TIME-DEPENDENT ANALYSIS OF COMPOSITE STEEL-CONCRETE BEAMS USING INTEGRAL EQUATION OF VOLTERRA, ACCORDING EUROCODE-4

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The paper presents analysis of the stress changes due to creep in statically determinate composite steel-concrete beam. The mathematical model involves the equation of equilibrium, compatibility and constitutive relationship, i.e. an elastic law for the steel part and an integral-type creep law of Boltzmann-Volterra for the concrete part. For determining the redistribution of stresses in beam section between concrete plate and steel beam with respect to time t, Volterra integral equations of the second kind have been derived, on the basis of the theory of the viscoelastic body of Arutyunian-Trost-Bazant. Numerical method, which makes use of linear approximation of the singular kernal function in the integral equationis is presented. Example with the model proposed is investigated. The creep functions is suggested by the 'CEB-FIP' models code 1990. The elastic modulus of concrete $E_c(t)$ is assumed to be constant in time t.

Keywords: *composite steel-concrete section, Volterra integral equations, rheology, EUROCODE-4*

1. Introduction

Steel-concrete composite beams are a popular and economical form of construction in both buildings and bridges. A reinforced concrete slab is mechanically connected to the top flange of a rolled or fabricated steel beam, thereby forming a composite member that is considerably stronger and stiffer than the steel beam acting on its own.

In sagging or positive bending, the concrete slab is most effective forming a wide compressive flange and raising the position of the neutral axis so that most of the steel section is available to carry tension.

The time-varying behavior of composite steel-concrete members under sustained service loads drawn the attention of engineers who have been dealing with the problems of their design for more than 60 years [56].

It is known that while in the steel beam, under the effect of the serviceability loads, we see only elastic deformations, in the concrete plate during the time significant inelastic deformation takes place as a consequence of creep and shrinkage of concrete.

These inelastic strains in the concrete deck cause redistribution of stress and significant increases in deformation.

The first works, which give the answer to this problem, are based on the Law of Dischinger [49, 50] (theory of aging), who had first formulated a time-dependent stress-strain

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differential relationship for concrete, using the following equation:

$$\frac{\mathrm{d}\varepsilon_{\mathrm{ct}}}{\mathrm{d}t} = \frac{\sigma_{\mathrm{ct}}}{E_{\mathrm{c0}}}\,\frac{\mathrm{d}\varphi_t}{\mathrm{d}t} + \frac{1}{E_{\mathrm{ct}}}\,\frac{\mathrm{d}\sigma_{\mathrm{ct}}}{\mathrm{d}t}\ , \tag{1}$$

where φ_t is called creep function.

These books and papers connected with the names of Fröhlich [56], Esslinger [54], Klöppel [64], Sonntag [94], Kunert [67], Müller [70], Dimitrov [48], Mrazik [69] and Bujňák [39] represent one independent group for which it is characteristic that by writing equilibrium and compatibility equations and the constitutive laws for the two materials, the problem is governed by a system of two simultaneous differential equations, which have been derived and solved.

As known in this differential equations it exists a group of normal forces $N_{\mathrm{c,r}}(t)$, $N_{\mathrm{a,r}}(t)$ and bending moments $M_{\mathrm{c,r}}(t)$, $N_{\mathrm{a,r}}(t)$, which influence the general stress conditions of the statically determinate composite plate beam is expressed by the decrease of the stresses in the concrete plate and in the increase of stresses in the steel beam (Fig. 1).

All these methods have been collected and analyzed by Sattler [87] and by the first author of this paper [72].

In parallel with the developed analytical methods, Blaszkowiak [36], Bradford [37], Fritz [55] and Wippel [101] have developed approximate methods, which use Dischinger's idea for applying in the calculation the ideal (fictitious) modulus of elasticity [49, 50]:

$$E_{\mathrm{c}i} = \frac{E_{\mathrm{c0}}}{1+\varphi_{\mathrm{n}}}\ , \tag{2}$$

where φ_{n} is the ultimate value of creep.

Another method of the estimate design calculation as described in [90] has been based on the creep fibred method by Busemann [40].

With Wippel's methods [101] the first stage of the development of the analytical methods, based entirely on the works of Dischinger [49, 50], has been completed.

Further development of rheology as a fundamental science and its application to concrete [2, 5, 81, 86, 98] as well as a great number of investigations in the field of creep of concrete have led to new formulations of the time-dependent behavior of concrete [22, 41, 80].

These new formulations that give the relationship between $\sigma_{\mathrm{c}}(t)$ and $\varepsilon_{\mathrm{c}}(t)$ are formulated by integral equations, which present the basis of the theory of linear viscoelastic bodies.

The integral-type creep law, i.e., the superposition equation for stepwise prescribed stress history $\sigma(t)$, is expressed by:

$$\varepsilon_{\mathrm{c}}(t,t_0) = \varepsilon^{\mathrm{sh}}(t) + \sigma(t_0)\,J(t,t_0) + \int_{t_0}^{t} \frac{\mathrm{d}\sigma(\tau)}{\mathrm{d}\tau}\,J(t,\tau)\,\mathrm{d}\tau\ . \tag{3}$$

By using algebraic methods, simpler forms for (3) are obtained. These methods are based on the hypothesis that the strain in the concrete fibers can be considered as a linear function of the creep coefficient (Trost [96], Bažant [8], Knowles [43]). This permits transforming (3) into

$$\varepsilon_{\mathrm{c}}(t,t_0) = \varepsilon^{\mathrm{sh}} + \sigma_{\mathrm{c}}(t_0)\left[\frac{1}{E_{\mathrm{c}}(t_0)} + \frac{\varphi(t,t_0)}{E_{\mathrm{c}}}\right] + \left[\sigma_{\mathrm{c}}(t) - \sigma_{\mathrm{c}}(t_0)\right]\left[\frac{1}{E_{\mathrm{c}}(t_0)} + \frac{\chi(t,t_0)\,\varphi(t,t_0)}{E_{\mathrm{c}}}\right]\ , \tag{4}$$

where
$$\chi(t,t_0) = \frac{E_c(t_0)}{E_c(t_0) - R(t,t_0)} - \frac{E_c}{E_c(t_0)\,\varphi(t,t_0)} \tag{5}$$
is the aging coefficient; $\varphi(t,t_0)$ – the creep coefficient; $R(t,t_0)$ – relaxation function, i.e., the stress response to a constant unit strain applied at the time t_0; E_c – the elastic modulus of concrete at 28 days.

The age-adjusted effective method (AAEM) directly assumed the expression provided by (5) for the aging coefficient. In this case, it is necessary to evaluate previously the relaxation function $R(t,t_0)$. This function is calculated numerically by applying the step-by-step procedure of the general method to the integral type relation between the creep and the relaxation function (Bažant [7]). However, for some standard parameters, diagrams of the χ coefficient are available from model codes (Chiorino, Carreira [43]). Moreover, a number of empiric expressions were recently proposed that provide final values of the χ coefficient with sufficient precision (Lacidogna and Tarantino [43]).

Using the effective modulus method (EMM), (4) becomes
$$\varepsilon_c(t,t_0) = \varepsilon^{sh}(t) + \sigma_c(t)\,J(t,t_0)\ , \tag{6}$$
where $\chi(t,t_0) = 1$; and $E_c(t_0) = E_c$. In this case, the variation of the stress in the interval $(t-t_0)$ is neglected and the stress is always considered equal to its final value. Consequently, this method underestimates the creep effects when the stress decreases with time. The time dependent analysis can be performed as an equivalent elastic analysis, where Young's modulus E_c is multiplied by the coefficient $1/[1+\varphi(t,t_0)]$.

When the Mean Stress Method (MSM) is applied (4) can be written as
$$\varepsilon_c(t,t_0) = \varepsilon^{sh}(t) + \sigma_c(t_0)\,J(t,t_0) + \big[\sigma_c(t) - \sigma_c(t_0)\big]\,\frac{J(t,t) + J(t,t_0)}{2}\ , \tag{7}$$
where $\chi(t,t_0) = 0.5$; and $E_c(t_0) = E_c$.

Equations (4), (6) and (7) represent the essence of the algebraic methods. It needs to be pointed out, however, that these algebraic equations used in structural analysis as constitutive laws for concrete in substitution of the integral-type creep law, as presented still cannot give a realistic picture of the stresses and deflections.

However, in order to avoid the mathematical problems in solving of the integral equations of Volterra for treating the problem connected with the creep of concrete structures, Trost [96] and Zerna [99], have revised the integral relationship into new algebraic stress-strain relationship:
$$\varepsilon_{ct} = \frac{\sigma_{c0}}{E_{c0}}\,[1+\varphi_t] + \frac{\sigma_{ct} - \sigma_{c0}}{E_{c0}}\,[1+\varrho\,\varphi_t]\ ,$$
where ϱ is the relaxation coefficient. From the same considerations another revision of integral relationship into new algebraic stress-strain relationship have been made by Krüger [65] and Wolff [103]:
$$E_{c0}\,\varepsilon_{c\varphi,t} = \sigma_{c0}\,\frac{\varphi_{t0} - \varphi_{t1}}{2} + \sigma_{ct}\left[1 + \frac{\varphi_{t(t-1)}}{2}\right] + \sum_{i=1}^{t-1} \sigma_{ci}\,\frac{\varphi_{t,i-1} - \varphi_{t,i+1}}{2}\ .$$

On the basis of that algebraic stress-strain relationship, new methods have been developed connected with the names Wappenhans [100], Wolff [103], Trost [97], Heim [62], Amadio [1], Dezi [43–47, 95] (by preposition that the connectors are deformationsable) and Gilbert [58, 59], for solving the problem raised by Fröhlich [56].

