TESTING OF DYNAMICS OF BLADE WHEEL WITH DOUBLE PERIODICITY

Luděk Pešek, František Vaněk, Ladislav Půst, Vítězslav Bula, Jan Cibulka*

The dynamics of the mistuned model of the test bladed disc with double periodicity was experimentally investigated. The mistuning arised due to the implementation of two bunches of blades with damping heads fixed on opposite ends of the diameter. The dry friction effect in the heads was treated. The scale for measurement of the electromagnetic force acting on the disc was designed. It enables to analyse the real electromagnetic excitation of the disc under rotation and evaluation of the FRF characteristics from measured blade responses under rotation. The identified eigenfrequencies and damping constants served as indicators for assesment of stiffening and damping effects of the friction couplings in the heads.

Keywords : *dynamics, bladed wheel, rotation, mistuning*

1. Introduction

The mistuning efect of imperfect disc-blade systems on its dynamic behaviour is still under investigation of many authors, e.g. [1], [2], [3], [4]. The main goal of this research is mostly connected with a goal of higher resonance damping. The analytical solution of imperfect bladed discs shows the splitting of double-multiple eigenvalues of such structures. The multiple eigenvalues are typical for the ideal rotary symmetry case [5]. By the eigenvalue split the resonace spectra become double dense than the spectra of ideal discs. Therefore the problem of blade disc flexural vibration was solved both theoretically and experimentally and basic concepts for diagnostic and identification method that help to ascertain structural behaviour of turbine discs and their test models were elaborated.

This paper deals mainly with experimental research of dynamics of the test model with double periodicity that arised due to insertion of two bunches of five different blades turned mutually by 180° into the disc formerly eqquiped with sixty prismatic beam blades. It causes cyclic imperfection of mass, stiffeness and damping in their circumferential distribution along the disc [4]. For friction damping tests the mentioned blades were ended with heads with interblade slots, where the friction elements were inserted. Under rotation these heads lock together by the elements that move radially in the slots due to centrifugal force action. The normal and hence also friction forces in contacts increase with revolution frequency that increase the stiffness of the blades like a continuous shroud but decrease the relative displacements and damping effect.

Experimental research was based on measurement and analysis of forced vibration (radial, axial and circumferential) of blade disc using strain gauges and contactless displacement sensors under excitation both by electromagnet and permanent magnets [6]. Since a strain

* Ing. L. Pešek, CSc., Ing. F. Vaněk, CSc., Ing. L. Půst, DrSc., Ing. V. Bula, J. Cibulka, Institute of Thermomechanics AS CR, v.v.i., Dolejškova 5, 182 00 Prague 8

gauge dynamic scale was rigged for measurement of electromagnet excitation force it enabled to evalute the frequency response function (FRF) of the model disc under revolution.

2. Stationar dynamic behaviour of disc with double periodicity under harmonic excitation

Imperfection of the disc due to the implementation of bunches of new blades into the disc causes free vibration of two orthogonal modes, so-called sine and cosine, having the same number of node radials with different frequencies Ω_{an}, Ω_{bn}. The modes are orientated in the disc: sine mode has one node radial placed into the centre of imperfection and on the contrary the cosine mode has one anti-node radial in this centre. The splitting of double eigenvalues evokes qualitatively different behaviour at the mistuned discs compared to the rotary symmetric discs. The resonance disc vibration with the travelling wave due to combination of both orthogonal modes could not for example arise at a harmonic excitation since due to the eigenfrequency splitting the only one of these modes is excited in resonance.

For description of the disc dynamic behaviour under rotation, two coordination systems are introduced:

1) the first r, θ_{a} connected with a stationary space,
2) the second r, θ connected with the rotating space rotating with angular frequency ν in opposite direction to the direction of θ_{a}.

Then the transformation relation between the spaces 1) and 2) is $\theta_{\mathrm{a}} = \theta - \nu\, t$. Let the disc lie in the rotationg space 2) and let the force $F_1(t) = F_0 \cos\omega t$ act at the point P ($\theta = \theta_{\mathrm{a}} = 0$ for $t = 0$) connected with the space 1) then the force in space 2) is defined as

$$F(\theta, t) = F_1(t)\, \delta[\theta - \nu\, t] \ , \tag{1}$$

where δ is a delta function defined by expressions

$$\delta[\theta - \nu\, t] = 0 \quad \text{for} \quad \theta \neq \nu\, t \ , \qquad \delta[\theta - \nu\, t] = 1 \quad \text{for} \quad \theta = \nu\, t \ . \tag{2}$$

Then the generalized force for selected sine vibration mode shape with n nodal diameters caused by moving harmonic excitation force $F_0 \cos\omega t$ along disc circumference can be evaluated by the integral over whole circumference [7]

$$Q(t) = \int_0^{2\pi} F_0 \cos(\omega\, t)\, \delta[\theta - \nu\, t] \sin(n\, \theta)\, \mathrm{d}\theta = F_0 \cos(\omega t) \sin(n\, \nu\, t) \ . \tag{3}$$

