THE GENERATORS OF CHAOTIC NOISE AND ITS USAGE IN MODELING DYNAMIC PROPERTIES OF TECHNICAL SYSTEMS

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Chaos is present in many aspect of life. It is very difficult to detect and control chaotic behavior in nonlinear engineering dynamical systems. This contribution deals about some devices for generation of chaotic signals, for example about Chua's circuit, chaos module and analog chaotic oscillators.

Keywords : *chaos, dynamical system*

1. Introduction

Lately a lot of attention was brought to study of chaos in technical systems. The conditions of its occurrence [1], its visualization [2] and control [3], [4] were studied. Chaos can be used in many case both in machinery and communication technology [5] and other scientific fields. For dynamical system to be classified as chaotic, it must have following properties :

– it must be sensitive to inicial conditions,
– it must be topologically mixed, and
– its periodic orbits must be dense.

Sensitivity to initial conditions means that each point a system is arbitrarily closely approximated by other points with significantly different future trajectories. Thus, an arbitrarily small perturbation of the current trajectory may lead to significantly different future behaviour. Even for bounded systems, sensitivity to initial conditions is not identical with chaos [6].

Topologically mixed means that the system will evolve over time so that any given region or open set of this phase space will eventually overlap with any other given region. Here, 'mixed' is really meant to correspond to the standard intuition : the mixing of colored dyes or fluids is an example of a chaotic system.

Some dynamical systems are chaotic everywhere (see e.g. Anosov diffeomorphisms) but in many cases chaotic behaviour is found only in a subset of phase space. The cases of most interest arise when the chaotic behaviour takes place on an attractor, since then a large set of initial conditions will lead to orbits convergent to this chaotic region.

An easy way to visualize a chaotic attractor is to start with a point in the basin of attraction of the attractor, and then simply plot its subsequent orbit. Because of the topological transitivity condition, this is likely to produce a picture of the entire final attractor.

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2. The chaos noise generators

Therefore the question of design of suitable generators of chaotic processes becomes actual. The contribution deals with utilization of Chuať's circuit, chaos module and analog chaotic generator. This devises are made from standard components (resistors, capacitors, inductors) and must satisfy three criteria before it can display chaotic behaviour. It must contain:

– one or more nonlinear elements
– one or more locally active resistors and (1)
– three or more energy-storage elements.

3. Chua's circuit

Chua's circuit is a simple electronic circuit that exhibits classic chaos theory behaviour. It was introduced in 1983 by L. O. Chua. The ease of construction of the circuit has made it a ubiquitous real-word example of a chaotic systems, leading some to declare it 'a paradigm for chaos'. A version of Cha's circuit withoud Chua's diode is shown on Fig. 1.

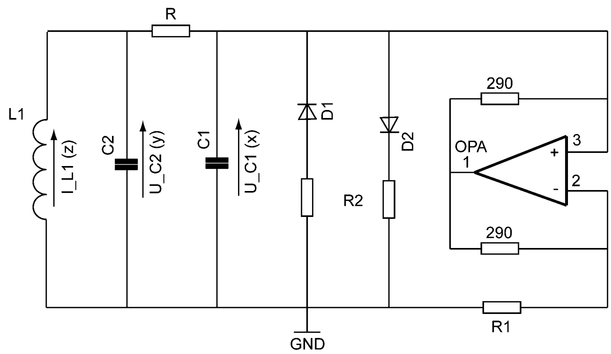


Fig.1: A version of Chua's circuit

Chua's circuit is the simplest electronic circuit meeting criteria (1). As shown in the Fig. 1, the energy storage elements are two capacitors (labeled C1 and C) an an inductance (labeled L1). There is an active resistor (labeled R). There is a nonlinear resistor made of two linear resistors and two diodes. At the far right is a negative impedance converter made from the linear resistors and an operational amplifier.

By means of the application of the laws of electromagnetism, the dynamics of Chua's circuit can be accurately modeled by means of a system of three nonlinear ordinary differential equations in the variables $x(t)$, $y(t)$ and $z(t)$, which give the voltage in the capacitors C1 and C2, and the intensity of the electrical current in the inductance L1, respectively. These equations are:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= \alpha\,[y - x - f(x)]\ , \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= x - y + z\ , \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= -\beta\, y\ . \end{aligned} \tag{2}$$

The function $f(x)$ describes the electrical response of the nonlinear resistor, and its shape depends on the particular configuration of this components. The parameters α and β are determined by the particular values of the circuit components.

