher stress in the matrix, which is a critical most loaded part of the high pressure device. The influence of the strip damage on the stress and strain state in the vessel has been studied.

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2. Problem formulation

The strip cracking has been investigated with the help of the equivalent orthotropic material model in the damaged area. Regarding the geometrical shape, outer loading and material characteristics, the problem can be considered as an axial symmetric one. The cylindrical coordinates r, z, t are also the principal orthotropic axes – longitudinal L, transversal T and transversal T', see Fig. 1.

The constitutive equations can be written as follows

$$\varepsilon_{\mathrm{r}} = \frac{\sigma_{\mathrm{r}}}{E_{\mathrm{r}}} - \mu_{\mathrm{zr}} \frac{\sigma_{\mathrm{z}}}{E_{\mathrm{z}}} - \mu_{\mathrm{tr}} \frac{\sigma_{\mathrm{t}}}{E_{\mathrm{t}}} \ , \tag{1}$$

$$\varepsilon_{\mathrm{z}} = -\mu_{\mathrm{rz}} \frac{\sigma_{\mathrm{r}}}{E_{\mathrm{r}}} + \frac{\sigma_{\mathrm{z}}}{E_{\mathrm{z}}} - \mu_{\mathrm{tz}} \frac{\sigma_{\mathrm{t}}}{E_{\mathrm{t}}} \ , \tag{2}$$

$$\varepsilon_{\mathrm{z}} = -\mu_{\mathrm{rt}} \frac{\sigma_{\mathrm{r}}}{E_{\mathrm{r}}} - \mu_{\mathrm{zt}} \frac{\sigma_{\mathrm{z}}}{E_{\mathrm{z}}} + \frac{\sigma_{\mathrm{t}}}{E_{\mathrm{t}}} \ , \tag{3}$$

$$\gamma_{\mathrm{rz}} = \frac{\tau_{\mathrm{rz}}}{G_{\mathrm{rz}}} \ , \tag{4}$$

where σ_{r}, σ_{z}, σ_{t} are radial, axial and tangential normal stresses, τ_{rz} is shear stress, E_{r}, E_{z}, E_{t} are radial, axial and tangential elasticity moduli, symbol G_{rz} denotes shear modulus and finally μ_{zr}, μ_{tz} and μ_{tr} mean orthotropic Poisson's ratii.

The set of equations (1)–(4) can be described in a symbolic form

$$\boldsymbol{\varepsilon}_{\mathrm{p}} = \mathbf{S}\,\boldsymbol{\sigma}_{\mathrm{p}} \ , \tag{5}$$

where $\boldsymbol{\sigma}_{\mathrm{p}}$ and $\boldsymbol{\varepsilon}_{\mathrm{p}}$ denote the principal orthotropic stress and strain arrays, $\mathbf{S}$ is a compliance matrix

$$\boldsymbol{\sigma}_{\mathrm{p}} = [\sigma_{\mathrm{r}}, \sigma_{\mathrm{z}}, \sigma_{\mathrm{t}}, \tau_{\mathrm{rz}}]^{\mathrm{T}} \ , \tag{6}$$

$$\boldsymbol{\varepsilon}_{\mathrm{p}} = [\varepsilon_{\mathrm{r}}, \varepsilon_{\mathrm{z}}, \varepsilon_{\mathrm{t}}, \gamma_{\mathrm{rz}}]^{\mathrm{T}} \ . \tag{7}$$

To simplify the mathematic treatment, the symmetry condition of the compliance matrix has been utilized

$$\frac{\mu_{\mathrm{rz}}}{E_{\mathrm{r}}} = \frac{\mu_{\mathrm{zr}}}{E_{\mathrm{z}}} \ , \tag{8}$$

$$\frac{\mu_{\mathrm{rt}}}{E_{\mathrm{r}}} = \frac{\mu_{\mathrm{tr}}}{E_{\mathrm{t}}} \ , \tag{9}$$

$$\frac{\mu_{\mathrm{zt}}}{E_{\mathrm{z}}} = \frac{\mu_{\mathrm{tz}}}{E_{\mathrm{t}}} \ . \tag{10}$$