Therefore the moving harmonic force evokes a division of the excitation frequency ω into two frequency componenets $(\omega \pm n\, \nu)$ due to amplitude modulation of excitation force by n-multiple of revolution frequency as described by (3). By consequence of that we get two terms in the stationary solutions u_{an}, u_{bn} of imperfect disc to the harmonic excitation in generalized displacements. The solution in non-dimensional form is (see [4]):

$$u_{an} = \frac{\sin n\, \theta \sin[(\omega + n\, \nu)\, t - \psi_{an1}]}{2\sqrt{(\Omega_{an}^2 - (\omega + n\, \nu)^2)^2 + \beta_{an}^2\, (\omega + n\, \nu)^2}} - \\ - \frac{\sin n\, \theta \sin[(\omega - n\, \nu)\, t - \psi_{an2}]}{2\sqrt{(\Omega_{an}^2 - (\omega - n\, \nu)^2)^2 + \beta_{an}^2\, (\omega - n\, \nu)^2}} \ , \tag{4}$$

$$u_{bn} = \frac{\cos n\,\theta\,\sin[(\omega + n\,\nu)\,t - \psi_{bn1}]}{2\,\sqrt{(\Omega_{bn}^2 - (\omega + n\,\nu)^2)^2 + \beta_{bn}^2\,(\omega + n\,\nu)^2}} + \\ + \frac{\cos n\,\theta\,\sin[(\omega - n\,\nu)\,t - \psi_{bn2}]}{2\,\sqrt{(\Omega_{bn}^2 - (\omega - n\,\nu)^2)^2 + \beta_{bn}^2\,(\omega - n\,\nu)^2}}\,, \tag{4}$$

where $\psi_{an1,2}$, $\psi_{bn1,2}$ are phase angle differences of the modes with respect to the excitation force and β_{an}, β_{bn} are doubles of damping constants b_{an}, b_{bn} – real parts of eigenvalues of the corresponding vibration modes. The displacements u_{an}, u_{bn} in (4) describe the vibration components of sine and cosine modes, respectively, with n nodal diameters.

So the resonances of the sine or the cosine modes as shown by (4) do not arise for excitation frequencies equal to Ω_{an}, Ω_{bn}, corresponding to the eigenmodes but equal to $\Omega_{an} \pm n\,\nu$. Therefore in literature the term 'apparent' resonant frequency is used since the frequency $\omega = \Omega_{an} \pm n\,\nu$ and not $\omega = \Omega_{an}$ corresponds to the resonance excitation. Since in the experiment under certain conditions both cases, actual and apparent, resonance excitations appear and for their searching the 'sweep' frequency band excitation was used we introduce for clarity also two next terms, i.e. 'main' and 'side' bands of excitation. The main band of resonance excitation means that excitation frequencies lie in the surrounding of actual resonant frequency and the side band that the excitation frequencies lie in the surrounding of apparent resonant frequency.

3. Numerical simulation of dynamic behavior for identification

For proposal of eigenvalue identification method of the disc under rotation and electromagnetic sweep excitation, the numerical simulations based on analytical solution (4) were performed. It comes from the modal model of the disc in the non-dimensional form that is defined by its eigenfrequencies and damping constants. Experimental eigenfrequencies of cosine and sine modes with one nodal diameter ($n = 1$) in a steady state are 52 Hz and 54.8 Hz, with two nodal radials ($n = 2$) 71.6 Hz and 77 Hz and with three nodal radials ($n = 3$) 102 Hz and 113 Hz, respectively.

For the numerical simulation a case of sine and cosine modes with $n = 2$ and eigenfrequencies 72.7 Hz a 78 Hz (FE model) at the same damping constants $b_{an} = b_{bn} = 1.5$ (i.e. $\beta_{an} = \beta_{bn} = 3$) was chosen. Sweep excitation signal was generated in the band (50–60) Hz with a sweep rate 0.02 Hz/s. The FRF was calculated from the time characteristics of the excitation signal and its response $u_{an} + u_{bn}$ according to (4) as function $H_1 = G_{\mathrm{FX}}/G_{\mathrm{FF}}$ (G_{FX} crosspower spectrum of excitation and response signals, G_{FF} autospectrum of excitation signal) minimizing uncorelated noise on the input. The response was evaluated in the circumference location ($\theta = 30°$) in the band (70–80) Hz (Fig. 1). The excitation band (50–60) Hz was for this numerical case the side one. The spectra line number 16384 and averaging with overlaping 70 % were used for the FRF's calculations. Due to sampling frequency 2 kHz the spectra resolution was 0.03 Hz. The amplitudes of the spectra represent mean power values of frequency components in a sweep time interval that was 500 s. Identification of eigenvalues of the numerical simulation by the fitting 'Least Square Frequency Domain' (LSFD) method gave the eigenfrequencies 72.9 and 78.2 Hz and damping ratios 0.37 % and 0.39 %, respectively.