A chaotic attractor, known as 'The Double Scroll', because of its shape in the (x, y, z) space, was first observed in a circuit containing a nonlinear element such that $f(x)$ was a 3-segment piecewise linear function [7]. The easy experimental implementation of the circuit, combined with existence of a simple and accurate theoretical model, makes Chua's circuit a useful system to study many fundamental and applied issues of chaos theory, for example for fuzzy modeling of chaotic systems [8].

4. Chaos module

4.1. Chaos module characteristics

Chaos module is electronic device developed by Yamakawa's Lab & FLSI for modeling and analysis of chaotic states of discrete nonlinear dynamic systems using storage oscilloscope and computers with PSpice program with respect to the changes of selected parameters of dynamic systems [2]. The device uses chaos chip connection enabling activation of chaos module electronic circuit. The device was designed in a way that minimum number of external equipment is required. For example in the simplest wiring it only needs clock signal (rectangular voltage generator), two channels storage oscilloscope and system to be measured. For higher precision measurements one can add external resistors, precise power sources, voltmeters and potentiometers. Internal structure of chaos chip circuit is shown on Figure 2 [10], [11].

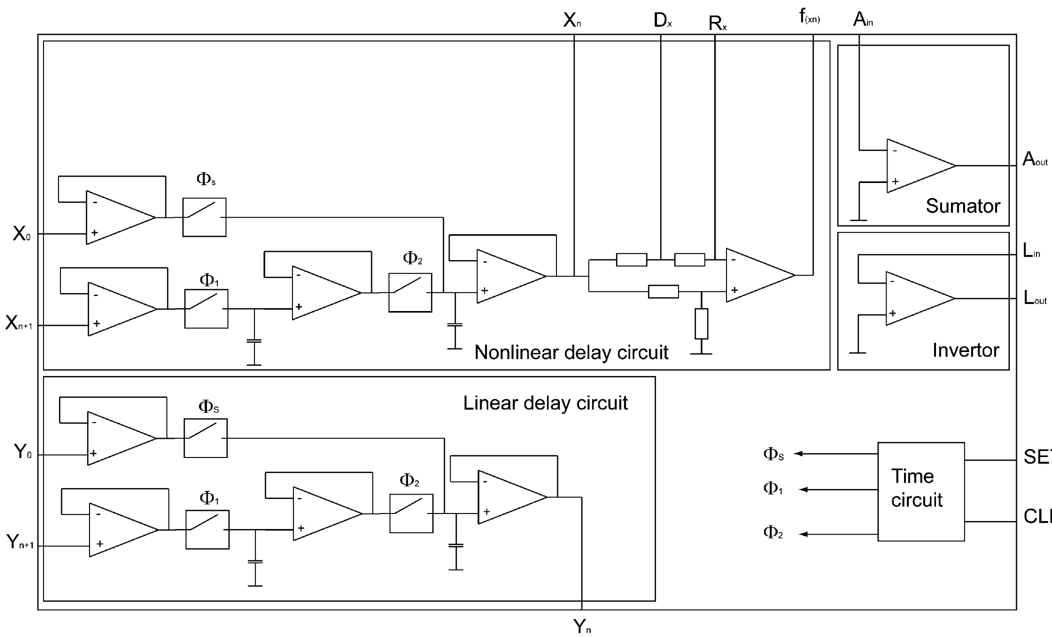


Fig.2: Internal structure of chaos chip

On Figure 2 one can see the basic structural elements of the chaos chip circuit : delay circuit, inverter and timing circuit. We will further focus on possibility to visualize chaotic states using bifurcation diagrams and using chaotic attractors via storage oscilloscope.

5. Realization of chaos chip wiring into measuring system

Two variants were implemented using the chaos chip system:
- Modeling of bifurcation diagrams (on 1-D system)
- Modeling of chaotic attractors (on 2-D system)

The block diagrams and some of the results of the experiments are shown in following paragraphs.

5.1. Implementation of 1-D nonlinear dynamic system

Fig. 3 shows the block diagram of 1-D system with chaos chip. The equation describing such circuit is of form:

$$x_{n+1} = \alpha\,\beta\,f(x_n) \tag{3}$$

for $n = 0, 1, 2, \ldots$, where α and β are the gains.

