DISASTER PROPAGATION MODELS

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To build a general model for the propagation of destructive events during disasters, we consider a networked system. The model involves network nodes as single objects and delayed interactions along directed links.

In this work, a disaster is understood as a sequence of dynamic destructive events which cause nonreversible changes, spreading in cascade-like manner. We define a global state of the system and suppose that the Markovian property holds. Hence, we can describe any object's first affect using the phase type distribution. This model can be used to improve disaster awareness and anticipated disaster response management.

Keywords: *propagation model, disaster*

1. Introduction

This work deals with some type of events, which we can denote as 'disasters'. In opposite to an usual failure, disaster has the ability of dynamic spreading, however as a rule it causes nonreversible changes (it is nonrepairable) and not least, consequences are often deep-going. There are no doubts that an impact of disasters on human society is very important.

In what follows, we present a model for the dynamic spreading of disastrous events in networked systems. We consider a disaster as a time sequence of single events, which spreads from a focus to other nodes of the network in a cascade-like manner. The complex network can represent some production system, factory, organization, infrastructure or communication system, geographic area and so on. As the nodes we assume system components as buildings, storehouses, tubes, conduits, servers, communication lines, but also natural objects as forest, underground water, river or air. In contrast to epidemic infection networks, interaction and infrastructure networks are often directed networks. Links between nodes in the network describe possible interactions or the functional and structural dependencies between components, causal dependence from the point of view of possible disastrous events.

While the occurrence of disasters comes mostly unexpected, we postulate that its propagation can be described using mathematical model (see e.g. [1], [2], [3], [5], [6]). In this paper, we consider a disaster occurrence within the framework of some complex system of objects, which border upon one another or are mutually connected by any way. The system of objects creates an oriented network. The objects are nodes and the oriented edges represent the directions of possible events propagation (transitions). The disaster starts by a strong initial event on some one of these objects and it spreads with some probability to an other contiguous object during a random time. This propagation continues in a cascade-like manner to other objects. Often, it reminds of a domino-effect. From another point of view, the disaster propagation can be consider as a random process, the states of which represents a number of attached objects along with the level of their deterioration.

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The transition probabilities of disastrous events from one given node to another depend on the global state of the network in time and not on the way, by which the system approached this state – the history of system. Evidently, this assumption is not general, but acceptable in most cases.

We suppose that the behavior of the system can be described by a Markov process with irreducible transition intensity matrix. This assumption provides us to use the phase-type distribution for description of the behavior of the first passage time. The phase-type distribution has been introduced by Neuts in 1975, along with a detailed discussion of its properties (see [7]). In studied case, we obtain an explicit formula for evaluation of the probability that the system, which occurs in an operational state, will reach some of the failure states at time t. Some notes on evaluation of phase-type distribution can be found in [4].

2. The models

Consider a system of objects (appointments), which can be or may not be mutually connected. In the following, we consider a random event, which occurs on these objects in succesive steps, on each object only once. The state of this system at the time t can be characterized by n-dimensional vector

$$\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_n(t)) \ , \quad t \geq 0 \ ,$$

where $\omega_i(t) = 0$ if the event did not occur on i-th object until time t, $\omega_i(t) = 1$ in the case when the event occurred on ith object at the time t already. Recall that n is the number of objects in the system.

In the our model, the disaster propagation process is realized by the following manner:

a) At the beginning, the system is in the state $(0, \ldots, 0)$.

b) The process starts by deterioration of an object i with probability π_i, $i = 1, 2, \ldots, n$.

c) When at a time t an object i was affected, there was a random time period τ after which the event moved onto some of the unaffected objects.

d) The process moves to the next object with a probability which depends only on the recent state, not on the path leading to the recent state (the time sequence of events).

The assumption d) enables us to use a Markov model for description of the disaster propagation process. Although the assumption represents some kind of 'memorylessness', it means an independence of the future on the history in the following sense: the probability of future event depends only on the set of already affected objects in the past but not on the time ordering of disastrous events. This assumption is acceptable in a lot of situations.