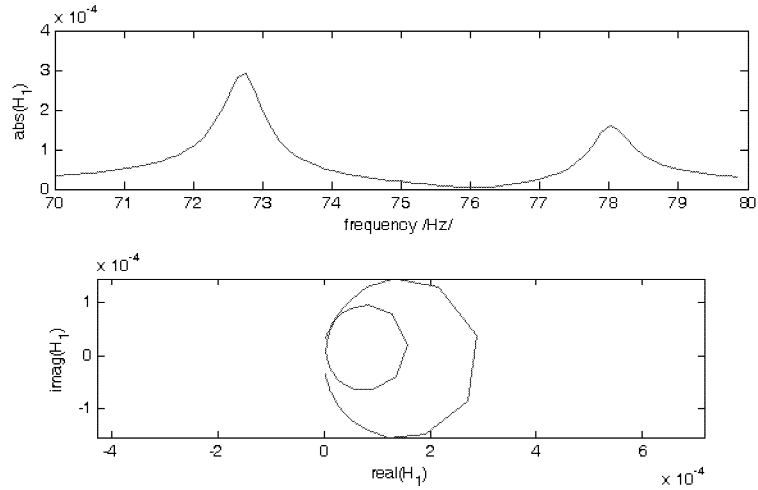

Fig.1: Numerically simulated FRF at the sweep excitation
a) amplitude spectrum, b) Nyquist diagram

4. Force description of electromagnetic excitation

The description of force $F(t)$ of electromagnet acting on the disc in steady non-rotating state can be expressed as

$$F(t) = \frac{1}{2}\, i^2(t)\, \frac{\Delta L}{\Delta x} \ , \tag{5}$$

where $x\,[\mathrm{m}]$ is a distance of electromagnet from the disc, $L\,[\mathrm{H}]$ coil inductivity and $i \approx I\, \sin(\omega\, t/2)$ is a supply current. Since the dependence of force on current is governed by second power the current frequency of electromagnet is half to the force excitation frequency and the next harmonic components of excitation frequency arise in the force spectrum. Dependence of force on supply current (Fig. 2) was ascertained by calibration measurement of supply current i and coil voltage U_{B} at different distances between the coil and disc. The coil inductivity was evaluated from the coil impedance for different excitation frequencies (50 Hz, 100 Hz).

5. Rotating disc excited by electromagnet

The cut view of the tested bladed disc with electromagnet is shown in Fig. 3. Diameter of the disc was ⌀ 505 mm. Double periodicity of the blading was caused by insertion of two bunches of five different blades (220 mm length) turned mutually by 180° into the disc formerly eqquiped with sixty prismatic beam blades (190 mm length). The inserted blades were ended with heads with interblade slots. For friction damping tests the friction elements (FES) were placed into the slots. Since the attention was aimed at the behavior of the inserted blades, strain gauge measurement of axial forced vibration of the bunch blades, i.e. the blades L50 (midle of the bunch), L51 a L21 (first side blades from the midle) was performed. The strain-gauges were sticked at the root of the blades. Electromagnet (EM) acted on the disc perimeter and over the disc all blades were excited. For more accurate description of this force, the dynamic scale that holds the electromagnet and transmits its reaction force was rigged.

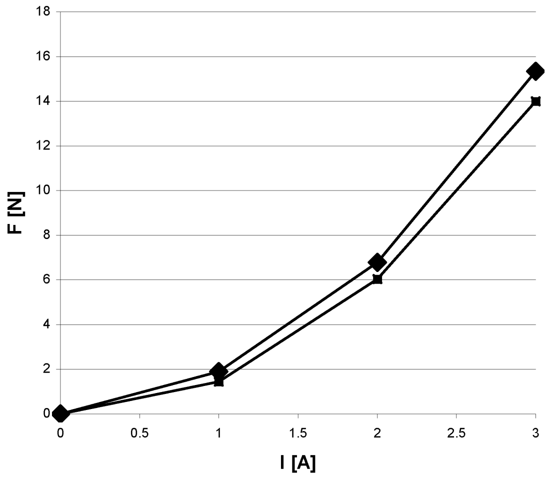

Fig.2: Dependence of electromagnetic force on supply current (◆ 50 Hz, ■ 100 Hz)

