A COMPARISON OF S-N CURVES OF UNIAXIAL AND COMBINED RANDOM LOADING

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The paper deals with results of special fatigue life tests. Random processes of different power spectral densities loaded tube specimens made of a mild carbon steel ČSN 411523.1 (11523.1), notched by a perpendicular hole. It has been found that fatigue lives build similar S-N curves like by harmonic loadings, when the standard deviation $\hat{s}_d$ of peaks of an effective damaging stress and number of loading blocks N_b are used instead of amplitudes σ_a and number of cycles N_a, respectively. S-N curves of uniaxial and multiaxial loading are compared in the paper.

Keywords: *fatigue test, combined loading, S-N curves, spectral density*

1. Introduction

The article is a substantially extended and re-worked contribution of the authors presented at the colloquium of the Institute of Thermomechanics of the Academy of Sciences of the Czech Republic [2] and provides information about the current state of the issue of combined random loading solution.

At the Applied and Computational Mechanics Conference in the year 2009 [1], we dealt with the issue of possible utilization of random process spectral properties for the calculation of life under multiaxial loading. It resulted from experimental works performed on tube specimens of 11523.1 steel subjected to a combination of random tension-pressure and torsion loading that, analogous to harmonic loading, the *S-N* curves can also be constructed for random loading if we substitute damaging stress standard deviation s_d for stress amplitude σ_a and a number of loading blocks N_b for a number of cycles N_a. It has been found that the resulting life N_b under combined loading does not depend on the sequence of individual partial process loading levels but only on their magnitudes, represented by standard deviations s_σ and s_τ. In order to find out how these *S-N* curves will differ in the case of single loading methods, we extended the obtained results by some other works, which are included in this contribution.

2. Experimental works and their results

The experimental works were carried out using tube specimens of the same shape with a diameter of 30 mm and wall thickness of 2 mm (see Fig. 1). The specimen material, 11523.1 steel, was also the same as well as a single-sided through-thickness hole with a diameter of 3 mm, which by its notch effect delimited the place for monitoring the fatigue crack propagation.

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A total number of 12 test specimens was tested by means of the following loading methods:

- specimens Nos. 16, 17 and 18 – random tension-pressure with a decreasing shape of power spectral density (PSD),
- specimens Nos. 19, 20 and 21 – random torsion with a pyramidal PSD shape,
- specimens Nos. 22, 23 and 24 – random tension-pressure with a pyramidal PSD shape,
- specimens Nos. 25, 26 and 27 – random torsion with a decreasing PSD shape.

As we were monitoring the effect of spectral properties on life during the tests, the loading processes of individual groups of specimens were generated analogously to combined loading so that the PSD shape of each of the group of the specimens could remain the same.

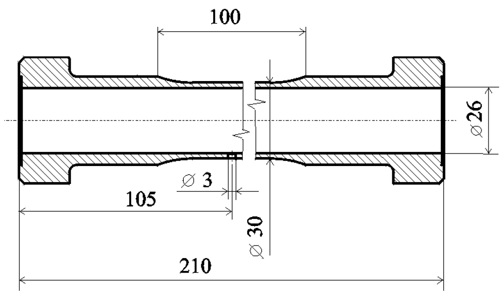

Fig.1: Test specimens shape

For the purpose of mutual comparison, we focused on the same PSD shapes, namely the decreasing and the pyramidal ones. The only difference was the magnitude of these processes characterized by the standard deviation s_d of effective damaging stress σ_d or its peaks.

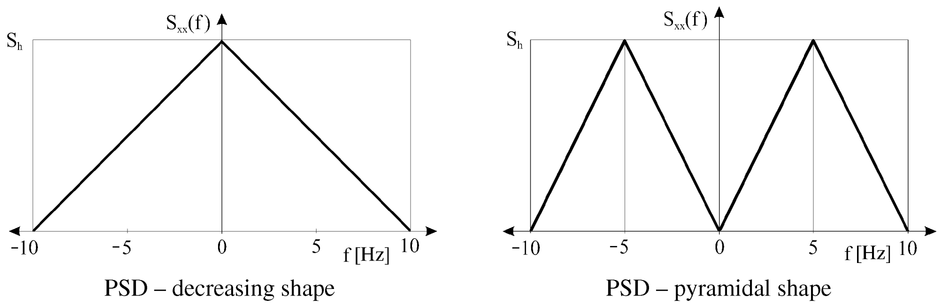

Fig.2: The applied power spectral density shapes

Let us define damaging effective stress for the damaging effect of both components s_σ and s_τ of the combined loading (normal and shear) in complex form