The inverse form of relation (5) represents the constitutive law which can be written as follows

$$\boldsymbol{\sigma}_{\mathrm{p}} = \mathbf{S}^{-1}\,\boldsymbol{\varepsilon}_{\mathrm{p}} = \mathbf{C}\,\boldsymbol{\varepsilon}_{\mathrm{p}} \ , \tag{11}$$

where $\mathbf{C}$ means the stiffness matrix

$$\mathbf{C} = \begin{bmatrix} c_{\mathrm{rr}} & c_{\mathrm{rz}} & c_{\mathrm{rt}} & 0 \\ c_{\mathrm{rz}} & c_{\mathrm{zz}} & c_{\mathrm{zt}} & 0 \\ c_{\mathrm{rt}} & c_{\mathrm{zt}} & c_{\mathrm{tt}} & 0 \\ 0 & 0 & 0 & G_{\mathrm{rz}} \end{bmatrix} \ . \tag{12}$$

Here again the symmetry condition has been applied. The stiffness matrix components are to be expressed using the s.c. technical material parameters – elasticity moduli E_i, Poison's ratii μ_{ij} $(i, j = r, z, t)$ with the clear physical meaning. The solution of equations (1)–(4) with respect to the stresses has been utilized for this purpose

$$c_{\mathrm{rr}} = D\left(1 - \frac{E_{\mathrm{z}}}{E_{\mathrm{t}}}\,\mu_{\mathrm{tz}}^2\right) \;, \tag{13}$$

$$c_{\mathrm{rz}} = D\left(\frac{E_{\mathrm{z}}}{E_{\mathrm{t}}}\,\mu_{\mathrm{tz}}\,\mu_{\mathrm{tr}} + \mu_{\mathrm{zr}}\right) \;, \tag{14}$$

$$c_{\mathrm{rt}} = D\left(\mu_{\mathrm{zr}}\,\mu_{\mathrm{tz}} + \mu_{\mathrm{tr}}\right) \;, \tag{15}$$

$$c_{\mathrm{zz}} = D\left(\frac{E_{\mathrm{z}}}{E_{\mathrm{r}}} - \frac{E_{\mathrm{z}}}{E_{\mathrm{t}}}\,\mu_{\mathrm{tr}}^2\right) \;, \tag{16}$$

$$c_{\mathrm{zt}} = D\left(\mu_{\mathrm{tr}}\,\mu_{\mathrm{zr}} + \frac{E_{\mathrm{z}}}{E_{\mathrm{r}}}\,\mu_{\mathrm{tz}}\right) \;, \tag{17}$$

$$c_{\mathrm{tt}} = D\left(\frac{E_{\mathrm{t}}}{E_{\mathrm{r}}} - \frac{E_{\mathrm{t}}}{E_{\mathrm{z}}}\,\mu_{\mathrm{zr}}^2\right) \;. \tag{18}$$

Parameter D is defined by relation (19)

$$D = \frac{E_{\mathrm{r}}}{1 - 2\,\dfrac{E_{\mathrm{r}}}{E_{\mathrm{t}}}\,\mu_{\mathrm{tr}}\,\mu_{\mathrm{zr}}\,\mu_{\mathrm{tz}} - \dfrac{E_{\mathrm{r}}}{E_{\mathrm{t}}}\,\mu_{\mathrm{tr}}^2 - \dfrac{E_{\mathrm{z}}}{E_{\mathrm{t}}}\,\mu_{\mathrm{tz}}^2 - \dfrac{E_{\mathrm{r}}}{E_{\mathrm{z}}}\,\mu_{\mathrm{zr}}^2} \;. \tag{19}$$