In what follows, we shall consider two models. The first model describes a situation, in which an affected object cannot be repaired or replaced in relatively short time. For example, when a river is contaminated by toxic material, there will take a long time to 'repair' it, from the point of view of the velocity of spreading disastrous events. The second model assumes that an affected object can be repaired or replaced by as-new one in a short random period. Already affected and repaired object can be affect once again. In both models, we assume that the time period τ has the exponential distribution with the intensity δ.

Model 1

Assumptions :

(i) at the beginning (at the time 0), a disastrous event affects i-th system's object with probability π_i, $i = 1, 2, \ldots, n$,

(ii) an event can only affect one object at a time,

(iii) an event can only occur once on a particular object,

(iv) states of the system in time t create a Markovian process $\{X(t),\, t \geq 0\}$ in continuous time. Values of this process lie within the set $\Omega = \{0,1\}^n - \{(0,0,\ldots,0)\}$.

The process $X(t)$ finishes always in the state $\omega^N = (1,1,\ldots,1)$, which is at the same time an absorption state. All else states are transient. The process can be understood as *the process of event spreading* in the system.

The set Ω consists of totally $N = 2^n - 1$ elements. Hence, a transient intensity matrix $\mathbf{S}$ of the process $X(t)$ is of size $N \times N$. Let us denote that $|\omega| = \omega_1 + \omega_2 + \cdots + \omega_n$. $|\omega|$ gives us the number of objects, on which some disastrous event holds yet. We can call this number a *size of deterioration* of the system.

Proposition 1. *Let us order states of the system in ascending order by the size of deterioration. Then, under assumptions (i)–(iv), the transition intensity matrix* $\mathbf{Q}$ *of the process* $\{X(t),\, t \geq 0\}$ *is upper triangular of the size* $(2^n - 1) \times (2^n - 1)$. *The matrix* $\mathbf{Q}$ *can be written in the block-form as*

$$\mathbf{Q} = \begin{pmatrix} \mathbf{D}_{1,1} & \mathbf{P}_{1,2} & \mathbf{O}_{1,3} & \cdots & \mathbf{O}_{1,n} \\ \mathbf{O}_{2,1} & \mathbf{D}_{2,2} & \mathbf{P}_{2,3} & \cdots & \mathbf{O}_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{O}_{n,1} & \mathbf{O}_{n,2} & \mathbf{O}_{n,3} & \cdots & \mathbf{0} \end{pmatrix}$$

where $\mathbf{P}_{i,j}$ *is a rectangular matrix of size* $\binom{n}{i} \times \binom{n}{j}$, *the symbol* $\mathbf{O}_{ij}$ *denotes a null matrix and* $\mathbf{D}_{ii}$ *are square diagonal matrices for* $i = 1, \ldots, n$. *Moreover,*

$$-\mathbf{D}_{i,i} = \mathbf{e}\,\mathbf{P}'_{i,i+1}\,\mathbf{I}_{i,i}$$

where $\mathbf{I}_{i,i}$ *is the unit matrix of size* i, $\mathbf{e}$ *is the row vector of* $\binom{n}{j}$ *ones.*

The set of all possible states Ω can be decomposed into n disjoint subsets $\Omega_1, \ldots, \Omega_n$, which involve states with $|\omega| = 1, \ldots, n$. It means, that the set Ω_j contains all states, in which the pursued event holds exactly on j objects. The number of such states is equal to $\binom{n}{j}$. The i-th column of the matrix $\mathbf{Q}$ corresponds to states from Ω_i.

When the process $X(t)$ is in the state $\omega \in \Omega_j$, then under assumptions (ii) and (iii), in the oncoming time it can either to stay in the state ω or to move in any of states of the set Ω_{j+1}. From it follows that

– blocks $\mathbf{O}_{j,j-l}$, $l = 1, \ldots, j$ are null matrices,

– matrix $\mathbf{D}_{jj}$ is diagonal and

– matrices $\mathbf{O}_{j,j+k}$, $k = 2, \ldots, n-j$ are null matrices.