$$\sigma_\mathrm{d}(t) = \sigma(t) + \mathrm{i}\, k_\mathrm{c}\, \tau(t) \tag{1}$$

where $k_\mathrm{c} = \sigma_\mathrm{c}/\tau_\mathrm{c}$ is a ratio of fatigue limits in tension-pressure and torsion respectively.

Then it applies for the damaging stress dispersion that

$$s_\mathrm{d}^2 = s_\sigma^2 + k_\mathrm{c}^2\, s_\tau^2 \ . \tag{2}$$

In our case of single loading we get

– for tension-pressure ($s_\tau = 0$)

$$s_\mathrm{d} = s_\tau \ , \tag{3}$$

– for torsion ($s_\sigma = 0$)

$$s_\mathrm{d} = k_\mathrm{c}\, s_\tau = 1.5\, s_\tau \ , \tag{4}$$

as we have experimentally found out that $k_\mathrm{c} = \sigma^*_\mathrm{c}/\tau^*_\mathrm{c} = 120/80 = 1.5$.

The Tab. 1 shows the lives N_b for individual groups of specimens. The values s_d correspond to standard deviations of process damaging stress, the values $\hat{s}_\mathrm{d}$ to their peak values.

If we compare the lives corresponding to approximately identical magnitudes s_d separately for the case of tension-pressure loading, then we can find that the obtained lives for decreasing and pyramidal PSD shapes do not almost differ for the random tension-pressure. For the case of torsion, the difference in PSD shape was only observed in specimens Nos. 20 and 26, less in specimens Nos. 19 and 25. The pyramidal shape proved to be a little more aggressive particularly in the first two specimens. However, the influence of damaging stress level s_d on life is unequivocal which could be anticipated.

Specimen No.	$\hat{s}_\mathrm{d}$ [MPa]	s_d [MPa]	Life N_b	Loading method
16	118.94	100.57	15.3	Random tension-pressure, decreasing PSD
17	98.86	84.43	24.69	
18	79.1	69.32	103.99	
19	116.68	99.12	12.45	Random torsion, pyramidal PSD
20	104.13	83.50	30.44	
21	93.81	70.50	127.50	
22	111.32	88.66	15.29	Random tension-pressure, pyramidal PSD
23	82.14	64.99	117.13	
24	82.92	65.81	91.36	
25	116.28	112.53	9.23	Random torsion, decreasing PSD
26	98.12	79.41	61.33	
27	61.76	51.35	?930	

Tab.1: The influence of PSD shape on life under single loading

For the purpose of comparison, we plotted the *S-N* curve for peak values of combined stress together with the *S-N* curve for the peak values of single loading in Fig. 3. All the values indicated in Tab. 1 regardless of the PSD shape and the types of loading were included in the set. The regression curve is passing through these points. This curve for the single stresses lies somewhat lower than the regression curve for the combined loading as it is evident from the Fig. 3. Both the curves can be considered as parallel. The loading processes are coded in the figure legends. The letters F and M correspond to force and moment respectively, K to decreasing PSD, P to pyramidal one and 0 to zero. In heading, 'T' means tube, '115231' material, and 'V' notched specimens.

The right-hand curve corresponds to the random combined torsional and axial loading with the decreasing and the pyramidal PSD shapes (10 points correspond to the decreasing PSD shape for tension-pressure and to the pyramidal shape for torsion; the PSD shapes were interchanged for other 5 points).