In parallel with the methods developed by Furtak [57], Kindman [63], Lapos [68], Pachla [71], Partov [75], on the basis of the theory of linear viscoelastic bodies, Sattler [88], Haenzel [60], and Profanter [79] have recently developed new methods, which are based on the 'modified theory' of Dischinger, called also the theory of Rüsch-Jungwirt [85]. This theory is described by the following equations:

$$\frac{\mathrm{d}\varepsilon_{\mathrm{ct}}}{\mathrm{d}t} = \frac{\sigma_{\mathrm{ct}}}{E_{\mathrm{cv}}}\,\frac{\mathrm{d}\varphi_{\mathrm{f,v}}}{\mathrm{d}t} + \frac{1}{E_{\mathrm{cv}}}\,\frac{\mathrm{d}\sigma_{\mathrm{ct}}}{\mathrm{d}t}\ ,$$

where $E_{\mathrm{cv}} = E_{\mathrm{c}}(t_0)/1.4$, $\varphi_{\mathrm{f,v}} = \varphi_{\mathrm{f,0}}\,|K_{\mathrm{f}}(t) - K_{\mathrm{f}}(t_0)|/1.4$.

Different approach to the solving of the formulated problems is applying the FEM by Hering [61], Cumbo [42] and Wissman [102].

Since the theory of Rüsch-Jungwirt [85] has been subjected to serious criticism in the works of Alexandrovski-Arutyunyan [2, 3, 52, 93] and [6–24, 27–35] the authors of the present paper make an attempt for a new step toward deriving more precise solution of the problem. An effort is made to give an answer to the dispute between Bažant and Rüsch-Jungwirt in [25, 26].

The first works [73–76], which give the answer to this dispute [25,26], using the integral equation of Volterra , are based on the Law of by Bolztmann-Volterra [2, 21, 91] who first formulated a time-dependent stress-strain differential relationship for concrete, described by the following integral equation:

$$\varepsilon_{\mathrm{c}}(t) = \frac{\sigma_{\mathrm{c}}(t_0)}{E_{\mathrm{c}}(t_0)}\left[1 + \phi(t - t_0)\right] + \int_{t_0}^{t} \frac{\mathrm{d}\sigma_{\mathrm{c}}(\tau)}{\mathrm{d}\tau}\,\frac{1}{E_{\mathrm{c}}(\tau)}\left[1 + \phi(t - \tau)\right]\mathrm{d}\tau\ , \tag{8}$$

where $\phi(t - \tau) = \varphi_{\mathrm{N}}\,K(\tau)\,f(t - \tau)$ is the so called the creep function and φ_{N} the ultimate value of creep coefficient, $K(\tau)$ depends on the age increase of concrete. It is called the function of aging, and it characterizes the process of the aging. The increase of τ makes $K(\tau)$ monotonously decrease. The functions

$$K(\tau) = \begin{cases} \dfrac{10.28}{5 + \sqrt{\tau}} & \text{for } \tau \le 857 \\ 0.3 & \text{for } \tau > 857 \end{cases} \qquad \text{and} \qquad f(t - \tau) = 1 - \mathrm{e}^{\left[-0.6\left(\frac{t-\tau}{30} + 0.0025\right)^{0.4} - 0.091\right]} \tag{8a}$$

(where t is the time interval during which the structure is under observation, τ is the running coordinate of time) – characterizes the process of creeping.

A practical method for solving of composite constructions based on Volterra integral equations are reported in [73]. A new idea for development of the above mentioned method is the investigation of the tangent modulus of concrete elasticity besides invariant in time t i.e. $E_{\mathrm{c}}(\tau) = E_{\mathrm{c}}(t_0) = E_{\mathrm{const}}$ and also for the case when it depends on time t [38, 92]:

$$E_{\mathrm{c}}(\tau) = E_{\mathrm{c}}(t_0)\sqrt{\frac{\tau}{4 + 0.86\,\tau}}\ . \tag{9}$$

A practical example with time-dependent elasticity modulus of concrete is considered in [78].

However the new norms suggested by EUROCODE-4 [52, 53] in analysis of composite steel-concrete beams regarding rheology, required a new 'CEB-FIP' creep models code 1990, which leads to completely different approach for solving of the above formulated problems [77]. In this paper we try to reformulate and solve these problems taking into account the new mathematical formulas.

2. Basic equations for determining the creep coefficient

The creep (compliance) function proposed by the 1990 CEB Model Code ('CEB-FIP' 1991), defined as the strain at time t caused by a constant unit stress acting from time τ to time t, is given by the relationship:

$$J(t,t_0) = \frac{1}{E_\mathrm{c}(t_0)} + \frac{\phi(t,t_0)}{E_\mathrm{c28}} \ ,$$

where $\phi(t,t_0)$ is the creep coefficient related to the elastic deformation at 28 related to E_c28; and $E_\mathrm{c}(t_0)$ and E_c28 – modulus of elasticity at the age of t_0 and 28 days, respectively. The creep coefficient is evaluated with the following formula:

$$\phi(t,t_0) = \phi_0\, \beta_\mathrm{c}(t-t_0) \ ,$$

where

$$\phi_0 = \phi_\mathrm{RH}\, \beta(f_\mathrm{cm})\, \beta(t_0)$$

or

$$\phi(t,t_0) = \phi_\mathrm{RH}\, \beta(f_\mathrm{cm})\, \beta(t_0)\, \beta_\mathrm{c}(t-t_0) \ ,$$
$$\phi(t,\tau) = \phi_\mathrm{RH}\, \beta(f_\mathrm{cm})\, \beta(\tau)\, \beta_\mathrm{c}(t-\tau) \ ,$$

where

$$\phi_\mathrm{RH} = 1 + \frac{1 - \dfrac{RH}{100}}{0.46\,\sqrt[3]{\dfrac{h_0}{100}}}$$

is a factor to allow for the effect of relative humidity on the notional creep coefficient. RH is the relative humidity of the ambient environment in %.

$$\beta(f_\mathrm{cm}) = \frac{5.3}{\left(\dfrac{f_\mathrm{cm}}{10}\right)^{0.5}}$$

is a factor to allow for the effect of concrete strength on the notional creep coefficient.

$$\beta(t_0) = \frac{1}{0.1 + (t_0)^{0.2}}$$

is a factor to allow for the effect of concrete age at loading on the notional creep coefficient (for continuous process we consider the function).

$$\beta(\tau) = \frac{1}{0.1 + \tau^{0.2}}$$

is a function of aging, depending on the age of concrete and it characterizes the process of aging.

$$\beta_{\mathrm{c}}(t-t_0)=\left[\frac{t-t_0}{\beta_{\mathrm{H}}+(t-t_0)}\right]^{0.3}$$

is a function to describe the development of creep with time after loading.

$$\beta_{\mathrm{H}}=150\left[1+\left(1.2\,\frac{RH}{100}\right)^{18}\right]\frac{h_0}{100}+250\leq 1500$$

is coefficient depending on the relative humidity (RH in %) and notional member size (h_0 in mm), where $f_{\mathrm{cm}}=f_{\mathrm{ck}}+8$ – the mean compressive strength of concrete at the age of 28 days (megapascals); and $h_0=2\,A_{\mathrm{c}}/u$ – the notional size of member (millimeters) (A_{c} – the cross section; and u – the perimeter of member in contact with the atmosphere).

Constant Young's modulus is given by:

$$E_{\mathrm{c}}=10^4(f_{\mathrm{cm}})^{\frac{1}{3}}\ .$$

Variable Young's modulus is given by:

$$E_{\mathrm{c}}(t)=\beta_{\mathrm{cc}}^{0.5}\,E_{\mathrm{c}}\ ,$$

where $E_{\mathrm{c}}=10^4(f_{\mathrm{cm}})^{1/3}$ and

$$\beta_{\mathrm{cc}}=\exp\left[s\left(1-\frac{5.3}{t^{0.5}}\right)\right]\ ,$$

where $s=0.25$ for normal and rapid hardening cements. So

$$E_{\mathrm{c}}(t)=336190\,\mathrm{e}^{0.5\left[0.25\left(1-\frac{5.3}{\sqrt{t}}\right)\right]}\ ,$$

$\phi(t_\infty,t_0)$ is a final creep coefficient of concrete.

3. Basic assumption and material constitutive relationship

The hypotheses (essentially based on those introduced in initial studies of [6, 56, 59, 63, 64, 79, 89]) in the elastic analysis of composite steel-concrete sections with stiff (rigid) shear connectors are assumed as following:

a) Bernoulli's concerning plane strain of cross-sections (Preservation of the plane cross section for the two elements considered compositely).
b) No vertical separation between parts, in other words identical vertical displacement at the slab-beam interface is assumed.
c) The connection system is distributed continuously along the axis of the beam.
d) The cross sections are free to deform (because they belong to statically determinate structures).
e) Concrete is not cracked $\sigma_{\mathrm{c}}\leq(0.4\div 0.5)\,R_{\mathrm{c}}$.
f) For the service load analysis of these cross sections the stress levels are small and, therefore, linear elastic behavior may be assumed for the steel beam, in another words Hooke's law applies to steel as well as to concrete under short-time loads.

g) Moreover , for the concrete part, if the dependence of strains and stresses upon histories of water content and temperature is disregarded, with the exclusion of large strain reversals, and under normal environment conditions, the strain can be considered as a linear functional of the previous stress history alone. This linearity implies the principle of superposition [8, 9, 80, 82, 83, 84, 92, 97], which states that strain response due to stress increments applied at different times may be added.