Moreover, the time period τ, for which the process stay in k-th state $\omega^k \in \Omega_j$ before it moves to another state in Ω_{j+1}, has the exponential distribution with the parameter $\delta = (D_{jj})_{kk}$, the k-th diagonal element of $\mathbf{D}_{jj}$.

Model 2

Assumptions:

(i) at the beginning (at the time 0), the system is in the state $\omega^0 = (0, 0, \dots, 0)$,

(ii) an event can only affect one object at a time,

(iii) states of the system in time t create a Markovian process $\{X(t), t \geq 0\}$ in continuous time. Values of this process lie within the set $\Omega = \{0, 1\}^n$.

The process $X(t)$ can be considered as a random walk between sets $\Omega_0, \dots, \Omega_n$. When the process in in a state $\omega \in \Omega_j$, $0 < j < n$, the only transitions to some states in Ω_{j-1} or Ω_{j+1} are allowed, whereas ω^0 and Ω^N are reflection states. In the model, all states are transient. The process can be understood as the process of event spreading in the system with repair.

Proposition 2. *In the model 2, the transition intensity matrix* $\mathbf{Q}$ *of the process* $\{X(t), t \geq 0\}$ *has the following block-form*

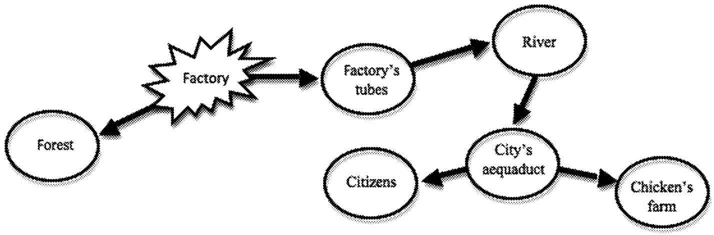
$$\mathbf{Q} = \begin{pmatrix} \mathbf{D}_{0,0} & \mathbf{P}_{0,1} & \mathbf{O}_{0,2} & \mathbf{O}_{0,3} & \cdots & \mathbf{0} \\ \mathbf{R}_{1,0} & \mathbf{D}_{1,1} & \mathbf{P}_{1,2} & \mathbf{O}_{1,3} & \cdots & \mathbf{O}_{1,n} \\ \mathbf{O}_{2,0} & \mathbf{R}_{2,1} & \mathbf{D}_{2,2} & \mathbf{P}_{2,3} & \cdots & \mathbf{O}_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{O}_{n,1} & \mathbf{O}_{n,2} & \mathbf{O}_{n,3} & \cdots & \mathbf{D}_{n,n} \end{pmatrix}$$

where $\mathbf{P}_{i,j}$ *is a rectangular matrix of size* $\binom{n}{i} \times \binom{n}{j}$, $\mathbf{R}_{j,i}$ *is a rectangular matrix of size* $\binom{n}{j} \times \binom{n}{i}$, *the symbol* $\mathbf{O}_{ij}$ *denotes a null matrix and* $\mathbf{D}_{ii}$ *are square diagonal matrices for* $i, j = 1, \dots, n$. *Moreover,*

$$-\mathbf{D}_{i,i} = (\mathbf{e}^i \, \mathbf{R}'_{i-1,i} + \mathbf{e}^{i+1} \, \mathbf{P}'_{i,i+1}) \, \mathbf{I}_{i,i}$$

where $\mathbf{I}_{i,i}$ *is the unit matrix of size* i, $\mathbf{e}^j$ *is the row vector of* $\binom{n}{j}$ *ones.*

Example 1. *Let us suppose a system of objects with a chemical factory in their centre. We consider a situation, where an explosion occurs in the factory and the following fire endanger a near-by forest. Simultaneously, we can expect a deterioration of factory's tubes and an outflow of toxic material into air and a contiguous river. As a consequence, there would be a contaminated drinking water source (aqueduct), which represents a big danger for towns of people, who are dependent on this water source. A chicken farm supplied from the same water source is in danger as well. For the example, there is reasonable to assume the model 1.*