The other curve corresponds to independently acting random loadings in tension-pressure or torsion, with decreasing and pyramidal PSD shapes (3 points correspond to tension-

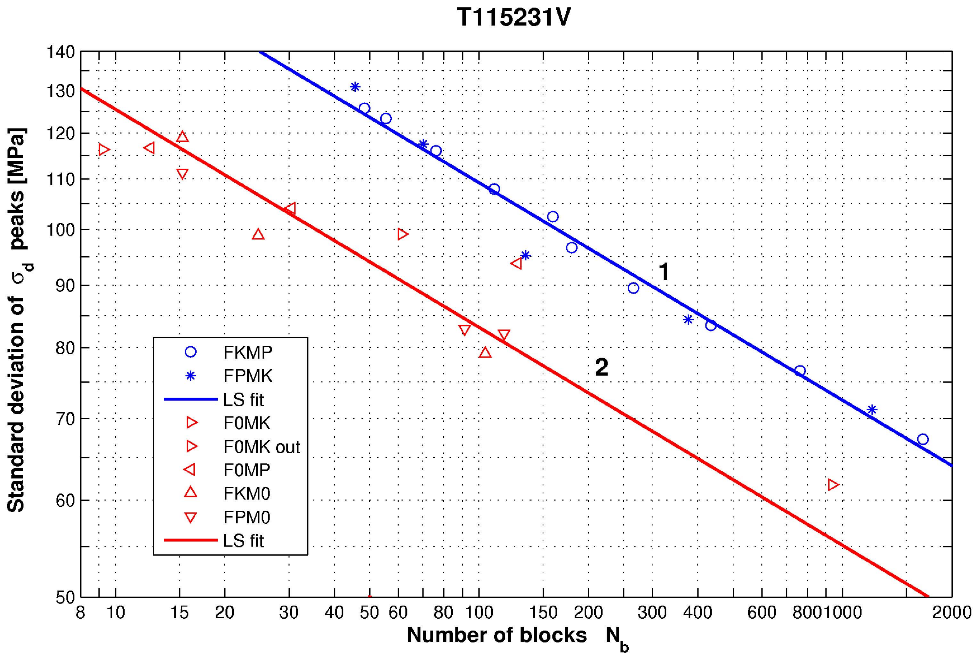

Fig.3: Comparison of S-N curves for peak values of combined and single random loading

pressure with the decreasing PSD shape, 3 points with the pyramidal shape, and similarly for the torsion).

The regression curves can be expressed in a logarithmic form as follows:

– curve No. 1 for combined random loadings

$$\log N_\mathrm{b} = 13.4194 - 5.6022 \log \hat{s}_\mathrm{d} \ , \tag{5}$$

– curve No. 2 for single random loadings

$$\log N_\mathrm{b} = 12.7553 - 5.6022 \log \hat{s}_\mathrm{d} \ . \tag{6}$$

The regression curves related to the process damaging stress standard deviations s_d of the same types of loading are plotted in Fig. 4. The formulae of regression curves slightly differ in this case:

– curve No. 1 for combined random loadings

$$\log N_\mathrm{b} = 12.8507 - 5.5500 \log s_\mathrm{d} \ , \tag{7}$$

– curve No. 2 for single random loadings

$$\log N_\mathrm{b} = 12.1965 - 5.5500 \log s_\mathrm{d} \ . \tag{8}$$

Both the curves in Figs. 3 and 4 are parallels where the curve for single loading is displaced lower into the areas of lower lives N_b in comparison with the curve for combined random loading. In the case the standard deviation is related to the process damaging stress, this displacement is smaller as evident in Fig. 4.

The equations of displaced regression curves (5), (6) and (7), (8) differ from each other only in the first member usually designated with a letter A which determines the curve position in the system of coordinates while the other constant corresponding to exponent w for the fatigue curve for harmonic loading determines the incline of the regression curves. As the shape of regression curves (S-N curves), as found out, does not particularly influence the PSD shape but only a so called magnitude of the applied loading processes characterized by a damaging stress standard deviation s_{d}, it is possible to determine the dependence between constants A in equations (9) and (10) and to find a dependence for recalculation of both types of random loading.