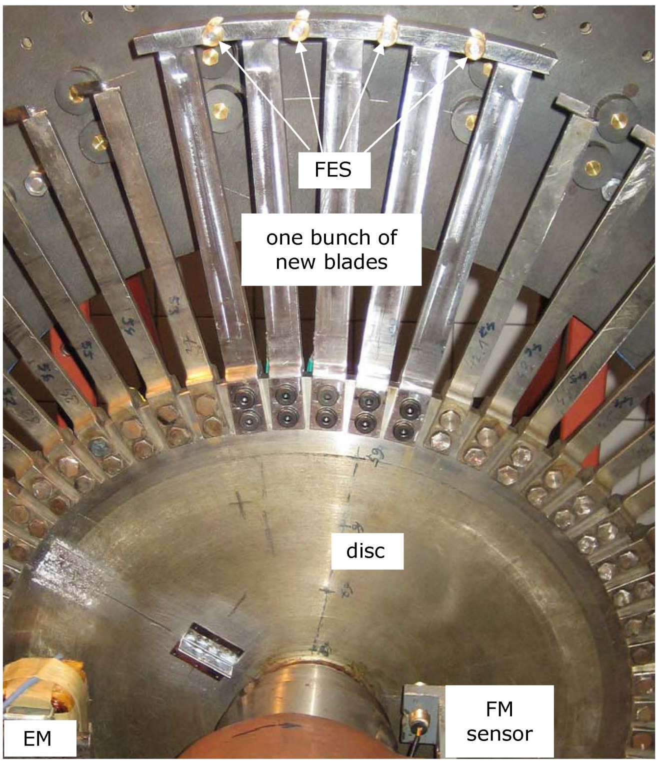

Fig.3: Picture of the experimental disc set-up (EM – electromagnet, FM – phase mark, FES – friction elements)

Besides the reaction force the scale registers inertia of electromagnet mass, too. This effect profiles however mainly in the resonance of the scale when the inertia force of electromagnet mass increases and the phase of excitation force is skewing from 0 up to 180°. This resonance frequence was 98.8 Hz. Frequency spectrum of excitation force measured by the scale and contemporary response of the blade L51 under rotation 8.3 Hz are depicted in Fig. 4. The excitation frequency was tuned to the resonance frequency of bladed wheel corresponding to the mode with $n = 1$. It is clear from the Fig. 4 that due to amplitude modulation of the excitation frequency by the revolution frequency and by occurence of harmonics of revolution frequency a quite dense spectrum of excitation arises. This fact is esppecially valid for lower revolution frequencies. The periodic cursors with a period of the revolution frequency 8.3 Hz denoted by the dotted line and relative distance cursors outspread out of excitation frequency 55.4 Hz by the revolution frequence by dashed line are depicted in the Fig. 3. Besides the cursors, some peaks in the spectra are specified by datatips with the frequency position.

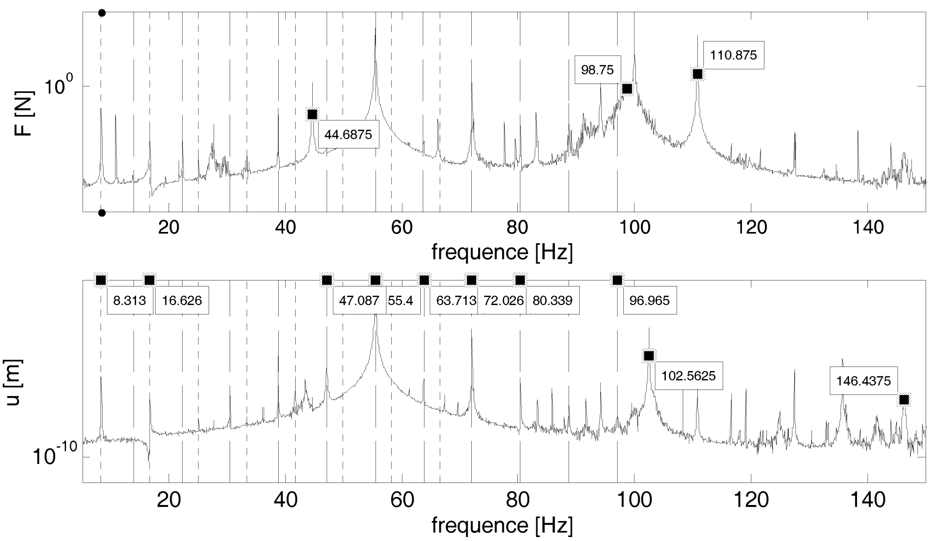

Fig.4: Frequence spectra of force (top) and blade L51 response (bottom)

The autospectrum of the blade response shows that most of frequency components of force are contained also in the blade response and that maximum amplitudes are achieved for the excitation frequency that is equal directly to the resonance frequency. When the apparent resonance excitations were used the disc vibration was very low. Therefore for the disc resonance excitation the sweep excitation in main bands of the modes with $n = 1, 2$ and 3 was used. Bandwidth was 10 Hz and sweep rate 0.02 Hz/s.