h) In the range of serviceability loads concrete behaves in a way allowing to be treated as a linear viscoelastic body. On the basis of our assumptions for the purpose of structure analysis the total strain for concrete subjected to initial loading at time t_0 with a stress $\sigma(t_0)$ and subjected to subsequent stress variations $\Delta\sigma(t_i)$ at time t_i may be expressed as follows:

$$\varepsilon_{\rm tot}(t,t_0) - \varepsilon^{\rm sh}(t,t_0) = \sigma(t_0)\,J(t,t_0) + \int\limits_{t_0}^{t} \frac{{\rm d}\sigma(\tau)}{{\rm d}\tau}\,J(t,\tau)\,{\rm d}\tau$$

where t is the time elapsed from casting of concrete; $\varepsilon_{\rm tot}(t,t_0)$ – total axial strain; $\varepsilon^{\rm sh}(t,t_0)$ – strain due to shrinkage, i.e. an elastic strain. Then the stress-strain behavior of concrete can be described with sufficient accuracy by the integral equations (1) by Bolztmann-Volterra [2, 21]

$$\varepsilon_{\rm c}(t) = \frac{\sigma_{\rm c}(t_0)}{E_{\rm c}(t_0)}\big[1+\phi(t,t_0)\big] + \int\limits_{t_0}^{t} \frac{{\rm d}\sigma_{\rm c}(\tau)}{{\rm d}\tau}\,\frac{1}{E_{\rm c}(\tau)}\big[1+\phi(t,\tau)\big]\,{\rm d}\tau\ ,$$

or according to ENV 1992-1-1 we get

$$\begin{aligned}\varepsilon_{\rm c}(t) = {} & \frac{\sigma_{\rm c}(t_0)}{E_{\rm c}(t_0)}\big[1+\phi_{\rm RH}\,\beta(f_{\rm cm})\,\beta(t_0)\,\beta(t-t_0)\big] + \\ & + \int\limits_{t_0}^{t} \frac{{\rm d}\sigma_{\rm c}(\tau)}{{\rm d}\tau}\,\frac{1}{E_{\rm c}(\tau)}\big[1+\phi_{\rm RH}\,\beta(f_{\rm cm})\,\beta(\tau)\,\beta(t-\tau)\big]\,{\rm d}\tau\ ,\end{aligned} \tag{10}$$

where $\phi_{\rm RH}\,\beta(f_{\rm cm})\,\beta(\tau)\,\beta(t-\tau)$ is the so called the creep function and $\varphi_{\rm N}$ the ultimate value of creep coefficient, $\beta(\tau)$ depends on the age increase of concrete. It is called the function of aging, and it characterizes the process of the aging. The increase of τ makes $\beta(\tau)$ monotonously decrease. The function $\beta_{\rm c}(t-\tau)$ – (where t is the time interval during which the structure is under observation, τ is the running coordinate of time) – characterizes the process of creeping. The constitutive law expressed by (10), represents the stress-strain-time relationship for the concrete slab.

i) The modulus of concrete elasticity is invariant in time t [38, 92] i.e.

$$E_{\rm c}(\tau) = E_{\rm c}(t_0) = E_{\rm const} = 10^4\,(f_{\rm cm})^{\frac{1}{3}}$$

and depending on time t

$$E_{\rm c}(t) = 336190\,{\rm e}^{0.5\left[0.25\left(1-\frac{5.3}{\sqrt{t}}\right)\right]}\quad {\rm daN/cm^2}\ . \tag{11}$$

j) According to a proposal by Sonntag [53], the influence of the development of the bending moment $M_{\mathrm{c,r}}(t)$ in the concrete member, upon the redistribution of the normal force of concrete $N_{\mathrm{c,r}}(t)$ can be neglected.
k) For the service load analysis no slip and uplift effects occur between the steel and concrete.
l) A single theory of interaction ignoring shear lag effects is considered [66].

4. Basic equations of equilibrium

Let us denote both the normal forces and the bending moments in the cross-section of the plate and the girder after the loading in the time $t = 0$ with $N_{\mathrm{c},0}$, $M_{\mathrm{c},0}$, $N_{\mathrm{a},0}$, $M_{\mathrm{a},0}$ and with $N_{\mathrm{c,r}}(t)$, $M_{\mathrm{c,r}}(t)$, $N_{\mathrm{a,r}}(t)$, $M_{\mathrm{a,r}}(t)$ a new group of normal forces and bending moments, arising due to creep and shrinkage of concrete.

For a composite bridge girder with

$$J_{\mathrm{c}} = \frac{A_{\mathrm{c}}(n\, I_{\mathrm{c}})\, n}{A_{\mathrm{s}}\, I_{\mathrm{s}}} \le 0.2$$

according to the suggestion of Sonntag [94] we can write the equilibrium conditions in time t as follows

$$N(t) = 0\ , \qquad N_{\mathrm{c,r}}(t) = N_{\mathrm{a,r}}(t)\ , \tag{12}$$

$$\sum M(t) = 0\ , \qquad M_{\mathrm{c,r}}(t) + N_{\mathrm{c,r}}(t)\, r = M_{\mathrm{a,r}}(t)\ . \tag{13}$$

Due to the fact that the problem is a twice internally statically indeterminate system, the equilibrium equations (12), (13) are not sufficient to solve it.

It is necessary to produce two additional equations in the sense of compatibility of deformations of both steel girder and concrete slab in time t (Fig. 1).

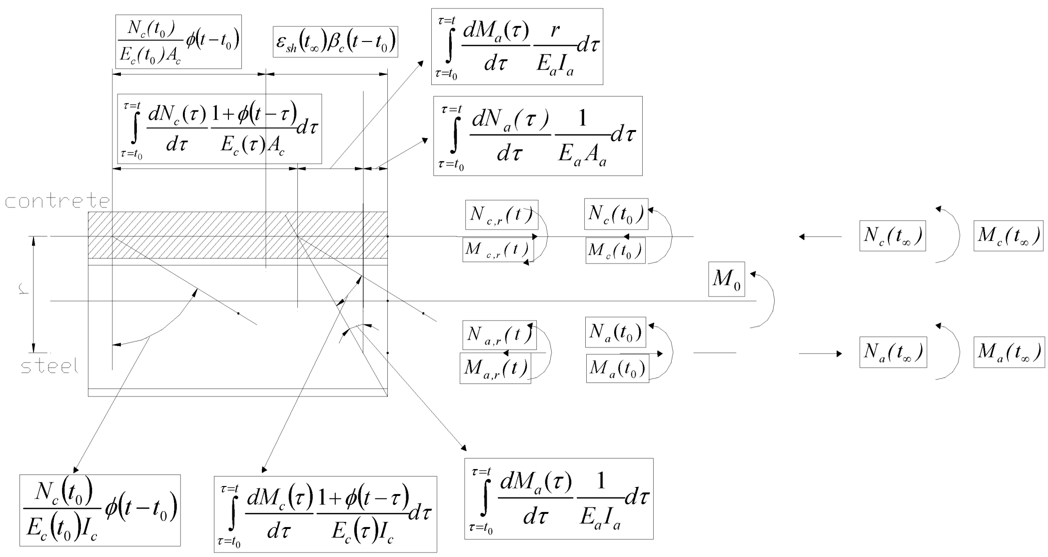

Fig.1: Mechano-mathematical model for deformations in cross-section in composite steel-concrete beam, regarding creep of the concrete

5. Deriving of the generalised mechano-mathematical model

5.1. Strain compatibility on the contact surfaces between the concrete and steel members of composite girder

$$\begin{aligned}
&\frac{N_{\mathrm{c},0}}{E_\mathrm{c}(t_0)\,A_\mathrm{c}}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(t_0)\,\beta(t-t_0)\right]-\\
&\quad-\frac{1}{A_\mathrm{c}}\int_{t_0}^{t}\frac{1}{E_\mathrm{c}(\tau)}\,\frac{\mathrm{d}N_{\mathrm{c,r}}(\tau)}{\mathrm{d}\tau}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(\tau)\,\beta(t-\tau)\right]\mathrm{d}\tau+\\
&\qquad+\frac{N_{\mathrm{a},0}}{E_\mathrm{a}\,A_\mathrm{a}}-\frac{1}{E_\mathrm{a}\,A_\mathrm{a}}\int_{t_0}^{t}\frac{\mathrm{d}N_{\mathrm{a,r}}(\tau)}{\mathrm{d}\tau}=\\
&\qquad\quad=\frac{M_{\mathrm{a},0}}{E_\mathrm{a}\,I_\mathrm{a}}\,r+r\,\frac{1}{E_\mathrm{a}\,I_\mathrm{a}}\int_{t_0}^{t}\frac{\mathrm{d}M_{\mathrm{a,r}}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau\ .
\end{aligned}\tag{14}$$

Using

$$\frac{N_{\mathrm{c},0}}{E_\mathrm{c}(t_0)\,A_\mathrm{c}}+\frac{N_{\mathrm{a},0}}{E_\mathrm{a}\,A_\mathrm{a}}=\frac{M_{\mathrm{a},0}}{E_\mathrm{a}\,I_\mathrm{a}}\,r$$

and integrating the equation (14) by parts we get

$$\begin{aligned}
&\frac{N_{\mathrm{c},0}}{E_\mathrm{c}(t_0)\,A_\mathrm{c}}\left[\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(t_0)\,\beta(t-t_0)\right]-\\
&\quad-\left[\frac{N_{\mathrm{c,r}}(\tau)}{E_\mathrm{c}\,A_\mathrm{c}}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(\tau)\,\beta(t-\tau)\right]\right]\Bigg|_{t_0}^{t}+\\
&\qquad+\frac{1}{A_\mathrm{c}}\int_{t_0}^{t}N_{\mathrm{c,r}}(\tau)\,\frac{\mathrm{d}}{\mathrm{d}\tau}\left\{\frac{1}{E_\mathrm{c}(\tau)}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(\tau)\,\beta(t-\tau)\right]\right\}\mathrm{d}\tau-\\
&\qquad\quad-\frac{1}{E_\mathrm{a}\,A_\mathrm{a}}\,N_{\mathrm{a,r}}(t)=r\,\frac{1}{E_\mathrm{a}\,I_\mathrm{a}}\,M_{\mathrm{a,r}}(t)\ ,
\end{aligned}\tag{15a}$$

$$\begin{aligned}
&\frac{N_{\mathrm{c},0}}{E_{\mathrm{c},0}\,A_\mathrm{c}}\left[\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(t_0)\,\beta(t-t_0)\right]-\\
&\quad-\frac{N_{\mathrm{c,r}}(t)}{E_\mathrm{c}\,A_\mathrm{c}}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(t)\,\beta(t-t)\right]+\\
&\qquad+\frac{N_{\mathrm{c,r}}(t_0)}{E_\mathrm{c}\,A_\mathrm{c}}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(t_0)\,\beta(t-t_0)\right]+\\
&\qquad+\frac{1}{A_\mathrm{c}}\int_{t_0}^{t}N_{\mathrm{c,r}}(\tau)\,\frac{\mathrm{d}}{\mathrm{d}\tau}\left\{\frac{1}{E_\mathrm{c}(\tau)}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(\tau)\,\beta(t-\tau)\right]\right\}\mathrm{d}\tau-\\
&\qquad\quad-\frac{1}{E_\mathrm{a}\,A_\mathrm{a}}\,N_{\mathrm{a,r}}(t)=r\,\frac{1}{E_\mathrm{a}\,I_\mathrm{a}}\,M_{\mathrm{a,r}}(t)\ .
\end{aligned}\tag{15b}$$