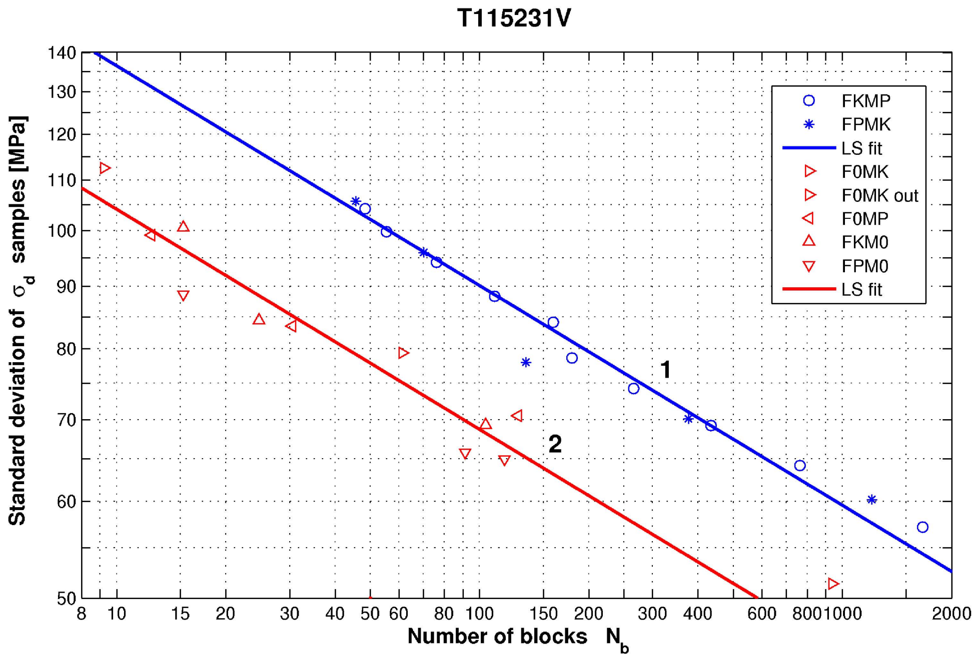

Fig.4: Comparison of S-N curves of combined and single random loading processes

If we write both the equations in general form, we get:

– for the curve No. 1

$$\log N_{\mathrm{b}1} = A_1 - w \log s_{\mathrm{d}1} \; , \tag{9}$$

– for the curve No. 2

$$\log N_{\mathrm{b}2} = A_2 - w \log s_{\mathrm{d}2} \; . \tag{10}$$

For the same number of blocks N_{b}, it is possible to get a relation for the difference of constants A_1, A_2 from the equality of the equation right-hand sides in the form

$$A_1 - A_2 = w \left(\log s_{\mathrm{d}1} - \log s_{\mathrm{d}2}\right) \; . \tag{11}$$

The difference of these constants is proportional to the difference of logarithms of damaging stress standard deviation s_{d} of combined and uniaxial loading processes under the same fatigue lives. Equation (11) can serve for the evaluation of the exponent w, if all other parameters be known.

The parallelism of regression curves for combined and uniaxial stresses in Figs. 3 and 4 was experimentally acknowledged for the case of the same damage of partial processes of normal and shear stress components in equation (2). General validity for various ratios of these components damage will have to be experimentally verified. It can be assumed that in case of differently damaging partial processes the new *S-N* curves will lie between the curves in Fig. 3, in some case 4.

3. Amount of supplied energy

It is generally known that there is a relation between the amount of energy supplied by a given process of loading and the extent of damage of a component subjected to dynamical loading. If the area below the PSD curve indicates the size of supplied power P in a given frequency range of one loading block the time duration of which is T_b, then the amount of supplied energy in this block can be expresses with the below equation

$$W_i = P\,T_\mathrm{b} = s_\mathrm{d}^2\,T_\mathrm{b}\ , \tag{12}$$

where s_d^2 is the damaging stress dispersion which can be determined from equation (2). It holds generally.

A frequency range $f = 0 \div 10\,\mathrm{Hz}$ and time duration of one loading block $T_\mathrm{b} = 5\,\mathrm{minutes}$, i.e. 300 s, was chosen in the case of our specimens subjected to testing.