The time characteristic of axial blade L51 vibration (A1) is shown in Fig. 5. It belongs to a case of the bladed disc without friction elements, excitation sweep band (57–61.7) Hz and revolution frequency 15 Hz. The windows I and II denotes the passages of two resonant zones. Frequency pictures of these windows are in Fig. 6 and 7. The Fig. 6 shows the dominant excitation of eigenfrequency 58.5 Hz corresponding the sine mode with $n = 1$. The pair frequencies $\pm k$ 15 Hz ($k = 1, 2, \ldots$) around this frequency arise herein, too. It is caused by ampitude modulation of the excitation (carrier) frequency 58.5 Hz by the revolution frequency 15 Hz. So, e.g. for $k = 3$ the next eigenfrequency 103.5 Hz of cosine mode ($n = 3$) is excited. The frequency spectra of window II (Fig. 7) shows an excitation of apparent resonance frequency since the dominant resonance frequency 45.3 Hz is excited by frequency 60 Hz. Furthermore, the excited frequencies around 140 Hz appear in Fig. 6 and 7 that correspond neither to multiples of revolution frequence nor to frequencies originated by the amplitude modulation of the excitation frequence. The first flexural eigenfrequence of clamped blade lies in this band. Aerodynamic forces coming out of airflow around the blades cause likely their excitation. The occurence of revolution frequence and its multiples is caused by unbalance and shimmy effects of the wheel. The Fig. 8 shows the amplitude spectrum of FRFs of all blades L50, L51 a L21 for in the band (140–150) Hz excited in the side band of excitation (50–60) Hz in the revolution speed 600 rpm. It can be seen that only one resonance (142 Hz) is excited in this band and all measured blades vibrate practically by same amplitudes, the characteristics of neighbour blades L50 and L51 overlap here. Since a design of the electromagnet disabled to excite efficiently this band by the main band excitation, the excitation by permanet magnet was used instead.

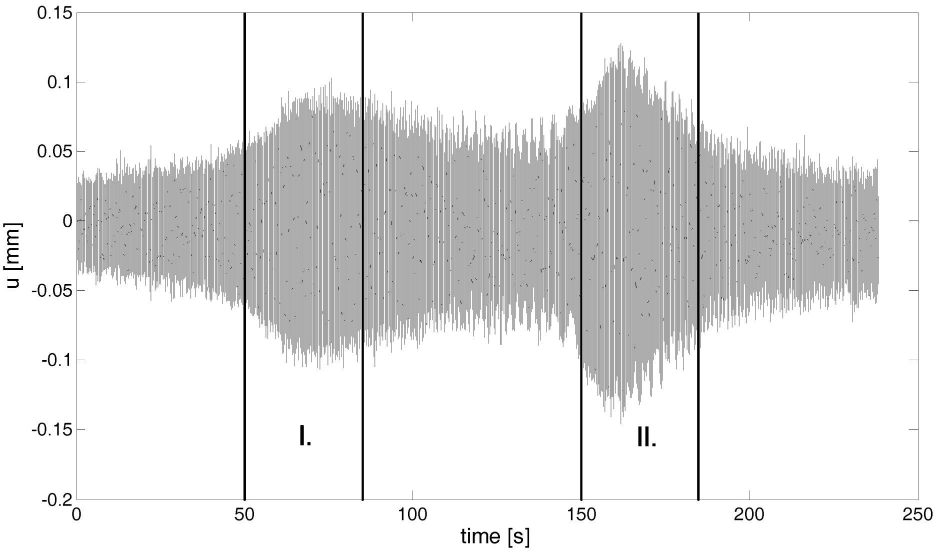

Fig.5: Time characteristic of the signal A1 (blade L51) at sweep excitation (57–61.7) Hz (57 Hz in time = 0)