Using relations (11)–(19) the expression for tangential stress σ_{t} can be obtained, which is necessary for further analysis

$$\sigma_{\mathrm{t}} = E_{\mathrm{r}}\,\frac{(\mu_{\mathrm{zr}}\,\mu_{\mathrm{tz}} + \mu_{\mathrm{tr}})\,\varepsilon_{\mathrm{r}} + \left(\mu_{\mathrm{zr}}\,\mu_{\mathrm{tr}} + \dfrac{E_{\mathrm{z}}}{E_{\mathrm{r}}}\,\mu_{\mathrm{tz}}\right)\varepsilon_{\mathrm{z}} + \left(\dfrac{E_{\mathrm{t}}}{E_{\mathrm{r}}} - \dfrac{E_{\mathrm{t}}}{E_{\mathrm{z}}}\,\mu_{\mathrm{zr}}^2\right)\varepsilon_{\mathrm{t}}}{1 - 2\,\dfrac{E_{\mathrm{r}}}{E_{\mathrm{t}}}\,\mu_{\mathrm{tr}}\,\mu_{\mathrm{zr}}\,\mu_{\mathrm{tz}} - \dfrac{E_{\mathrm{r}}}{E_{\mathrm{t}}}\,\mu_{\mathrm{tr}}^2 - \dfrac{E_{\mathrm{z}}}{E_{\mathrm{t}}}\,\mu_{\mathrm{tz}}^2 - \dfrac{E_{\mathrm{r}}}{E_{\mathrm{z}}}\,\mu_{\mathrm{zr}}^2} \;. \tag{20}$$

With respect to tensional character of σ_{t} in the wound vessel part, the strip cracking in the radial direction leads to the condition $\sigma_{\mathrm{t}} = 0$ at the damaged area. This condition can be modelled applying the orthotropic material in such a way, that the elastic moduli E_{r} and E_{z} correspond to the steel strip modulus $E_{\mathrm{s}} = 2.1{\times}10^5\,\mathrm{MPa}$ and the tangential elastic modulus equals zero or very small value $E_{\mathrm{t}} = E_{\mathrm{c}} \to 0$ to be able to solve the problem numerically. Further it has been assumed that the orthotropic Poisson's ratii μ_{tz}, μ_{tr}, μ_{zr} correspond to the steel strip Poisson's ratio $\mu_{\mathrm{s}} = 0.3$ and the shear modulus G_{rz} equals $G_{\mathrm{rz}} = E_s/[2\,(1+\mu_s)]$. Then the relation (20) for σ_{t} can be simplified

$$\sigma_{\mathrm{t}} = E_{\mathrm{s}}\,\frac{\left(\mu_{\mathrm{s}}^2 + \mu_{\mathrm{s}}\right)\varepsilon_{\mathrm{r}} + \left(\mu_{\mathrm{s}}^2 + \mu_{\mathrm{s}}\right)\varepsilon_{\mathrm{z}} + \left(\dfrac{E_{\mathrm{c}}}{E_{\mathrm{s}}} - \dfrac{E_{\mathrm{c}}}{E_{\mathrm{s}}}\,\mu_{\mathrm{s}}^2\right)\varepsilon_{\mathrm{t}}}{1 - 2\,\dfrac{E_{\mathrm{s}}}{E_{\mathrm{c}}}\,\mu_{\mathrm{s}}^3 - 2\,\dfrac{E_{\mathrm{s}}}{E_{\mathrm{c}}}\,\mu_{\mathrm{s}}^2 - \mu_{\mathrm{s}}^2} \;. \tag{21}$$

Because of $\lim E_{\mathrm{c}}/E_{\mathrm{s}} = 0$ and $\lim E_{\mathrm{s}}/E_{\mathrm{c}} = \infty$ and regarding the real values of E_{s}, μ_{s}, ε_{r}, ε_{z} and ε_{t} the following relation may be written for tangential stress σ_{t} in the defined cracked area Ψ_{c}

$$\sigma_{\mathrm{t}}(r, z \in \Psi_{\mathrm{c}}) \to 0 \tag{22}$$

what satisfies well the modelled situation at cracked strip packet.