The total supplied energy for each of the specimens subjected to cyclic loading was

$$W = \sum Wi = s_\mathrm{d}^2\,T_\mathrm{b}\,N_\mathrm{b}\ , \tag{13}$$

where N_b is the total life of a given specimen in the number of loading blocks to fracture.

The dependence between the resulting process magnitude related to the damaging stress loading is indicated by regression curves equations (7) and (8). However, we will be interested in the energy criterion, whether and how the amount of supplied energy will be of use in the case of multiaxial and uniaxial stresses. In order to get for both the monitored types of stress a dependence of the amount of supplied energy W on the number of loading blocks N_b, let us take the logarithm of the equation (13). We will get

$$\log W = 2\,\log s_\mathrm{d} + \log T_\mathrm{b} + \log N_\mathrm{b}\ . \tag{14}$$

Hence

$$\log s_\mathrm{d} = \frac{1}{2}\,[\log W - \log T_\mathrm{b} - \log N_\mathrm{b}]\ . \tag{15}$$

If we substitute $\log s_\mathrm{d}$ in the equations (7) and (8), and if we substitute our block duration of 300 s into T_b, we will get the following two equations for the amount of supplied energy:

– for combined random loading

$$\log W_1 = 7.108 + 0.6396\,\log N_\mathrm{b1}\ , \tag{16}$$

– for single random loading

$$\log W_2 = 6.872 + 0.6396\,\log N_\mathrm{b2}\ . \tag{17}$$

By taking the antilogarithm we can get the sought equations for the amount of supplied energy in exponential form

$$W_1 = 1.28233\times10^7\, N_{b1}^{0.6396} \tag{18}$$

and

$$W_2 = 0.74473\times10^7\, N_{b2}^{0.6396}\ . \tag{19}$$

These dependences are shown in Fig. 5. It is apparent from the figure that to achieve the same life it is needed to supply a higher amount of energy in the case of combined loading (see the curve 1) than in the case of single random loading (uniaxial tension-pressure, or torsion). The difference is rising up with the stress level reduction.

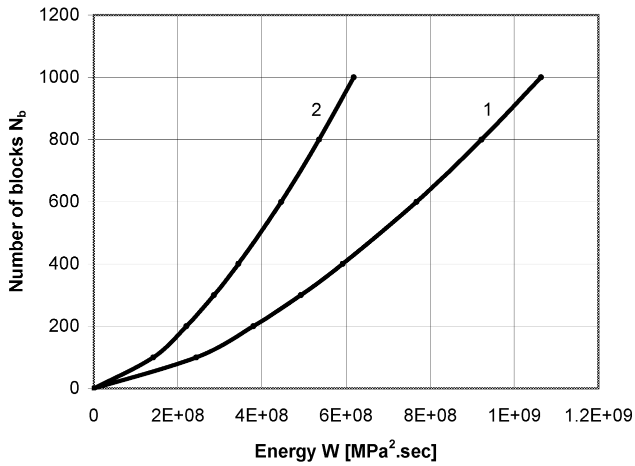

Fig.5: Dependence of life N_b on the amount of supplied energy W (1 ... combined loading, 2 ... single loading)

The relation for increase of difference in these energies in dependence on the number of loading blocks N_b can be derived from the difference of equations (18) and (19). The dependence is exponential

$$\Delta W = 0.53717\, N_b^{0.6396}\ . \tag{20}$$

This dependence applies for our case when we compared the supplied energies W under single and combined loadings with the same damaging effect of individual components. In the case these damaging effects will differ, the constants in equation (20) will obviously be different, too.