Since $\beta_\mathrm{c}(0)=0$ and $N_{\mathrm{c,r}}(t_0)=0$ for assessment of normal forces $N_{\mathrm{c,r}}(t)$ linear integral

Volterra equation of the second kind is derived

$$
\begin{aligned}
N_{\mathrm{c,r}}(t) = \lambda_{\mathrm{N}}(t) \int\limits_{t_0}^{t} N_{\mathrm{c,r}}(\tau) \frac{\mathrm{d}}{\mathrm{d}\tau} \left\{ \frac{1}{E_{\mathrm{c}}(\tau)} \left[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(\tau)\, \beta(t-\tau)\right] \right\} \mathrm{d}\tau \, + \\
+ \lambda_{\mathrm{N}}(t) \frac{N_{\mathrm{c},0}}{E_{\mathrm{c},0}} \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta(t-t_0) \ ,
\end{aligned}
\tag{16}
$$

where

$$
\lambda_{\mathrm{N}} = \left[\frac{1}{E_{\mathrm{c}}(t)} + \frac{A_{\mathrm{c}}}{E_{\mathrm{a}}\, A_{\mathrm{a}}} + \frac{A_{\mathrm{a}}\, r^2}{E_{\mathrm{a}}\, I_{\mathrm{a}}} \right]^{-1} \ . \tag{17}
$$

5.2. Compatibility of curvatures when $\tau = t$

$$
\begin{aligned}
& \frac{M_{\mathrm{c},0}}{E_{\mathrm{c}}(t)\, I_{\mathrm{c}}} \left[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta(t-t_0)\right] - \\
& \quad - \frac{1}{I_{\mathrm{c}}} \int\limits_{t_0}^{t} \frac{\mathrm{d}M_{\mathrm{c,r}}(\tau)}{\mathrm{d}\tau} \frac{1}{E_{\mathrm{c}}(\tau)} \left[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(\tau)\, \beta(t-\tau)\right] \mathrm{d}\tau = \\
& \quad = \frac{M_{\mathrm{a},0}}{E_{\mathrm{a}}\, I_{\mathrm{a}}} + \frac{1}{E_{\mathrm{a}}\, I_{\mathrm{a}}} \int\limits_{t_0}^{t} \frac{\mathrm{d}M_{\mathrm{a,r}}(\tau)}{\mathrm{d}\tau} \mathrm{d}\tau \ .
\end{aligned}
\tag{18}
$$

From

$$
\frac{M_{\mathrm{c},0}}{E_{\mathrm{c}}(t)\, I_{\mathrm{c}}} = \frac{M_{\mathrm{a},0}}{E_{\mathrm{a}}\, I_{\mathrm{a}}} \ ,
$$

after integrating the equation (18) by parts and using (13) for assessment of the bending moment $M_{\mathrm{c,r}}(t)$ linear integral Volterra equation of the second kind is derived:

$$
\begin{aligned}
& \frac{M_{\mathrm{c},0}}{E_{\mathrm{c}}(t)\, I_{\mathrm{c}}} \left[\phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta(t-t_0)\right] - \\
& \quad - \left[\frac{M_{\mathrm{c,r}}(\tau)}{E_{\mathrm{c}}\, I_{\mathrm{c}}} \left[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(\tau)\, \beta(t-\tau)\right] \right] \Bigg|_{t_0}^{t} + \\
& \quad + \frac{1}{I_{\mathrm{c}}} \int\limits_{t_0}^{t} M_{\mathrm{c,r}}(\tau) \frac{\mathrm{d}}{\mathrm{d}\tau} \left\{ \frac{1}{E_{\mathrm{c}}(\tau)} \left[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(\tau)\, \beta(t-\tau)\right] \right\} \mathrm{d}\tau - \\
& \quad - \frac{1}{E_{\mathrm{a}}\, A_{\mathrm{a}}} N_{\mathrm{a,r}}(t) = r \frac{1}{E_{\mathrm{a}}\, I_{\mathrm{a}}} M_{\mathrm{a,r}}(t) \ ,
\end{aligned}
\tag{18a}
$$

$$
\begin{aligned}
& \frac{N_{\mathrm{c},0}}{E_{\mathrm{c}}(t)\, A_{\mathrm{c}}} \left[\phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta(t-t_0)\right] - \\
& \quad - \frac{N_{\mathrm{c,r}}(t)}{E_{\mathrm{c}}\, A_{\mathrm{c}}} \left[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t)\, \beta(t-t)\right] + \\
& \quad + \frac{N_{\mathrm{c,r}}(t_0)}{E_{\mathrm{c}}\, A_{\mathrm{c}}} \left[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta(t-t_0)\right] + \\
& \quad + \frac{1}{A_{\mathrm{c}}} \int\limits_{t_0}^{t} N_{\mathrm{c,r}}(\tau) \frac{\mathrm{d}}{\mathrm{d}\tau} \left\{ \frac{1}{E_{\mathrm{c}}(\tau)} \left[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(\tau)\, \beta(t-\tau)\right] \right\} \mathrm{d}\tau - \\
& \quad - \frac{1}{E_{\mathrm{a}}\, A_{\mathrm{a}}} N_{\mathrm{a,r}}(t) = r \frac{1}{E_{\mathrm{a}}\, I_{\mathrm{a}}} M_{\mathrm{a,r}}(t) \ ,
\end{aligned}
\tag{18b}
$$

Since $\phi(0)=0$ and $N_\mathrm{c}(t_0)=0$ for assessment of normal forces $N_\mathrm{c,r}(t)$ linear integral Volterra equation of the second kind is derived

$$M_\mathrm{c,r}(t)=\lambda_\mathrm{M}(t)\int\limits_{t_0}^{t} M_\mathrm{c,r}(\tau)\,\frac{\mathrm{d}}{\mathrm{d}\tau}\left[\frac{1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(\tau)\,\beta(t-\tau)}{E_\mathrm{c}(\tau)}\right]\mathrm{d}\tau\,+$$

$$+\,\lambda_\mathrm{M}(t)\,\frac{M_\mathrm{c,0}}{E_\mathrm{c,0}}\,\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(t_0)\,\beta(t-t_0)-\lambda_\mathrm{M}(t)\,\frac{I_\mathrm{c}}{E_\mathrm{a}\,I_\mathrm{a}}\,N_\mathrm{c,r}(t)\,r\ ,$$

in which

$$\lambda_\mathrm{M}=\left[\frac{1}{E_\mathrm{c}(t)}+\frac{I_\mathrm{c}}{E_\mathrm{a}\,I_\mathrm{a}}\right]^{-1}\ . \tag{19}$$

In each of these equations the functions

$$N_\mathrm{c,0}\,\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(t_0)\,\beta(t-t_0)\ ,\qquad M_\mathrm{c,0}\,\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(t_0)\,\beta(t-t_0)\ ,$$
$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(\tau)\,\beta(t-\tau)\right]$$

are given.

5.3. Basic equations for the constant elasticity modulus of concrete

For constant elasticity module of concrete strain compatibility on the contact surfaces between the concrete and steel members of composite girder is

$$\begin{aligned}&\varepsilon_\mathrm{sh}(t_0)\,f(t-t_0)+\frac{N_\mathrm{c,0}}{E_\mathrm{c}(t_0)\,A_\mathrm{c}}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(t_0)\,\beta(t-t_0)\right]-\\&\quad-\frac{1}{E_\mathrm{c}(t_0)\,A_\mathrm{c}}\int\limits_{t_0}^{t}\frac{N_\mathrm{c,r}(\tau)}{\mathrm{d}\tau}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(\tau)\,\beta(t-\tau)\right]\mathrm{d}\tau\,+\\&\qquad+\frac{N_\mathrm{a,0}}{E_\mathrm{a}\,A_\mathrm{a}}-\frac{1}{E_\mathrm{a}\,A_\mathrm{a}}\int\limits_{t_0}^{t}\frac{\mathrm{d}N_\mathrm{a,r}(\tau)}{\mathrm{d}\tau}=\frac{M_\mathrm{a,0}}{E_\mathrm{a}\,I_\mathrm{a}}\,r+r\,\frac{1}{E_\mathrm{a}\,I_\mathrm{a}}\int\limits_{t_0}^{t}\frac{\mathrm{d}M_\mathrm{a,r}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau\ .\end{aligned} \tag{20}$$