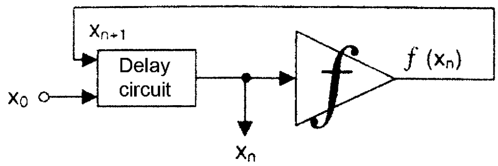


Fig.3: Block diagram of 1-D system

The goal of this arrangement is to model bifurcation diagrams [11]. As generally known, bifurcation diagrams show, how change of single parameter of the circuit can change behavior of the whole system. The values of parameter that is changed are on horizontal axis from left to right, the state of observed system x_n is on vertical axis.

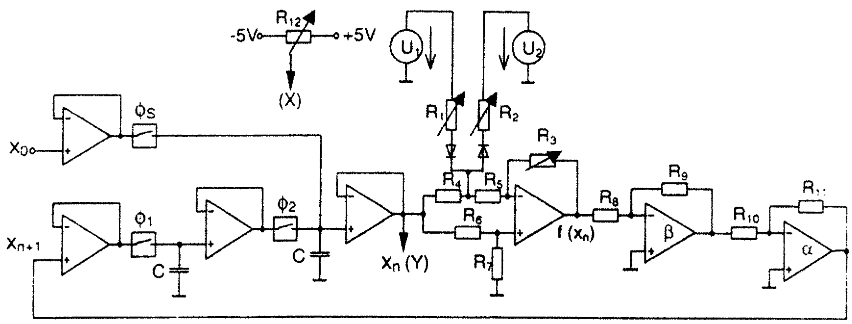


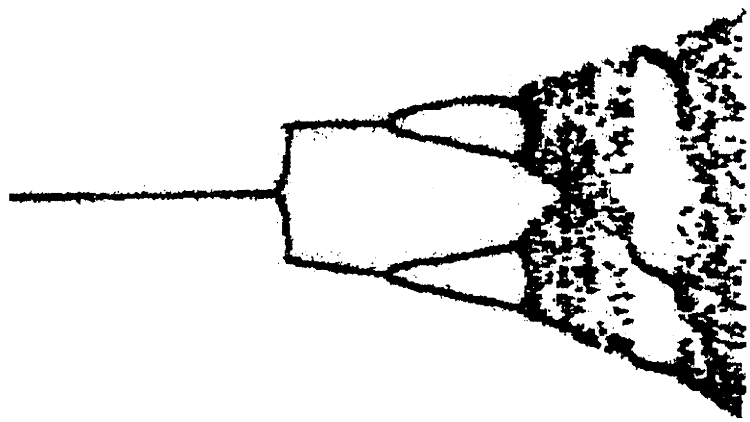
Fig.4: Circuit diagram of 1-D system

Circuit diagram of 1-D system is shown on Fig. 4. Prior to computational iteration process the SET terminal must be set to positive value and all parameters of nonlinear system must be set, that is R2, R3, U1, U2, gains α, β and initial value of iteration x_0. The parameter, whose influence on complete system behavior we observe (e.g. R1) is connected with resistor R12. Its output is the voltage that follows parameter value change. On oscilloscope we connect this voltage to horizontal axis X. Output observed variable (system state) is connected to vertical axis Y.

After setting all the values we bring negative voltage to SET terminal and start computational iteration process. After setting required ranges of X and Y inputs we start to record observed variables in connected storage oscilloscope. At the same time we very slowly change bifurcation parameter (R1 in our case) in given range. This way we obtain on screen bifurcation diagram we are searching for.

The examples of bifurcation diagrams calculated using chaos chip for various bifurcation parameters are shown on Fig. 5a and 5b.

a) U1=0.48V, U2=0.7V, R1=0–100kΩ, R2=8.2kΩ, R3=72kΩ, α=−1, β=−0.6



b) U1=0.24V, U2=0.19V, R1=6.8kΩ, R2=15kΩ, R3=50–200kΩ, α=−1, β=−1

Fig.5: Bifurcation diagram of the system with (a) R1, (b) R2 parameter [10]

5.2. Implementation of 2-D nonlinear dynamic system

Figure 6 shows the block diagram of 2-D system with chaos chip. The equations describing such circuit are of form:

$$\begin{aligned} x_{n+1} &= f(x_n) - \alpha\, y_n \ , \\ y_{n+1} &= x_n \end{aligned} \tag{4}$$

for $n = 0, 1, 2, \ldots$, where α is the gain.