The stress and strain analysis has been done with the help of the finite element method (FEM) code PROKOP [2], which takes into account the whole loading history starting with the vessel assembly including the winding process till the actual loading state. The strip damage can be modelled in an arbitrary loading state. The corresponding FEM equilibrium equation in an incremental form follows [3], [4]

$$\mathbf{K}^{\mathrm{ep}}\,\delta\boldsymbol{\Delta} = \delta\mathbf{f}^{\mathrm{v}} + \delta\mathbf{f}^{\mathrm{p}} + \delta\mathbf{f}^{\mathrm{d}} + \delta\mathbf{f}^{\mathrm{T}} + \delta\mathbf{f}^{\mathrm{r}} = \delta\mathbf{f}\ , \tag{23}$$

where $\mathbf{K}^{\mathrm{ep}}$ denotes the global elastic-plastic stiffness matrix, $\delta\boldsymbol{\Delta}$ nodal displacement incremental vector, $\delta\mathbf{f}^{\mathrm{v}}$, $\delta\mathbf{f}^{\mathrm{p}}$, $\delta\mathbf{f}^{\mathrm{d}}$ and $\delta\mathbf{f}^{\mathrm{T}}$ are transformed force incremental vectors caused by volume force, pressure loading, deformation loading, temperature loading and finally $\delta\mathbf{f}^{\mathrm{r}}$ means an increment of unbalanced forces caused by the removal of the construction part during the disassembly process or by the strip damage.

A method of unbalanced generalised forces acting at the contact surface between the damaged and undamaged areas has been applied for the numerical calculation in such a way that the material in the damaged area has been removed and substituted with the equivalent orthotrophic one and then loaded with the unbalanced forces.

The stress response of the strip packet damage at three different positions (Fig. 1) for two different packet thickness (1 mm and 10 mm) has been studied and the corresponding contact pressure p_{w} at contact radius r_{w} between the wound part and steel ring (Fig. 1) has been calculated for the most dangerous state, when the full loading $p = 6\,\mathrm{GPa}$ is acting.

3. Results and discussion

The damaged strip layer of thickness 1 mm at any radial position (Fig. 1) does not cause significant decrease of contact pressure p_{w}. In the case of 10 mm thickness the strip damage influence can not be neglected. The corresponding contact pressures were $p_{\mathrm{w}} = 1278\,\mathrm{MPa}$ for damaged area at the internal winding radius (Fig. 1), $p_{\mathrm{w}} = 1347\,\mathrm{MPa}$ for damaged area at the middle winding radius and $p_{\mathrm{w}} = 1384\,\mathrm{MPa}$ for position at the outer winding radius, compared with the values for the undamaged strip winding $p_{\mathrm{w}} = 1490\,\mathrm{MPa}$. These values agree well with the fact that the radial stiffness of the damaged strip winding is smaller than the undamaged one. Smaller contact pressure p_{w} leads to the higher loading of the vessel matrix and consequently to the lower vessel safety.

The radial distribution of the tangential stress σ_{t} along the vessel symmetry plane (Fig. 1) is presented in the Fig. 2 for the undamaged strip winding (full line) and strip layer damage of thickness 10 mm placed at inner radius (dotted line), at middle radius (dot and dash line) and at outer radius (dashed line). It is evident from the picture that the condition (22) is fulfilled very well at the damaged areas.

In the Fig. 3 the radial distribution of the radial stress σ_{r} at the same plane and for the same conditions as in the Fig. 1 is illustrated.

With respect to the equilibrium equation in radial direction the radial distribution of σ_{r} should be continuous. This condition is also satisfied very well, see Fig. 3.