And compatibility of Curvatures when $t=\tau$ is

$$\begin{aligned}&\frac{M_\mathrm{c,0}}{E_\mathrm{c}(t_0)\,I_\mathrm{c}}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(t_0)\,\beta(t-t_0)\right]-\\&\quad-\frac{1}{E_\mathrm{c}(t_0)\,I_\mathrm{c}}\int\limits_{t_0}^{t}\frac{\mathrm{d}M_\mathrm{c,r}(\tau)}{\mathrm{d}\tau}\left[1+\phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(\tau)\,\beta(t-\tau)\right]\mathrm{d}\tau=\\&\qquad=\frac{M_\mathrm{a,0}}{E_\mathrm{a}\,I_\mathrm{a}}+\frac{1}{E_\mathrm{a}\,I_\mathrm{a}}\int\limits_{t_0}^{t}\frac{\mathrm{d}M_\mathrm{a,r}(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau\ .\end{aligned} \tag{21}$$

After integrating the two equations by parts and using the (12) and (13) for assessment of normal forces $N_\mathrm{c,r}(t)$ and bending moment $M_\mathrm{c,r}(t)$ two linear integral Volterra equations of the second kind are derived.

$$N_{\mathrm{c,r}}(t) = \lambda_{\mathrm{N}} \int_{t_0}^{t} N_{\mathrm{c,r}}(\tau) \frac{\mathrm{d}}{\mathrm{d}\tau}\big[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(\tau)\, \beta(t-\tau)\big]\, \mathrm{d}\tau + \\ + \lambda_{\mathrm{N}}\, N_{\mathrm{c},0}\, \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta(t-t_0) + \lambda_{\mathrm{N}}\, N_{\mathrm{sh}}\, \beta(t-t_0)\ , \tag{22}$$

$$M_{\mathrm{c,r}}(t) = \lambda_{\mathrm{M}} \int_{t_0}^{t} M_{\mathrm{c,r}}(\tau) \frac{\mathrm{d}}{\mathrm{d}\tau}\big[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(\tau)\, \beta(t-\tau)\big]\, \mathrm{d}\tau + \\ + \lambda_{\mathrm{M}}\, M_{\mathrm{c},0}\, \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta(t-t_0) - \lambda_{\mathrm{M}} \frac{E_{\mathrm{c}}\, I_{\mathrm{c}}}{E_{\mathrm{a}}\, I_{\mathrm{a}}}\, N_{\mathrm{c,r}}(t)\, r\ . \tag{23}$$

in which

$$\lambda_{\mathrm{N}} = \left[1 + \frac{E_{\mathrm{c}}\, A_{\mathrm{c}}}{E_{\mathrm{a}}\, A_{\mathrm{a}}} \left(1 + \frac{A_{\mathrm{a}}\, r^2}{I_a}\right)\right]^{-1}\ , \tag{24}$$

$$\lambda_{\mathrm{M}} = \left[1 + \frac{E_{\mathrm{c}}\, I_{\mathrm{c}}}{E_{\mathrm{a}}\, I_{\mathrm{a}}}\right]^{-1}\ . \tag{25}$$

In each of these equations the functions

$$N_{\mathrm{c},0}\, \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta(t-t_0)\ , \qquad M_{\mathrm{c},0}\, \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta(t-t_0)\ , \\ \frac{\mathrm{d}}{\mathrm{d}\tau}\big[1 + \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(\tau)\, \beta(t-\tau)\big]$$

are given.

6. Numerical method

The integral equations (22), (23) are weakly singular Volterra integral equation of the second kind:

$$y(t) = g(t) + \lambda \int_{t_0}^{t} K(t,\tau)\, y(\tau)\, \mathrm{d}\tau\ , \qquad t \in [t_0, T]\ , \qquad 0 < T_0 < T < \infty\ ,$$

where

$$g(t) = \lambda_{\mathrm{N}}\, N_{\mathrm{c},0}\, \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta(t-t_0)\ , \qquad \lambda = \lambda_{\mathrm{N}} = \left[1 + \frac{E_{\mathrm{c}}\, A_{\mathrm{c}}}{E_{\mathrm{a}}\, A_{\mathrm{a}}} \left(1 + \frac{A_{\mathrm{a}}\, r^2}{I_a}\right)\right]^{-1}$$

for (22) and

$$g(t) = \lambda_{\mathrm{N}}\, N_{\mathrm{c},0}\, \phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(t_0)\, \beta(t-t_0) - \lambda_{\mathrm{M}} \frac{E_{\mathrm{c}}\, I_{\mathrm{c}}}{E_{\mathrm{a}}\, I_{\mathrm{a}}} N_{\mathrm{c,r}}(t)\ , \qquad \lambda = \lambda_{\mathrm{M}} = \left[1 + \frac{E_{\mathrm{c}}\, I_{\mathrm{c}}}{E_{\mathrm{a}}\, I_{\mathrm{a}}}\right]^{-1}$$

for (23), and

$$K(t,\tau) = \frac{\mathrm{d}}{\mathrm{d}\tau}\, \big[1{+}\phi_{\mathrm{RH}}\, \beta(f_{\mathrm{cm}})\, \beta(\tau)\, \beta(t{-}\tau)\big] = \varphi_{\mathrm{N}}\left[\beta(t-\tau)\, \frac{\mathrm{d}}{\mathrm{d}\tau}\big(\beta(\tau)\big) + k(\tau)\, \frac{\mathrm{d}}{\mathrm{d}\tau}\big(f(t-\tau)\big)\right]\ .$$

The singular kernel function $K(t,\tau)$ can be written in the form:

$$K(t,\tau) = L(t,\tau)\,(t-\tau)^{-0.7}\ ,$$

where

$$L(t,\tau) =$$
$$= -\phi_{\rm RH}\,\beta(f_{\rm cm})\left[\frac{0.2}{(0.1+\tau^{0.2})^2}\,\frac{1}{\tau^{0.8}}\,\frac{t-\tau}{(804.85+t-\tau)^{0.3}} - \frac{241.455}{(0.1+\tau^{0.2})\,(804.85+t-\tau)^{1.3}}\right]\ .$$

So in our case discontinuous kernel function $K(t,\tau)$ has an infinite singularity of type $(t-\tau)^{\gamma-1}$, $\gamma > 0$. In order to solve (1), we use the idea of product integration by considering the special case of:

$$y(t) = g(t) + \lambda \int\limits_{t_0}^{t} L(t,\tau)\,(t-\tau)^{\gamma-1}\,y(t)\,\mathrm{d}\tau\ , \qquad t\in[t_0,T]\ , \qquad 0<t_0<T<\infty\ , \qquad (26)$$

where the given functions $g(t)$ and $L(t,\tau)$ are sufficiently smooth which guarantee the existence and uniqueness of the solution (see Yosida (1960), Miller & Feldstein (1971)).

To solve (26) we use the method called product trapezoidal rule.

Let $n \geq 1$ be an integer and points $\{t_j = t_0 + j\,h\}_{j=0}^{n} \in [t_0,T]$. Then for general $y(t) \in C_{[t_0,T]}$ we define

$$\big(L(t,\tau)\,y(\tau)\big)_n = \frac{1}{h}\left[(t_j-\tau)\,L(t,t_{j-1})\,y(t_{j-1}) + (\tau-t_{j-1})\,L(t,t_j)\,y(t_j)\right] \qquad (27)$$

for $t_{j-1} \leq \tau \leq t_j$, $t\in[t_0,T]$.

This is piecewise linear in τ and it interpolates $L(t,\tau)\,y(\tau)$ at $\tau = t_0,\dots,t_n$. Using numerical approximation (27) we obtain the following method for solving the integral equation (26):

$$\tilde{y}(t_i) = g(t_i) + \lambda \sum_{j=0}^{i} \omega_{n,j}(t_i)\left[L(t_i,t_j)\,\tilde{y}_n(t_j)\right] \quad \text{for} \quad i=0,1,\dots,n\ , \qquad (28)$$

with weights

$$\omega_{n,0}(t_i) = \frac{1}{h}\int\limits_{t_0}^{t_1}(t_1-\tau)\,(t_i-\tau)^{\gamma-1}\,\mathrm{d}\tau\ ,$$

$$\omega_{n,n}(t_n) = \frac{1}{h}\int\limits_{t_{n-1}}^{t_n}(\tau-t_{n-1})\,(t_n-\tau)^{\gamma-1}\,\mathrm{d}\tau\ ,$$

$$\omega_{n,j}(t_i) = \frac{1}{h}\int\limits_{t_{j-1}}^{t_j}(\tau-t_{j-1})\,(t_i-\tau)^{\gamma-1}\,\mathrm{d}\tau + \frac{1}{h}\int\limits_{t_j}^{t_{j+1}}(t_{j+1}-\tau)\,(t_i-\tau)^{\gamma-1}\,\mathrm{d}\tau\ ,$$

for $i=0,1,\dots,n$.

Calculating analytically the weights, we compute the approximate solution values $y_n(t_i)$ from the system (28).

Theorem 1. Consider the numerical approximation defined with piecewise linear interpolation (18). Then for all sufficiently large n, the equation (17) is uniquely solvable and moreover if $y(t) \in C^2_{[t_0,\,T]}$, then we have

$$||y - y_n|| \leq \frac{ch^2}{8} \max_{t_0 \leq t,\, \tau \leq T} \left| \frac{\partial^2 L(t,\tau)\, y(\tau)}{\partial \tau^2} \right| . \tag{29}$$

Since $L(t,\cdot) \in C^2_{[t_0,\,T]}$, $t_0 \leq t \leq T$ the estimate (20) is immediate consequence of theorem 4.2.1 in Atkinson [4].

7. Numerical example

The method presented in the previous paragraph is now applied to a simply supported beam, subjected to a uniform load, whose cross section is shown in Fig. 2.

On the basis of numerous solved examples the optimal step of one day for solving the integral equations (22), (23) is found. The elapsed time for solving the problem (28) is about up to ten minutes.