Let us set up the objects into the sequence in the following order:

1. chemical factory
2. forest
3. factory's tubes
4. river
5. city's aqueduct
6. citizens
7. chicken farm

Among all 2^7 system states, several of them never will occur. For example, the state (0001000) represents the situation in which the river will be contaminated without preceding deterioration of any other object. It is clear, that such situation cannot occur in our example. The set of possible states involves the following vectors:

$$(1000000), (1100000), (1010000), (1110000), (1011000), (1111000), (1011100),$$
$$(1111100), (1011110), (1011101), (1111110), (1111101), (1011111), (1111111).$$

Hence, the matrix of transient intensities is of the size 14×14 and the initial probability vector is equal to $\pi = (1, 0, 0, 0, 0, 0, 0)$. The transient intensity matrix $\mathbf{S}$ is in the form

$$\begin{pmatrix}
u & a & b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -c & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & v & d & e & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -f & 0 & f & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & w & g & h & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -i & 0 & i & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & x & j & k & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -l & 0 & 0 & l & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & y & 0 & m & 0 & n & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z & 0 & p & q & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -r & 0 & 0 & r\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -s & 0 & s\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -t & t\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.$$

The transition intensity matrix has to fulfill the condition of $\mathbf{S}\,\mathbf{e} = \mathbf{0}$. This makes $u = -a-b$, $v = -d-e$, $w = -g-h$, $x = -j-k$, $y = -m-n$, $z = -p-q$. There remains 19 unknown parameters in all. These parameters correspond to conditional transient intensities:

$$\begin{array}{lll}
a = i(1 \to 2|1)\ ; & g = i(1 \to 2|1,3,4)\ ; & m = i(1 \to 2|1,3,4,5,6)\ ;\\
b = i(1 \to 3|1)\ ; & h = i(4 \to 5|1,3,4)\ ; & n = i(5 \to 7|1,3,4,5,6)\ ;\\
c = i(1 \to 3|1,2)\ ; & i = i(4 \to 5|1,2,3,4)\ ; & p = i(1 \to 2|1,3,4,5,7)\ ;\\
d = i(1 \to 2|1,3)\ ; & j = i(1 \to 2|1,3,4,5)\ ; & q = i(5 \to 6|1,3,4,5,7)\ ;\\
e = i(3 \to 4|1,2)\ ; & k = i(5 \to 6|1,3,4,5)\ ; & r = i(5 \to 7|1,2,3,4,5,6)\ ;\\
f = i(3 \to 4|1,3)\ ; & l = i(5 \to 6|1,2,3,4,5)\ ; & s = i(5 \to 6|1,2,3,4,5,7)\ ;\\
 & & t = i(1 \to 2|1,3,4,5,6,7)
\end{array}$$

where $i(a \to b|c)$ means the conditional intensity of transition from object a to object b, conditioned by previous affection of object c. In the case, where transitions between objects are independent of the previous path, the whole system can be reduced to 6 unknown parameters a, b, e, h, k, n. These hold

$$a = d = g = j = m = p = t \ , \quad b = c \ , \quad e = f \ , \quad h = i \ , \quad k = l = q = s \ , \quad n = r \ .$$

3. The life time of the system

Let T as *the time of the total deterioration of the system.* It means the length of time, in which the process $X(t)$ will attach the state ω^N. During this time, the pursued events will pass through all objects of the system.