The radial distribution of the stress intensity σ_{i} along the vessel symmetry plane for the same conditions as in Fig. 1 is presented in Fig. 4. The stress intensity σ_{i} according to Mises' theory of plasticity is defined as follows

$$\sigma_{\mathrm{i}} = \sqrt{\sigma_{\mathrm{r}}^2 + \sigma_{\mathrm{z}}^2 + \sigma_{\mathrm{t}}^2 - \sigma_{\mathrm{r}}\,\sigma_{\mathrm{z}} - \sigma_{\mathrm{z}}\,\sigma_{\mathrm{t}} - \sigma_{\mathrm{t}}\,\sigma_{\mathrm{r}} + 3\,\tau_{\mathrm{rz}}^2}\ . \tag{24}$$

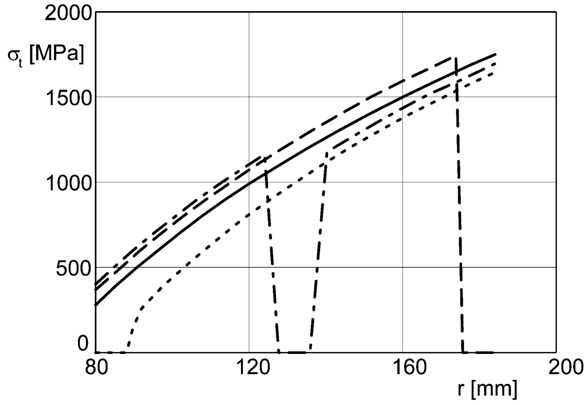

Fig.2: Radial distribution of tangential stress σ_{t}

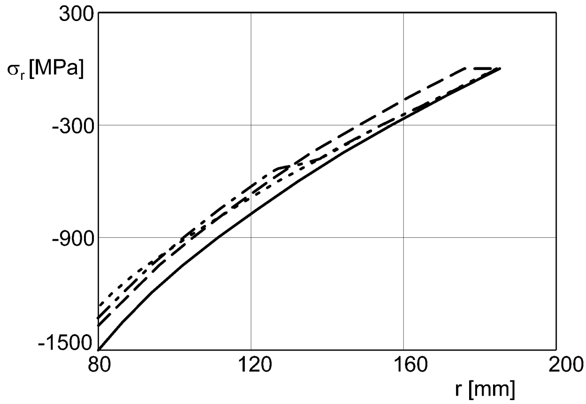

Fig.3: Radial distribution of radial stress σ_{r}

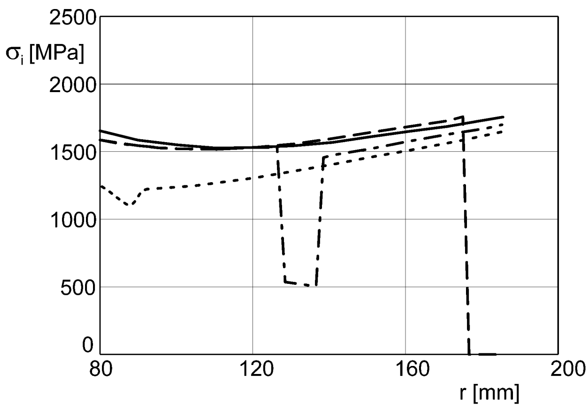

Fig.4: Radial distribution of stress intensity σ_{i}

The picture shows that stress intensity σ_{i} at the vessel wound part with damaged strip layers is smaller than in undamaged winding with exception of the case when the outer strip layers are damaged. Here the outer part of the winding is more stressed compared with the undamaged case.

4. Conclusion

The damaged strip layer of thickness 1 mm at any radial position (Fig. 1) does not cause significant stress increase at the most loaded inner vessel parts. The strip layer damage of 10 mm thickness influences negatively the compound vessel safety. The maximal calculated equivalent stress σ_{eq} according to the Mohr's hypothesis of brittle fracture in the matrix (made of sintered carbide WC-Co) was $\sigma_{\mathrm{eq}} = 3524\,\mathrm{MPa}$ for the strip damaged area at the inner winding radius, $\sigma_{\mathrm{eq}} = 3428\,\mathrm{MPa}$ for the damaged area at the middle radius and $\sigma_{\mathrm{eq}} = 3398\,\mathrm{MPa}$ for the damaged strip layer at the outer radius compared with the value $\sigma_{\mathrm{eq}} = 3024\,\mathrm{MPa}$ of the undamaged wound part.

It can be concluded that the strip fracture near the inner winding radius influences the stress and safety of the most loaded vessel inner components more negatively than the strip fracture near the outer winding radius.

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