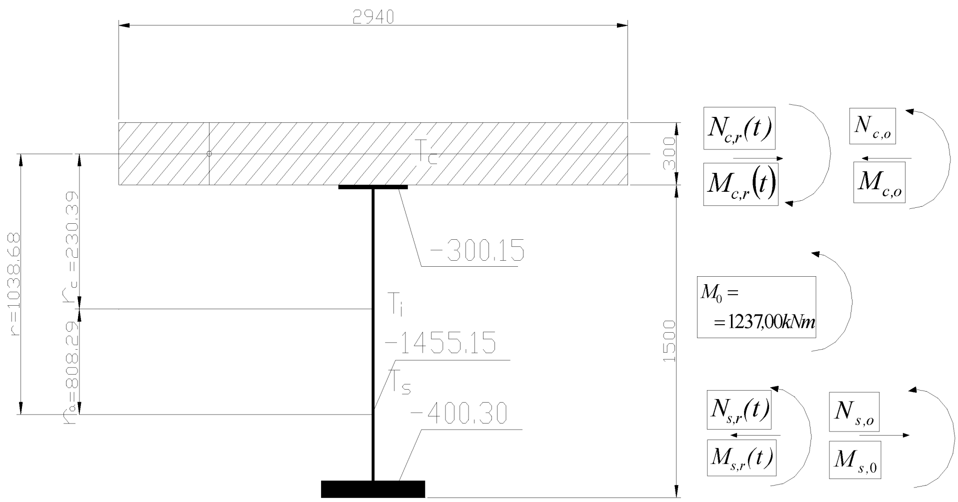

Fig.2: Composite beam with cross-section characteristic

$E_{\rm c} = 3.2{\times}10^4\,\text{MPa}$, $E_{\rm a} = 2.1{\times}10^5\,\text{MPa}$, $A_{\rm c} = 8820\,\text{cm}^2$, $A_{\rm a} = 383.25\,\text{cm}^2$,

$n = \dfrac{E_{\rm a}}{E_{\rm c}} = 6.56$, $I_{\rm c} = 661500\,\text{cm}^4$, $I_{\rm a} = 1217963.7\,\text{cm}^4$, $r_{\rm c} = 23.039\,\text{cm}$,

$r_{\rm a} = 80.829\,\text{cm}$, $r = 103.868\,\text{cm}$, $A_{\rm i} = 2453.05\,\text{cm}^2$, $I_{\rm i} = 4536360.758\,\text{cm}^4$,

$M_0 = 1237\,\text{kNm}$, $N_{\rm c,0} = 846.60\,\text{kN}$, $M_{\rm c,0} = 27.56\,\text{kNm}$, $M_{\rm a,0} = 330{,}13\,\text{kNm}$,

$$\lambda_{\rm N} = \left[1 + \frac{E_{\rm c}\,A_{\rm c}}{E_{\rm a}\,A_{\rm a}}\left(1 + \frac{A_{\rm a}\,r^2}{I_{\rm a}}\right)\right]^{-1} = 0.060545358 ,$$

$$\lambda_M = \left[1 + \frac{E_{\rm c}\,I_{\rm c}}{E_{\rm a}\,I_{\rm a}}\right]^{-1} = 0.922950026 ,$$

$$h_0 = \frac{2\,A\,C}{u} = 300\,\mathrm{mm}\ ,$$

$$\beta_\mathrm{H} = 150\left[1 + \left(1.2\,\frac{80}{100}\right)^{18}\right]\frac{h_0}{100} + 250 = 915.82 < 1500\ ,$$

$$\beta\,(f_\mathrm{cm}) = \left.\frac{5.3}{\left(\frac{f_\mathrm{cm}}{10}\right)^{0.5}}\right|_{f_\mathrm{cm}=30} = 3.06\ ,$$

$$\beta\,(t_0) = \left.\frac{1}{0.1 + (t_0)^{0.2}}\right|_{t_0=60} = 0.4223\ ,$$

$$\phi_\mathrm{RH} = 1 + \left.\frac{1 - \frac{RH}{100}}{0.46\,\sqrt[3]{\left(\frac{h_0}{100}\right)}}\right|_{RH=80,\,h_0=300} = 1.3014\ ,$$

$$\phi_0 = \phi_\mathrm{RH}\,\beta(f_\mathrm{cm})\,\beta(t_0) = 1.6817\ ,$$

$$\beta_\mathrm{c}(36500 - 60) = 0.9925811\ ,$$

$$\phi_{t=36500} = \phi_0\,\beta_\mathrm{c}(36500 - 60) = 1.669242\ .$$

8. Stress histories in midspan section according the received numerical results

In the concrete plate the normal component $N_\mathrm{c}(t_\infty)$ and the bending moment $M_\mathrm{c}(t_\infty)$ decrease by effect of creep. In the steel beam, the normal component $N_\mathrm{a}(t_\infty)$ decreases and the bending moment $M_\mathrm{a}(t_\infty)$ increases by the effect of creep.

The decrease of the stresses in concrete slab is accompanied by a gradual migration of stresses from the concrete slab to the steel beam.

This result in a very strong increase in the upper flange and a small increase of the stress in the bottom flange (less than 8 % of the initial stress) which is illustrated in Figures 8 and 9. Figure 8 shows how the stress at the top fibers of the steel section undergoes strong increases in time: the final values are four to six times higher than the initial values.

Consequently, the stress history in the top flange of the steel beam becomes the most interesting aspect of this study.

These graphs also shows how important is the age of concrete at loading.

Considering the stresses in the top flange of the steel beam, for low values of parameter $t_0 = 28$ days and $t_0 = 60$ days, we see that stresses increase more for young concrete and a little for old one.

Above all the influence of concrete age at loading time t_0 is significant only when its values are very low (i.e. with young concrete).

For the two standard cases assumed by CEB model code No 190 ('CEB-FIP' 1988) $RH = 50\,\%$ corresponding to dry conditions (inside) and $RH = 80\,\%$ corresponding to humid conditions (outside).

9. Time development of deflections

When the distribution of the bending moments in steel section $M_\mathrm{a,r}(t_\infty) = M_\mathrm{c,r}(t) + N_\mathrm{c,r}(t)$ is known, it is possible to calculate the change of the vertical deflections in time t.

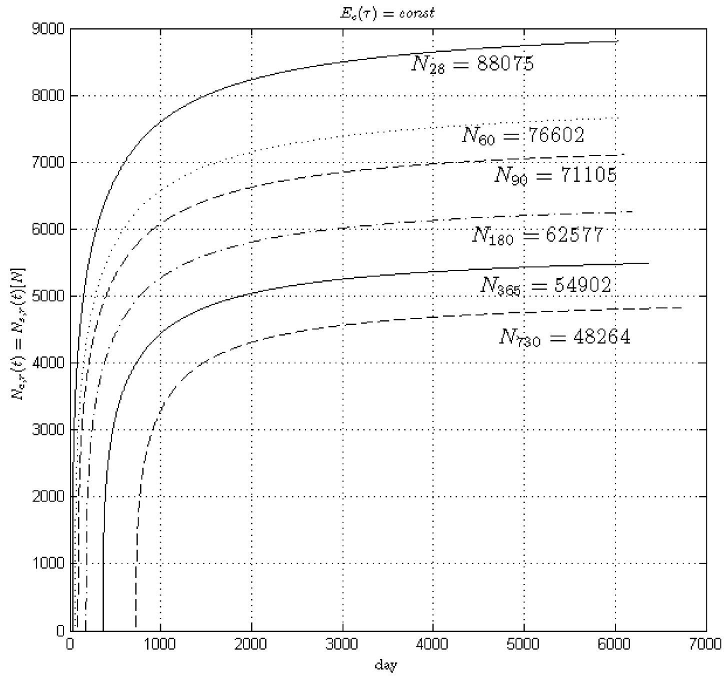

Fig.3: Values of normal forces $N_{\mathrm{c,r}}(t) = N_{\mathrm{a,r}}(t)$ in time t when loading is applied in time $t_0 = 28, 60, 90, 180, 365$ and 730 days

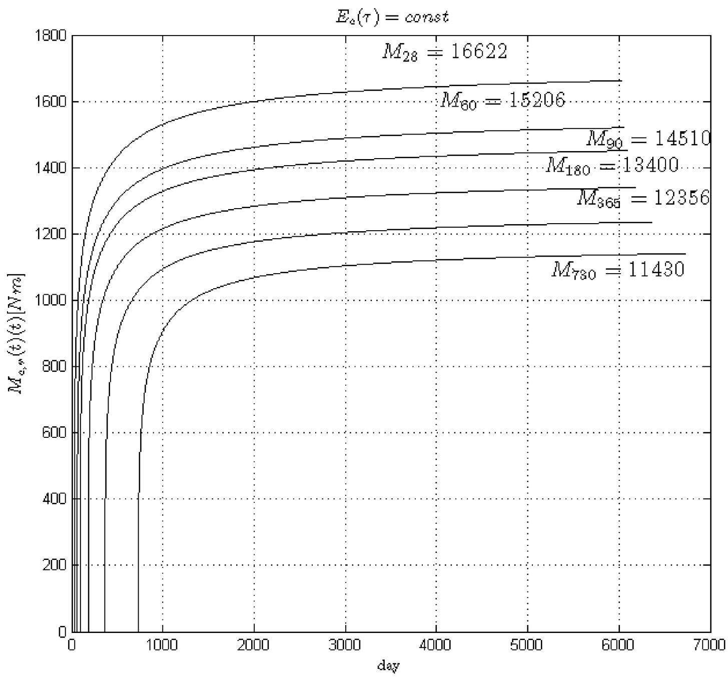

Fig.4: Values of bending moments $M_{\mathrm{c,r}}(t)$ in time t when loading is applied in time $t_0 = 28, 60, 90, 180, 365$ and 730 days