The goal of this arrangement is to model chaotic attractors. Let's note that we consider attractor as sets of responses gained by the state vector of dynamic system during sufficiently long time period from initialization in t_0 time. Attractors in its simplest form are so called

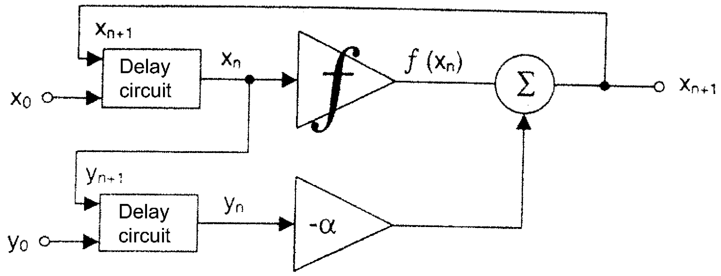


Fig. 6: Block diagram of 2-D system

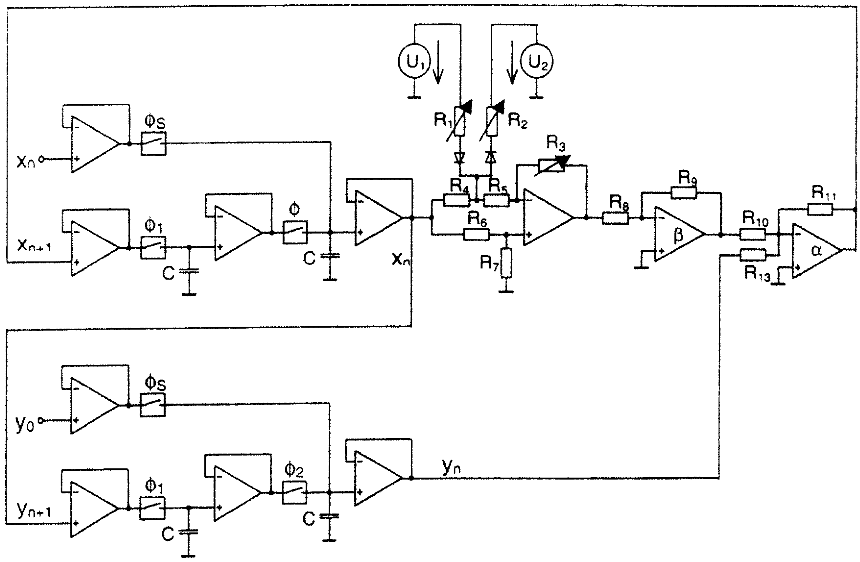


Fig.7: Circuit diagram of 2-D system

fixed points or limit cycles towards which the trajectories of the system are 'attracted'. Circuit diagram of 2-D system is shown on Fig. 7.

Setting the circuit parameters and initial conditions prior to iterative computation is done in the same way as in 1-D system. Moreover, apart from initial vector x_0 there is initial value of y_0 vector and gains α and β of particular signals can be set independently for vectors x and y. Output x_n is connected to X axis while y_n output is connected to Y axis. After bringing negative voltage to SET terminal the screen of oscilloscope shows the image which however does not have to be the attractor we are searching for. It strongly depends on setting all parameters of the circuit and setting the initial values of iteration process. Most commonly, it is required after starting iteration process, continuously to change circuit parameters to put system into chaotic state and therefore to obtain particular attractor. The parameters close to the unstable state must be changed gently, as with even very small change of one or more parameters in 'undesired direction' the system immediately gets into stable state. In such a case the iteration process must be stopped, its parameters set again together with initial values, computation must be restarted and 'tuned'.

Figures below show selected results of numerical experiments. All numerical values within these figures are final, meaning written in the moment of attractor appearance. Only the initial values of iteration correspond to the data regarding the iteration process, as the circuit parameters were 'tuned' during the process.

The case of particular interest is the portrait of the system with exhibited seven pseudo-steady limit cycles running around partial steady state (practically unstable). Even very small change of U2 voltage (from $-1.12\,\mathrm{V}$ to $-1.15\,\mathrm{V}$) led to creation of stable attractor, that was captured repeatedly – see Fig. 8a,b.