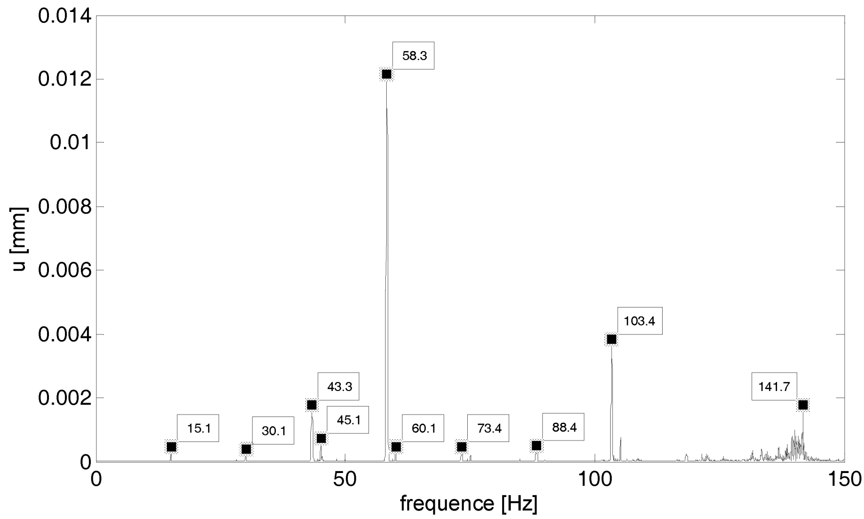

Fig.6: Amplitude-frequence spectrum of the resonance window I

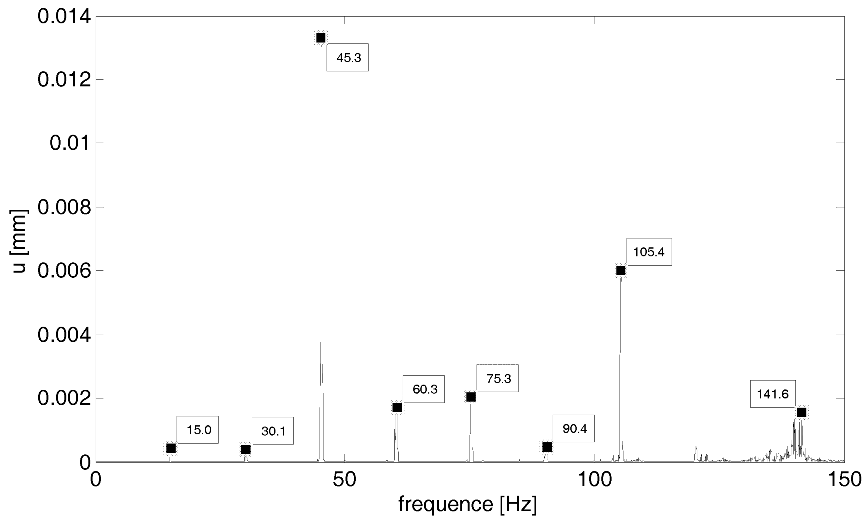

Fig.7: Amplitude-frequence spectrum of the resonance window II

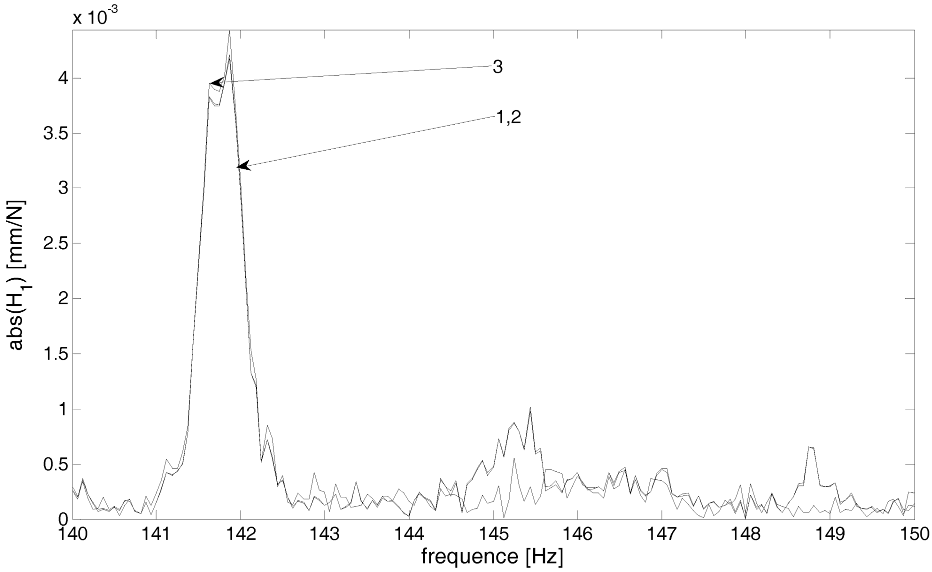

Fig.8: Amplitude-frequence FRF spectrum of the blade L51 (1), L50 (2) and L21 (3) in the band (140–142) Hz for excitation amplitude of 100 N is sweep range (50–60) Hz – 600 rpm, couplings with friction elements

6. Amplitude-frequence characteristics of axial blade disc vibration

In this chapter the amplitude-frequence characteristics of blade L51 vibration for cases of the wheel with friction elements and excitations both in the main band (70–80) Hz (Fig.9) and in the side band (90–100)Hz (Fig. 10) of the eigenmodes $n = 2$ at three different supply current ampitudes. Blade vibration responses on the different excitation amplitudes helped to assess a non-linear influence of the friction element on the resonance amplitudes. The excitation in different frequence bands showed the band where the resonances are most effectively excited. The characteristics of blade vibration are placed on the top and excitation force at the bottom of the figures. The characteristics are numbered according to excitation levels. The comparision of characteristics in Fig. 9 and 10 shows of cosine mode $n = 2$ is better excited in the main band (72–78) Hz than in the side band (92–98) Hz of excitation. Despite the characteristics have lower amplitudes and are not so smooth in the side band as in the main band they are qualitatively the same. This finding was confirmed also for the other resonances.

Contrary to Fig. 9 and 10 where vibration characteristics of one blade for three levels of excitation were plotted, the Fig. 11 shows characteristics of three blades L51 (1), L50 (2) and L21 (3) for just one excitation amplitude. In this case the disc was treated without insertion of the friction elements into the blade heads. The interesting phenomenon appears herein as the resonance 76.5 Hz is not excited at all blades at once. The blade L50 contrary to blades L51 and L21 does not vibrate. The reason is that the corresponding sine mode ($n = 2$) of vibration has the nodal radial in the place of L50. It proves a non-travelling character of the mode under rotation.