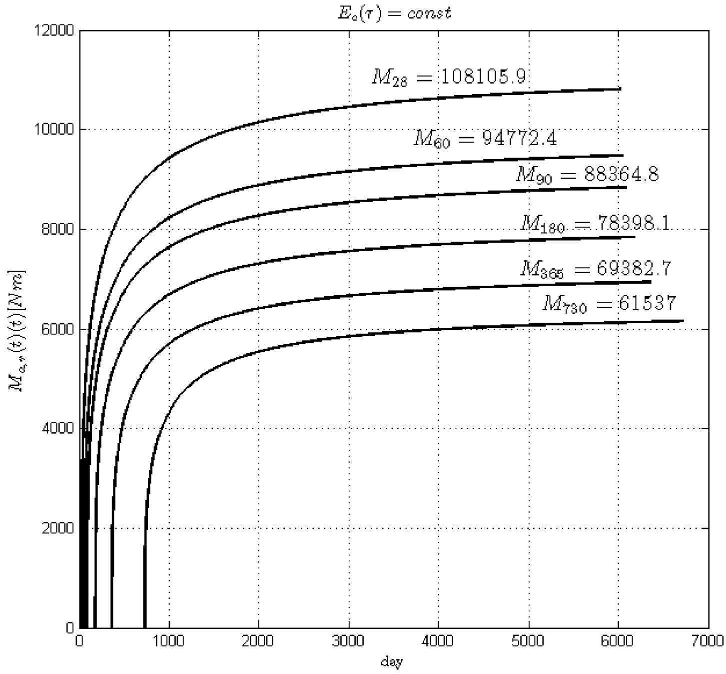

Fig.5: Values of bending moments $M_{\mathrm{a,r}}(t)$ in time t when loading is applied in time $t_0 = 28, 60, 90, 180, 365$ and 730 days

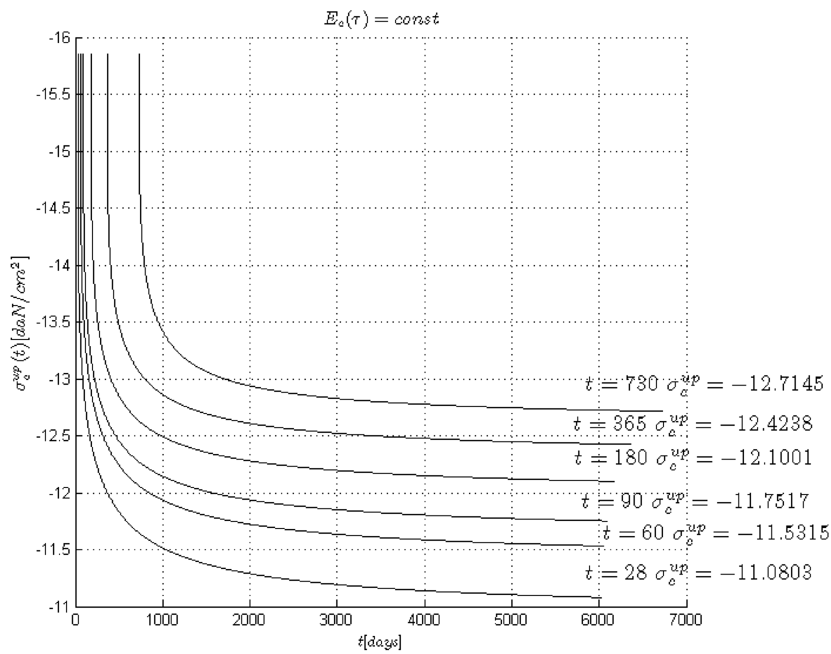

Fig.6: Values of normal stresses in upper fiber of concrete plate $\sigma_{\mathrm{c}}^{\mathrm{up}}(t)$ in time t_∞ when loading is applied in time $t_0 = 28, 60, 90, 180, 365$ and 730 days

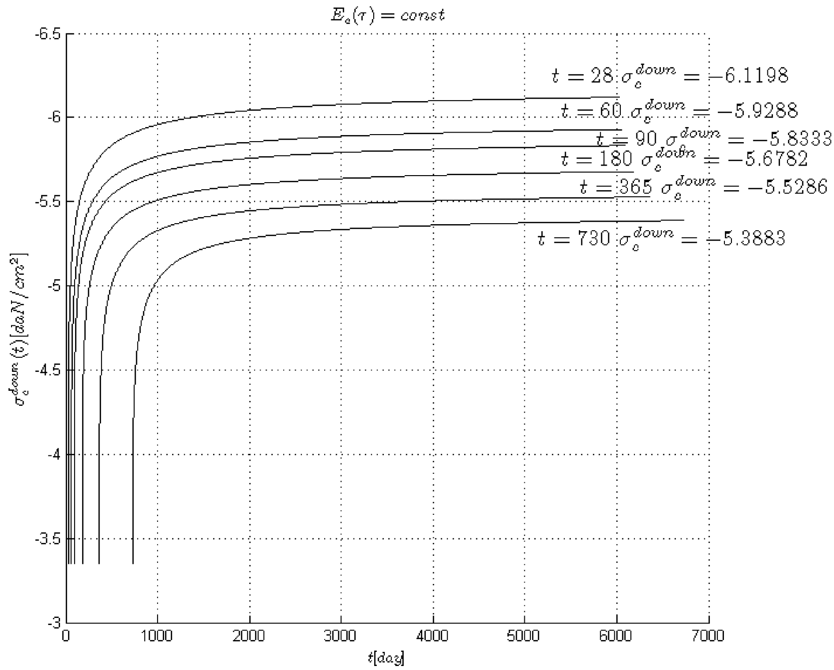

Fig.7: Values of normal stresses in down fiber of concrete plate $\sigma_c^{\mathrm{down}}(t)$ in time t_∞ when loading is applied in time $t_0 = 28, 60, 90, 180, 365$ and 730 days

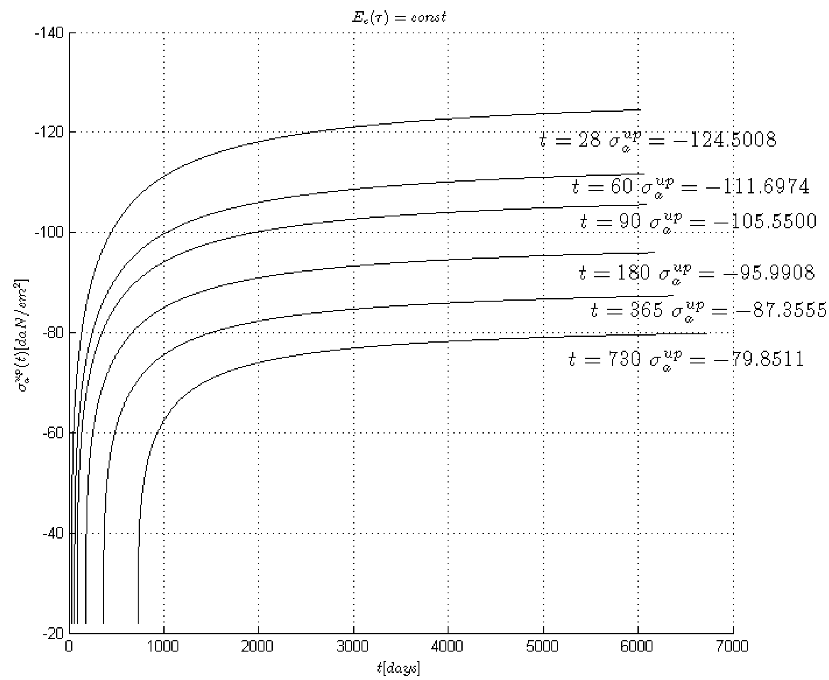

Fig.8: Values of normal stresses in upper fiber of steel girder $\sigma_a^{\mathrm{up}}(t)$ in time t_∞ when loading is applied in time $t_0 = 28, 60, 90, 180, 365$ and 730 days

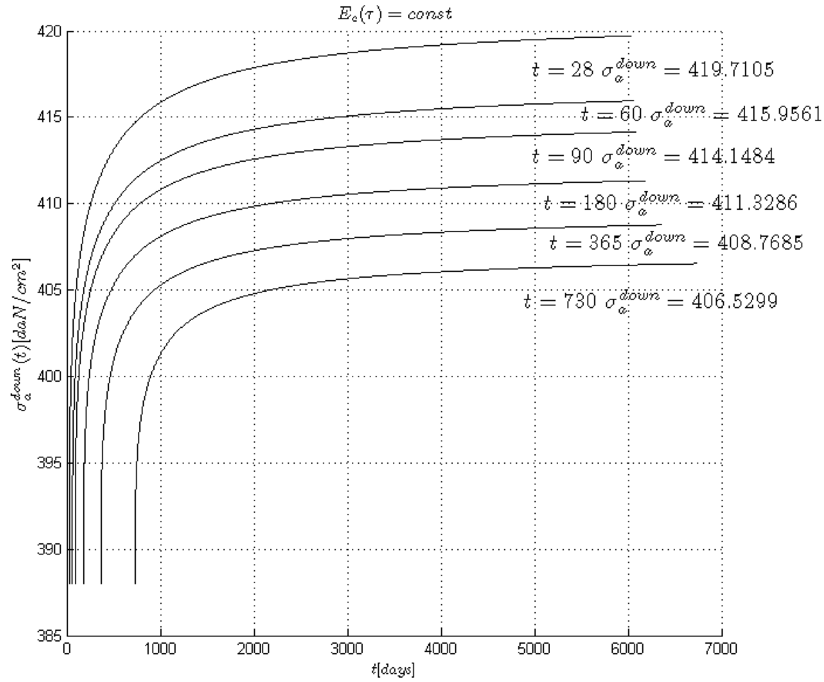

Fig.9: Values of normal stresses in down fiber of steel girder $\sigma_{\mathrm{a}}^{\mathrm{down}}(t)$ in time t_∞ when loading is applied in time $t_0 = 28, 60, 90, 180, 365$ and 730 days

	Humidity = 80 %	Humidity = 70 %	Humidity = 60 %	Humidity = 50 %
$M_{\mathrm{c,r}}$	15206	15154	15148	15147
$M_{\mathrm{a,r}}$	94772	95258	95302	95304
$N_{\mathrm{c,r}}$	76602	77121	77169	77172
$\sigma_{\mathrm{c}}^{\mathrm{up}}$	−1.15315	−1.15375	−1.15383	−1.1538321
$\sigma_{\mathrm{c}}^{\mathrm{down}}$	−0.59288	−0.59111	−0.59091	−0.590901
$\sigma_{\mathrm{a}}^{\mathrm{up}}$	−11.16974	−11.21910	−11.22359	−11.22387
$\sigma_{\mathrm{a}}^{\mathrm{down}}$	41.59561	41.60671	41.60768	41.60774

Tab.1: Values of normal forces, bending moments and normal stresses in time t_∞ when loading is applied in time $t_0 = 60$ for different humidity

The Figure 10 shows the values of deflection in midspan section of composite beams in time t_∞. As it can be observed the change of the initial time t_0 when the loading moment M_0 is applied, has very considerable influence in the time development of deflections.