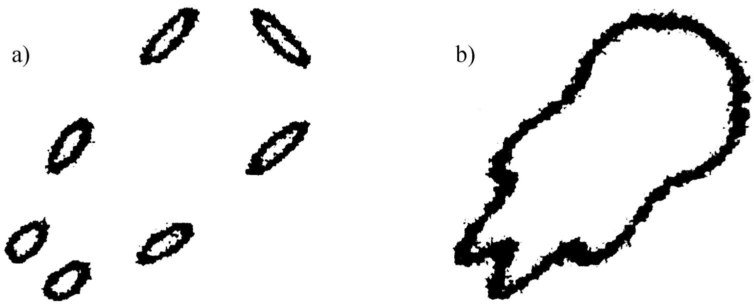


Fig.8: (a) Dynamic system state prior to attractor formation [10]
(b) Stabilized attractor of dynamic system

6. Universal and fully analog oscillator [12]

6.1. Utilization of chaos in communication systems

Lately the systems that utilize chaos gained importance together with applications that use nonlinear dynamics. Chaotic signals are used for information transfer for two main reasons. First one is hiding the information in chaos. Such application replaces conventional cryptography. The second one is the fact that chaotic signals are of wide spectrum, and that is advantageous for multiuser applications. Why use of chaotic signals in communications was examined? The original idea was to module information to chaotic signal so it is available for authorized user only. The receiver removes chaotic component and restores original information. Whoever captures the signal can only see chaotic, noisy signal, from which extracting the original information is extremely difficult. So wide possibilities emerges for various cipher methods. The second idea originates in ultrawide band communications (UWB), where pseudorandom signals are used for extending frequency band. The reason behind using such systems is in reducing interference of signals from various sources and reduction of delayed signals influence in wireless and mobile communication.

6.1.1. Chaotic system modulation

The goal of modulation in classical communication system is to join low-frequency information signal with high-frequency carrier signal, that transfers the information to the band suitable for transfer. Modulation using chaos can use two different approaches. In the first one the chaotic signal itself is the modulator, being suitably influenced by information signal. This method is simple, power saving, but on the other hand it is difficult to implement high- frequency chaotic system. Therefore this method is suitable for the research, not for practice. The second approaches mixes the chaotic and information signal on low frequency and resulting signal is transformed to high frequency using traditional modulator.

Simple implementations of chaotic modulator and demodulator are the advantages, higher complexity and energy requirements are the drawbacks.

6.1.2. Chaotic masking

Chaotic masking is mixing the information signal with chaotic one and its restoration by synchronization. The same chaotic systems are present in both transmitter and receiver and synchronization is secured with the receiver being controlled by received signal. Synchronization produces $A\,y(t)$ signal that is similar to chaotic signal $y(t)$ in the transmitter. Therefore the information signal $s(t)$ can be obtained by simply subtracting the synchronization output signal from received signal. However, such system works only when information signal is negligibly small towards chaotic signal, so it changes only a little. It is clear that the noise in channel makes restoring the information impossible and therefore this method is not used very often.

6.1.3. Chaotic shift keying (CSK)

In this method the information signal modulates certain parameter of chaotic system. In the simples case the information signal is binary and controls switching between to parameters vectors p and p'. Each bit is represented by chaotic curve of certain length. In the first method proposal the bit is detected as we try to synchronize both system with incoming signal, one with p parameters vector and the other with p' parameters vector. Once one of the systems is synchronized, received bit is indicated. Many chaotic shift keying methods were proposed. The simplest method uses the change of output function base on transferred bit, it so called chaos antipodal coding $b(t)$.

6.1.4. Differential chaos shift keying (DCSK)

This method was developed for removing the synchronization by transmitting non-modulated signal together with modulated one. More precisely, during sending of single bit the signal of given length is generated by chaotic system and that signal is send to transfer channel. If value '1' is transferred, the copy of the signal is send following the original signal. If value '0' is transferred, the copy with -1 added is send. Both parts are compared in the receiver to obtain the information. In this method the chaos properties are not used for demodulation, therefore any signal can be used. The drawback of the method is in necessity of using delay elements and triggers that causes implementation problems. Apart from that the half of the time is used for sending the reference signal. On the other hand it is the most efficient communication method that uses chaos, however, still worse than BPSK. Modifications of the method were proposed, in particular the CDSK (correlation delay shift keying), SCSK (symmetric chaos shift keying) and many others.