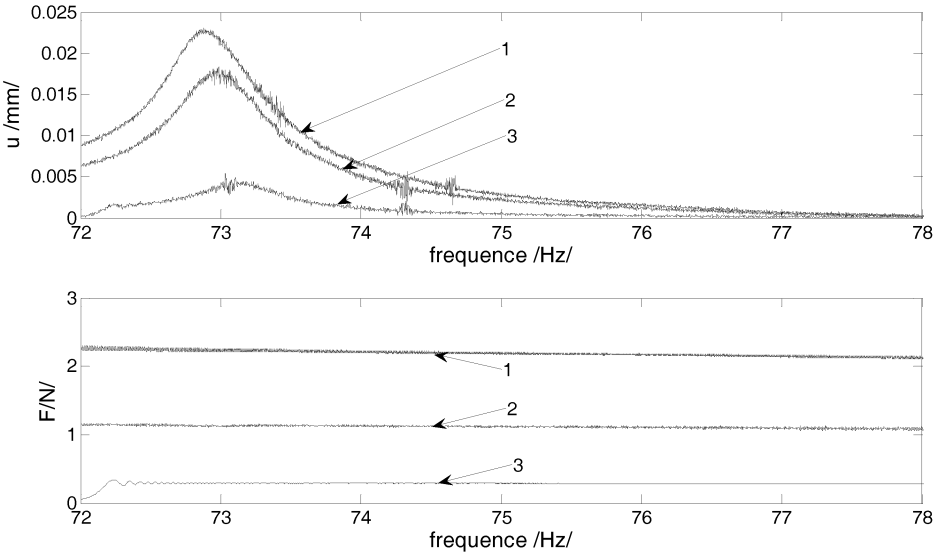

Fig.9: Amplitude-frequency vibration characteristics of blade L51 in the band (72–78) Hz for three levels of excitation 0.3 N (3), 1.1 N (2) a 2.2 N (1) in the sweep excitation range 72–78 Hz – 600 rpm, couplings with friction elements

7. Identification of eigenvalues of the disc under rotation

For quantitative assesment of dry friction influence of the friction elements on the wheel dynamics under rotation the selected eigenfrequencies and eigenvalues of the wheel were identified from the FRF functions. Since the series of measurement were performed for the wheel with and without friction elements inserted into blade inter-head spaces, these results enable to evaluate changes with respect to stiffness and mainly damping of the wheel vibration. The FRF functions evaluated from electromagentic sweep excitation and blade response time signals and eigenvalue identification were ascertained by the procedures described in chapter 2.

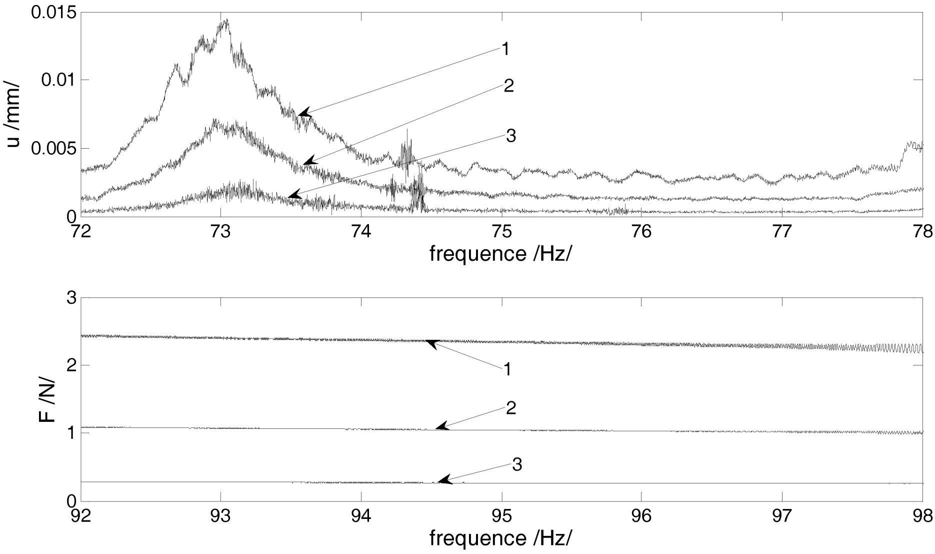

Fig.10: Amplitude-frequency vibration characteristics of blade L21 in the band (72–78) Hz for three levels of excitation 0.3 N (3), 1.1 N (2) and 2.5 N (1) in the sweep excitation range 92–98 Hz – 600 rpm, couplings with friction elements