4. Crack propagation under single and combined loading

We were monitoring the crack propagation in selected test specimens under single and combined methods of random loading. The characteristic course of propagation of fatigue cracks coming out of the cross-hole of tube specimens subjected to tension-pressure and torsion is apparent in Figs. 6–9. Fig. 6 shows typical crack propagation in a tube specimen subjected to combine random tension-pressure/torsion loading with a decreasing (tension-pressure) and pyramidal (torsion) PSD shapes. The cracks propagate under an angle of app. 30° uniformly in all four quadrants independently on the intensity of damaging stress standard deviation s_d. Fig. 7 shows the crack propagation in the case of the same loading method with an inverse PSD shape. The propagation is now different and proceeds in two opposite quadrants only. It also depends on the size of damaging stress standard deviation s_{d}. The torsion effect decreases with the decreasing value and the propagation approximates to the course under a single tension-pressure. Fig. 8 and 9 show the courses of fatigue crack propagation for a single random tension-pressure and torsion. The cracks run classically perpendicularly to the direction of the principal normal stress in the case of tension-pressure and under an angle of 45° in the case of torsion.

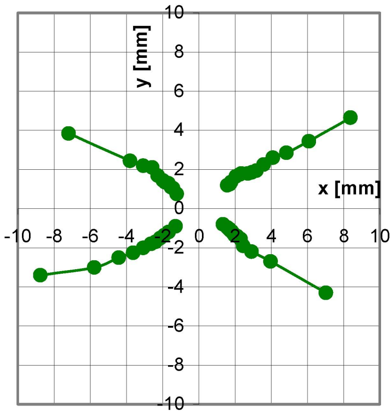

Fig.6: Crack propagation under combined random tension-pressure/torsion loading (decreasing and pyramidal PSD)

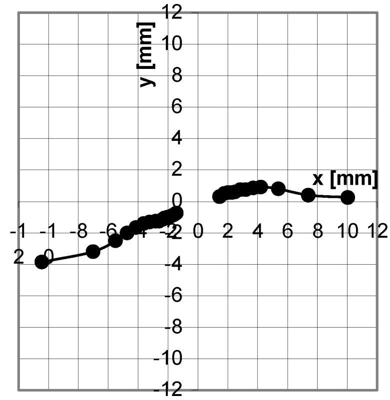

Fig.7: Crack propagation under combined random tension-pressure/torsion loading (pyramidal and decreasing PSD)

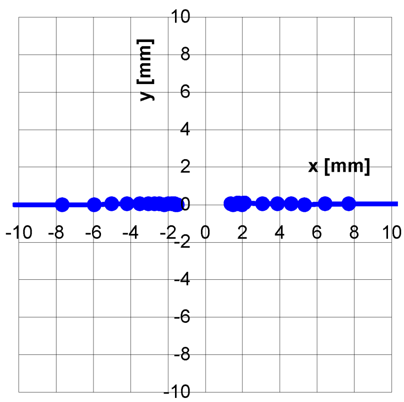

Fig.8: Crack propagation under single random tension-pressure loading (specimen No. 18 – see Tab. 1)

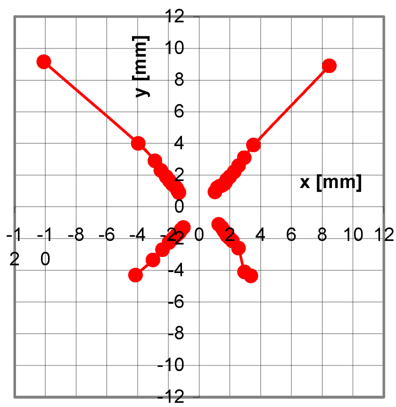

Fig.9: Crack propagation under single random torsion (specimen No. 21 – see Tab. 1)

In order to achieve fatigue fracture of testing specimens, we need to supply a certain amount of energy W that can be expressed by an equation (13). In this equation, s_{d}^2 means a damaging stress dispersion which can be calculated from equation (2), T_{b} value means the time duration of the loading block $N_{\mathrm{b}i}$ which represents a given random loading process and N_{b} a total life in the number of loading blocks until fracture. This energy is consumed for specimen material deterioration in its microstructure, and on specimen deterioration in the area of long cracks propagation. The long crack propagation can be described by fracture mechanics in the case of one main crack propagating with a certain growth rate. However, the cracks under combined random loading can have a different character round the notch as we can see in our case of the tubular specimen with a cross-hole. The deterioration process is usually such that initially, the first crack initiates in the weakest or the most stressed place and starts to propagate. Other cracks one by one follow and the life is then determined not only by the length of one crack but by a set of all arisen cracks which change the toughness of a given place. Let us have a look in this manner at our four specimens and write the needed parameters for them in Tab. 2.