In practice the deflection in time t_0 is determined by the following formulae:

$$\delta(t_\infty) = \frac{5}{48}\,\frac{M_0\,L^2}{E_{\mathrm{a}}\,I_{i,\mathrm{y}}} = \frac{5\cdot 1237\times 10^6\cdot 34000^2}{48\cdot 210000\cdot 35.289\times 10^9} = 20.10\,\mathrm{mm}$$

according to [51].

According to the described above numerical method we get the following formulae for calculating the deflection. If the moment M_0 and the inertia moment $I_{i,\mathrm{y}}$ are replaced with $M_{\mathrm{a}}(t_\infty)$ and I_{a} respectively we get:

$$\delta(t_\infty) = \frac{5}{48}\,\frac{M_{\mathrm{a}}(t_\infty)\,L^2}{E_{\mathrm{a}}\,I_{\mathrm{a}}} = \frac{5\cdot(330.13+94.7724)\times 10^6\cdot 34000^2}{48\cdot 210000\cdot 12.079\times 10^9} = 20.17\,\mathrm{mm}\ .$$

In every case considered above the elastic deflection $\delta(t)$ in time t_0 is the same we receive from the formulas:

$$\delta(t_0 = 0) = \frac{5}{48}\,\frac{M_0\,L^2}{E_{\mathrm{a}}\,I_{i,\mathrm{y}}(t_0)} = \frac{5 \cdot 1237{\times}10^6 \cdot 34000^2}{48 \cdot 210000 \cdot 45.260{\times}10^9} = 15.671929\,\mathrm{mm}\,,$$

according to [51],

$$\delta(t_0 = 0) = \frac{5}{48}\,\frac{M_{\mathrm{a},0}\,L^2}{E_{\mathrm{a}}\,I_{\mathrm{a}}} = \frac{5 \cdot (330.13){\times}10^6 \cdot 34000^2}{48 \cdot 210000 \cdot 12.079{\times}10^9} = 15.6718879\,\mathrm{mm}$$

according to our proposal.

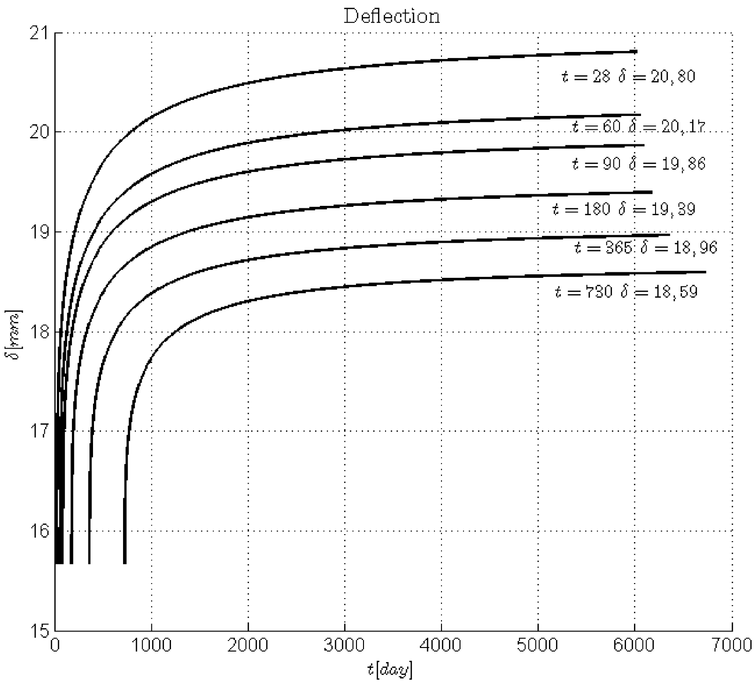

Fig.10: Values of deflection of steel girder (composite steel-concrete) $\delta(t)$ in time t_∞ when loading is applied in time $t_0 = 28, 60, 90, 180, 365$ and 730 days

10. Comparision with effective modulus methods (EMM)

This method uses the Dischinger's idea for applying in the calculation the ideal (fictitious) modulus of elasticity [52, 53]:

$$E_{\mathrm{ci}} = \frac{E_{\mathrm{cm}}}{1 + \psi_{\mathrm{L}}\,\phi_{\mathrm{t}}} = \frac{E_{\mathrm{cm}}}{1 + 1.1\,\phi_{\mathrm{t}}}\,,$$

where ψ_{t} is a final creep coefficient of concrete.

It is applied by Doleiš [51] to solve practical case shown in Figure 2. The results obtained by Doleiš are illustrated in Tables 2 and 3.

Type of beams		steel	Composite (in $t_0 = 0$)	Composite (in $t = \infty$)	
height	h_i	1500	1800	1800	mm
area	A_i	38325	172725	85689	mm^2
Static moment to down surface	$S_{\mathrm{y}0}$	23428688	245188688	101578534	mm^3
Gravity center	e_{top}	888.7	380.5	614.6	mm
Gravity center	e_{bottom}	611.3	1419.5	1185.4	mm
Moment of inertia	$I_{i,\mathrm{y}}$	12079015497	45260127815	35288674132	mm^4
Section modulus	$W_{i,\mathrm{y,ct}}$		−118959133	−57420939	mm^3
Section modulus	$W_{i,\mathrm{y,cb}}$		−562462122	−1121183853	mm^3
Section modulus	$W_{i,\mathrm{y,at}}$	−13592026	−562462122	−1121183853	mm^3
Section modulus	$W_{i,\mathrm{y,ab}}$	19759036	31883835	29768446	mm^3

Tab.2: Dimensions of steel and composite beams

Stress in time t_0	$t_0 = 60$ days	Stress in time t_∞	$t_\infty = 36500$ days
M_0	1237	M_0	1237
$n_0 = E_{\mathrm{a}}/E_{\mathrm{cm}}$	6.36	$n_{\mathrm{L}} = n_0(1+\psi_{\mathrm{L}}\,\phi_{\mathrm{t}})\,,\ \psi_{\mathrm{L}} = 1.1$	18.62
$\sigma_{\mathrm{c}}^{\mathrm{top}} = M/W_{i,\mathrm{y,ct}}/n_0$	−1.6	$\sigma_{\mathrm{c}}^{\mathrm{top}} = M/W_{i,\mathrm{y,ct}}/n_L$	−1.20
$\sigma_{\mathrm{c}}^{\mathrm{bottom}} = M/W_{i,\mathrm{y,cb}}/n_0$	−0.3	$\sigma_{\mathrm{c}}^{\mathrm{bottom}} = M/W_{i,\mathrm{y,ct}}/n_L$	−0.60
$\sigma_{\mathrm{a}}^{\mathrm{top}} = M/W_{i,\mathrm{y,at}}$	−2.2	$\sigma_{\mathrm{a}}^{\mathrm{top}} = M/W_{i,\mathrm{y,at}}$	−11.0
$\sigma_{\mathrm{a}}^{\mathrm{bottom}} = M/W_{i,\mathrm{y,ab}}$	38.8	$\sigma_{\mathrm{a}}^{\mathrm{bottom}} = M/W_{i,\mathrm{y,ab}}$	41.5

Tab.3

11. Conclusion

A numerical method for time-dependent analysis of composite steel-concrete sections according EUROCODE-4 is presented. Using MATLAB code a numerical algorithm was obtained and subsequently applied to a simple supported beam. These numerical procedures, suited to a PC, are employed to better understand the influence of the creep of the concrete in time-dependent behavior of composite section.

For the service load analysis, this method makes it possible to follow with great precisions the migration of the stresses from the concrete slab to the steel beam, which occurs gradually during the time as a result of creep of the concrete. At the same time, it is possible to calculate the deflections in the midspan section according to EUROCODE-4. Both these effects have a considerable importance in time-dependent response of composite beams.

The parametric analysis results are characterized by the following effects: the state of stress in the concrete slab depends on the age of the concrete at loading time t_0; the stress in the top flange of the steel section increases strongly with time, while the stress in the bottom flange undergoes small variations; the stress increases more for young concrete and less for old one.

Relative humidity causes not considerable variations in the final stress (see Table 1).

The most important conclusion of our investigation is that considering the creep effect and using the integral equations (22) and (23) an universal numerical method has been elaborated for statically determinate bridge composite plate girder. This method allows the use of a perfect linear theory of concrete creep i.e. the theory of the viscoelastic body of Boltzman-Volttera-Maslov-Arutyunyan-Trost-Bazant.

According to our results based on numerous practical examples we can state that maximum values of the stressed in concrete or steel in time t_∞ are reached after about eight years in comparison with the period of hundred years obtained by the EM Method [51, 52].

The results, obtained by this numerical method, are completely comparable with the results derived by Doleiš [51] based on EMM proposed by EUROCODE-4.

Finally, creep effect must be carefully evaluated in order to fully understand the behavior of the structure. The numerical methods proposed in this paper can be used to control the deflection in every test in composite beams sustained at service loads during the time t. It means that we can prove the regulars of the theory of the concrete creep.

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Received in editor's office: June 23, 2009
Approved for publishing: August 2, 2009