6.2. The concept of universal chaotic oscillator

Based on given differential equations

$$\begin{aligned}
\dot{x} &= a_{11}\,x + a_{12}\,y + a_{13}\,z + b_1\,h\left(\mathbf{w}^{\mathrm{T}}\,\mathbf{x}\right)\,, \\
\dot{y} &= a_{21}\,x + a_{22}\,y + a_{23}\,z + b_2\,h\left(\mathbf{w}^{\mathrm{T}}\,\mathbf{x}\right)\,, \\
\dot{z} &= a_{31}\,x + a_{32}\,y + a_{33}\,z + b_3\,h\left(\mathbf{w}^{\mathrm{T}}\,\mathbf{x}\right)\,,
\end{aligned} \tag{5}$$

where a_{ij}, b_i are mutually independent parameters and

$$\mathbf{w}^{\mathrm{T}}\,\mathbf{x} = w_1\,x + w_2\,y + w_3\,z\ ,$$

where w_i are independent parameters and using functional blocks

– inverting voltage amplifier – by cascade connecting the amplifiers we can set the values of universal oscillator coefficients including the sign,
– inverting sum integrating amplifier – the connection of particular blocks created by cascade circuit of inverting amplifiers,
– symmetrical diode limiter – PWL block implementation,
– differential amplifier

was built the universal chaotic oscillator, its circuit is shown on Fig. 9. The verification of the functionality was performed using simulations in PSpice program, see Fig. 10.

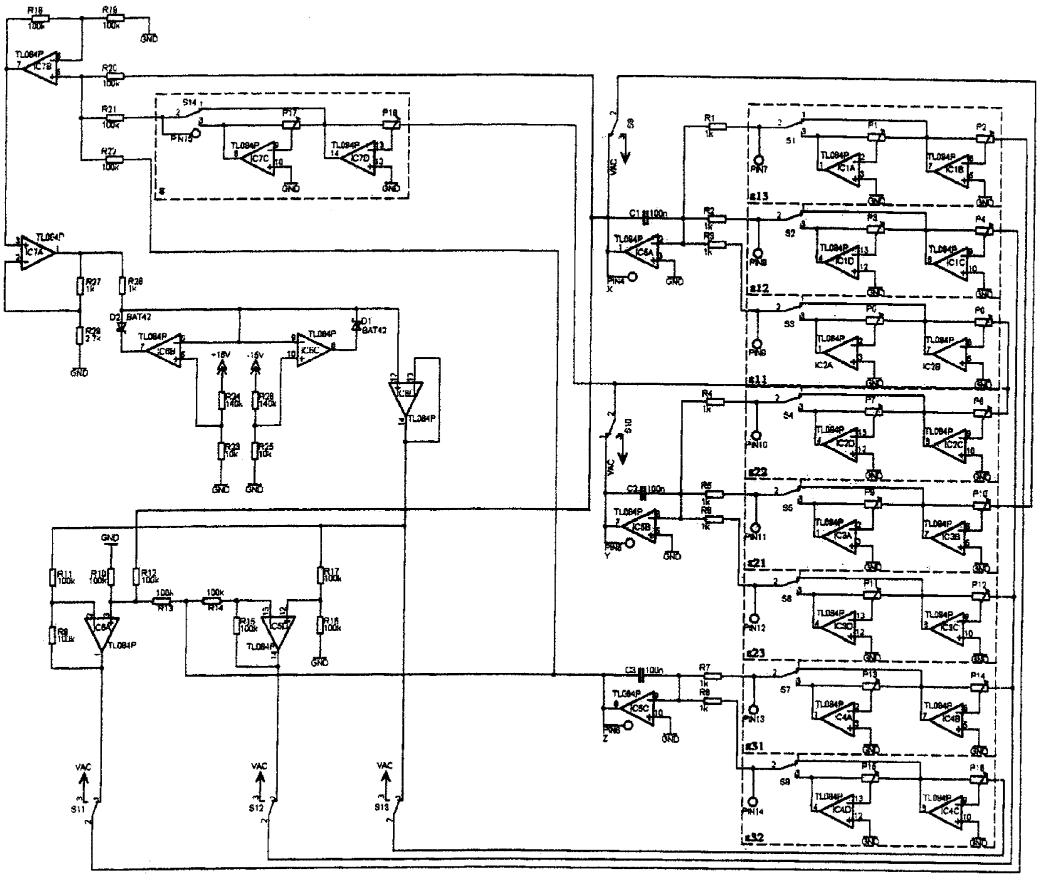
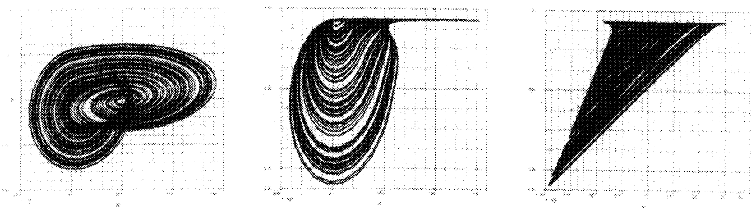