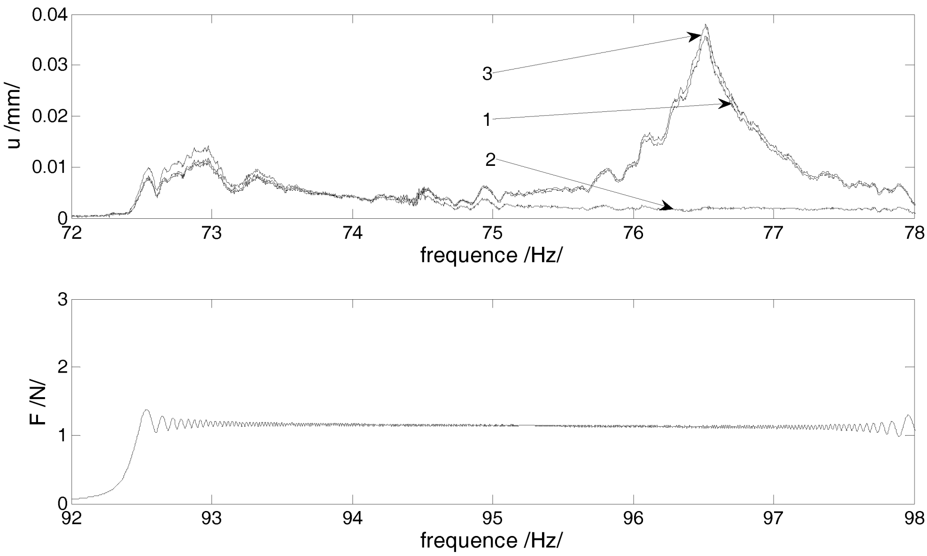

Fig.11: Amplitude-frequency vibration characteristic of of the blade L51 (1), L50 (2) and L21 (3) in the band (72–78) Hz for excitation amplitude of 1N is sweep range (92–98) Hz – 900 rpm, couplings without friction elements

FRFs were evaluated for the blades L50, L51 and L21 under different excitation amplitudes and different frequency bands. In the Fig. 12 there is an example of FRFs in a complex plane (the Nyquist diagram) for the blade L51 and both cases of wheel with and without friction elements. Measured sample frequency 20 kHz was downsampled to 4 kHz for the FRF evaluation.

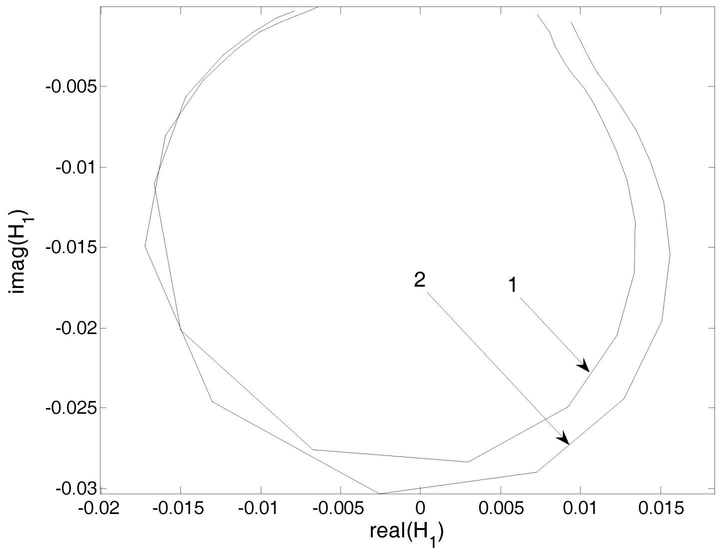

Fig.12: Nyquist diagram of FRFs in band (57–59) Hz, supply current 1.5 A and revolution frequency 900 rpm – coupling with (1) and without friction elements (2)

8. Conclusion

In the paper, some results of experimental research of the model bladed disc with double periodicity are presented. The description of electromagnetic force acting on the rotating disc is first modeled by analytical formula and then experimentally evaluated by the dynamic scale. It appeared that due to the occurence of revolution frequency and its multiples and amplitude modulation of the excitation carrier frequency by the revolution frequency the frequency spectra of excitation force is quite dense. Part of the amplitude modulations of the carrier frequency is caused by its interaction with the resonant vibrating disc as the analytical solution for harmonic excitation proves. In addition, the modulations are only partial so that besides the pair frequency ocurrence the carrier frequence component remains dominant. Therefore, the excitation of resonance frequencies is the most effective in so-called main frequency bands spreaded in the vicinity of corresponding resonance frequency.

If some of the pair modulation frequencies fall into the resonance frequency of the wheel, the so-called apparent resonant vibration occurs. This vibration mode is however stationar and does not travel around the disc.

The best excited eigenmodes were modes with number of nodal diameters $n = 1$ and 2 in the occurence band (50–60) Hz and (70–80) Hz, respectively. The identified eigenvalues of these modes were used for evaluation of dry friction influence on the wheel dynamics. The results were weakly dependent on the amplitude of electromagnetic excitation and the wheel behaved quasilinearly. The reason was the lower amplitude of excitation forces with respect to adhesion forces in contacts between the friction elements and blade heads in considered revolution frequencies. So, the friction elements were locked in the interblade space and therefore the damping effect was not strong. The slight increase of eigenfrequenies with increasing of revolution and their decrease with excitation amplitude was observed.

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