Specimen in Fig. No.	s_d [MPa]	s_d^2	Number of loading blocks			Energy W_i in 1 loading block	Total energy W until fracture
			micro	propagation	total		
6	99.77	9953.7	30	25.43	55.43	2986110	1.6552×10^8
7	95.92	9200.6	38	32.85	70.25	2760180	1.9390×10^8
8	69.32	4805.3	40	63.99	103.99	1441590	1.4991×10^8
9	70.50	4970.3	76	51.5	127.5	1491090	1.9011×10^8

Tab.2 Parameters of the above four tube specimens

The total energy W until fracture as indicated in the Tab. 2 gives rise to deterioration of specimens both in the microstructure area and in the area of long fatigue crack propagation indicated in Figs. 6–9. The equation (13) was used to calculate this energy, and the equation (2) to calculate dispersion s_d^2 of damaging stress. The table also shows experimentally determined lives expressed in a number of loading blocks N_b, both in the material microstructure until the rise of a so called technical crack and during its propagating, which we are going to focus on now.

First, it should be stated that random loading of tube specimens was carried out under soft loading with a controlled force and moment. All the loading blocks N_{bi} remained the same in individual specimens for the whole time of testing. It means that the supply of energy remained the same, too. If several fatigue cracks of different lengths a_j arose during one loading block N_{bi}, even in different quadrants, then the sum of lengths of all these cracks can be considered as the size of damage, on the rise of which was participating the energy of this block. If we plot a dependence a_s on the number of blocks N_{bi} for the four monitored specimens, we will get the curves plotted in Fig. 10.

It should be pointed out that the functions designated 8 and 9 in the Tab. 2 corresponding to single loading have app. 2/3 magnitude of functions Nos. 6 and 7. That is why these curves in Fig. 10 are shifted to the right and indicate a smaller damage. The paths of specimen's deterioration in Fig. 10 are not projected until fracture because the phase of total break usually occurred in a shorter time of one or two blocks and the crack length could not be measured. For the performed tests, we know the exact total number of blocks until fracture N_b.

Let us have a look now at the damaging of the monitored 4 specimens with regard to the amount of supplied energy W (see Fig. 11).

It results from the Fig. 11 that the biggest damage occurs in the specimen No. 18 in Fig. 8 subjected to single random tension-pressure (with a decreasing PSD). This specimen needed the smallest amount of energy for its deterioration. A smaller damage was in the specimen in Fig. 6 subjected to combine random stress of tension-pressure (a decreasing PSD) – torsion (a pyramidal PSD). The specimen in Fig. 9 subjected to single random torsion

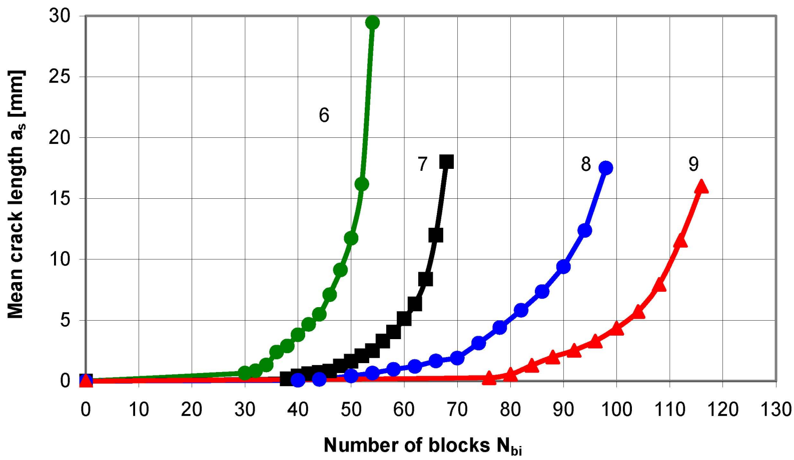

Fig.10: Cumulative damage a_s as function of fatigue life N_b for specimens in Figs. 6–9