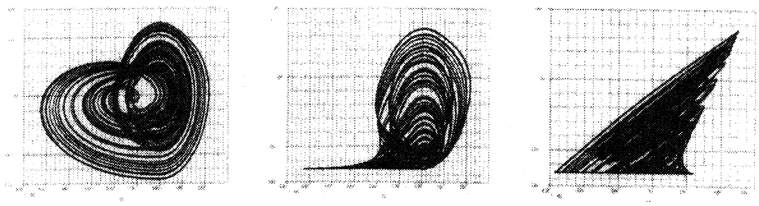
Fig.9: Universal analog oscillator circuit

7. Conclusion

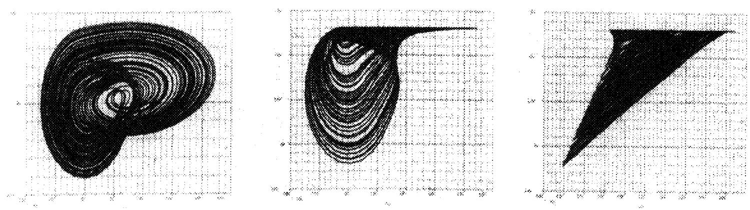
In mechanical engineering the application of chaos usually started as experimental demos for education purposes, for instance as the control of pendulum, plates or other technical devices. Recently, it has been shown that the study of chaos noise can be useful for to test of a dynamical property of mechanical systems. Chaotic behaviour we can observed in natural



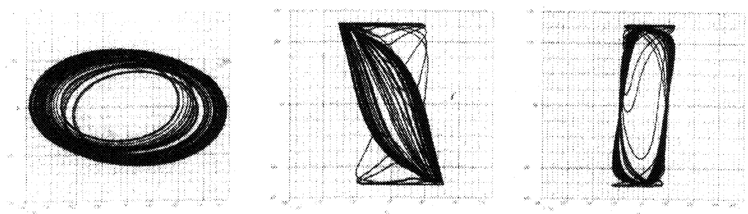
Chaotic atractor 1 – CDCD



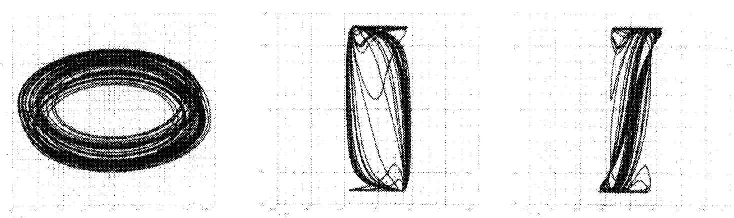
Chaotic atractor 2 – CDCD



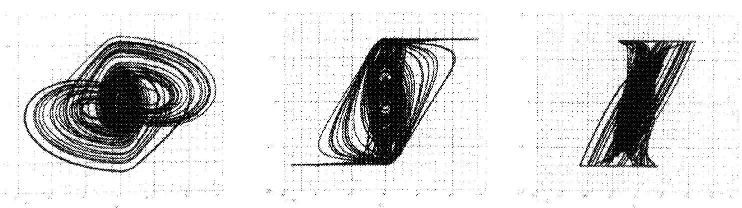
Chaotic atractor 2 – ECEC



Chaotic atractor 4 – CDCD



Chaotic atractor 4 – ECEC



Chaotic atractor 7 – CDCD

Fig.10: Simulation using PSpice program Various types of chaotic atractors – the symbols denote the type of structure and particular selection of universal oscillator parameters [12]

environment, such as the weather or water systems. But in the laboratory we need some generators of chaotic signals. This contribution deals about some devices for generation of chaotic signals, where are applicable for simulation behaviour of mechanical systems, for example Chua's circuit, chaos module and analog chaotic oscilators.

Acknowledgements

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