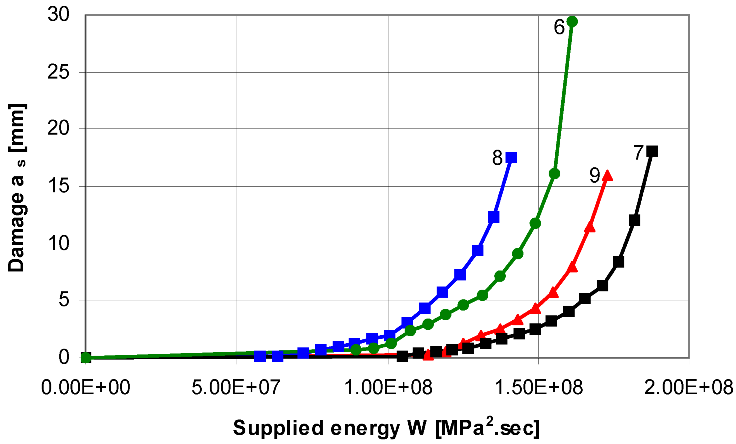

Fig.11: Cumulative damage a_s as function of supplied energy W for specimens in Figs. 6–9

(PSD a pyramid) showed approximately similar damage. The specimen in Fig. 7 which was subjected to a combination of random tension-pressure and torsion with exchanged PSD shapes (tension-pressure – pyramidal PSD, torsion – decreasing PSD) that showed itself to be the least aggressive, needed to be supplied the highest amount of energy for its damage.

It is apparent from the above that the evaluation of a random process aggressiveness and also the life according to the number of loading blocks and an amount of supplied energy need not always be in correspondence and so the validity is limited.

The experimental works resulted in the following conclusions.

5. Conclusion

It resulted from the carried out experimental works that for the single loading method it is also possible to construct a regression curve (an *S-N* curve) which is parallel with the regression curve for combined loading but shifted to the area of lower lives. When relating the standard deviation to the process damaging stress, this displacement is smaller than in case of relating the damaging stress standard deviation to the process peak values. The formulae of regression curves only differ from each other by the first member usually designated as A which determines the curve position in the system of coordinates while the other constant corresponding to exponent w in the fatigue curve for harmonic loading indicates the regression curves incline which is identical for both the curves. As it was derived, the difference of constants A for the same life N_b is proportional to the difference of logarithms of damaging stress standard deviations s_d of the process of combined and uniaxial stress. This can be used for recalculating the life from single to multiaxial loading and vice versa.

The monitoring of dependence of the supplied amount of energy W and the resulting life N_b allowed to find out that to achieve the same size of damage it is needed (in case of combined loading, see the curve 1) to supply a higher amount of energy than in the case of single random loading (uniaxial tension-pressure, or torsion) which shows evidence of higher aggressiveness of single loading. The difference increases with a decreasing level of stress.

During the fatigue testing we were also monitoring the fatigue crack propagation both in the specimens with a combined method of loading and with single loading. In both the cases the character and manner of fatigue crack propagation is different. As an example and for the purpose of comparison we used figures of crack propagation in four tubular specimens subjected to different loading with different magnitude of damaging stress. The parameters of these processes are indicated in Tab. 2. In other two figures, there are dependences of size of a so-called cumulative crack length a_s (independent on the fatigue crack quantity and propagation direction) on the number of loading blocks and also on the amount of supplied energy. It resulted from the above that the evaluation of random process aggressiveness and also the life according to the number of loading blocks and an amount of supplied energy needed not always be in agreement and so the validity is limited.

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References

[1] Svoboda J., Balda M., Frohlich V.: Utilization of random properties for the calculation of fatigue life under combined loading, Extended Abstracts Conf. Applied and Computational Mechanics 2009, Nečtiny, ISBN 978-80-7043-824-4

[2] Svoboda J., Balda M., Fröhlich V.: Comparison of *S-N* curves for uniaxial and combined random loading, Proceedings of Machine Dynamics Colloquium 2010, Prague, p. 73–78, ISBN 978-80-87012-24-6

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