Proposition 3. *In the model 1, the time T to the system deterioration is a random variable, which has phase type probability distribution with initial probability vector $\boldsymbol{\pi} = (\pi_1, \pi_2, \dots, \pi_n, 0, 0, \dots, 0)$ and upper triangular transition intensity matrix*

$$\mathbf{S} = \begin{pmatrix} \mathbf{D}_{1,1} & \mathbf{P}_{1,2} & \mathbf{O}_{1,3} & \cdots & \mathbf{O}_{1,n-1} \\ \mathbf{O}_{2,1} & \mathbf{D}_{2,2} & \mathbf{P}_{2,3} & \cdots & \mathbf{O}_{2,n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{O}_{n-1,1} & \mathbf{O}_{n-1,2} & \mathbf{O}_{n-1,3} & \cdots & \mathbf{D}_{n-1,n-1} \end{pmatrix} .$$

Recall that the phase type distribution is the probability distribution of the random variable W, which represents the time until absorption in a finite irreducible markov process with $(n-1)$ transient states and one absorption state. We shall denote the phase type distribution (PH-distribution) as $\mathrm{PH}(\boldsymbol{\pi}, \mathbf{S})$. The tuple $(\boldsymbol{\pi}, \mathbf{S})$ is usually called the representation of PH-distribution. (see [7], [4]). The cumulative distribution function of PH-distribution $\mathrm{PH}(\boldsymbol{\pi}, \mathbf{S})$ has the form of

$$F(w) = \begin{cases} 1 - \boldsymbol{\pi} \, \exp(\mathbf{S} w) \, \mathbf{e}' & w \geq 0 \ , \\ 0 & w < 0 \ , \end{cases}$$

It is clear that $F(w)$ is continuous if $\sum \pi_i = 1$. The density function of $\mathrm{PH}(\boldsymbol{\pi}, \mathbf{S})$ has the form

$$f(w) = \begin{cases} \boldsymbol{\pi} \, \exp(\mathbf{S} w) \, \mathbf{S}^{0\prime} & w \geq 0, \\ 0 & w < 0 \ , \end{cases}$$

where $\mathbf{S}^0 = \mathbf{S} \, \mathbf{e}'$, $\mathbf{e} = (1, 1, \dots, 1)$. If $\mathbf{S}$ is a regular matrix, it can be shown, that the random variable W has all its moments finite and they can be expressed in the following form

$$E(T^k) = (-1)^k \, k! \, \boldsymbol{\pi} \, \mathbf{S}^{-k} \, \mathbf{e}' \ , \quad k \in \mathbf{N}.$$

Whereas in the model 1, the state ω^N is absorption state, the life time of the system is well-defined. In the model 2 the situation is more complicated. One possibility is to say, that the system is *totally deteriorated* when reaches the state ω^N. In the case, the state ω^N can be assumed as absorbing and the following proposition holds.

Proposition 4. *In the model 2, the time T to the system deterioration is a random variable, which has the phase type probability distribution with initial probability vector $\boldsymbol{\pi} = (1, 0, \dots, 0)$ and transition intensity matrix*

$$\mathbf{S} = \begin{pmatrix} \mathbf{D}_{0,0} & \mathbf{P}_{0,1} & \mathbf{O}_{0,2} & \cdots & \mathbf{O}_{0,n-1} \\ \mathbf{R}_{1,0} & \mathbf{D}_{1,1} & \mathbf{P}_{1,2} & \cdots & \mathbf{O}_{1,n-1} \\ \mathbf{O}_{2,0} & \mathbf{R}_{2,1} & \mathbf{D}_{2,2} & \cdots & \mathbf{O}_{2,n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{O}_{n-1,0} & \mathbf{O}_{n-1,1} & \mathbf{O}_{n-1,2} & \cdots & \mathbf{D}_{n-1,n-1} \end{pmatrix} .$$

Another possibility is to define totally deteriorated system as the system, in which the given state ω^* is affected by disastrous event. The ω^* must not be equal to ω^N. This situation is discussed in the following section.

4. First affect time

Let us consider the ith object, say 'aqueduct'. We are interested in a first passage time probability distribution for this object. Using the knowledge of this distribution, we will be able to predict for example the mean time to attack the object or the probability that the object will not be affected until some given time and others characteristics.

Denote $S_0^i = \{\omega \in \Omega : \omega_i = 0\}$ the set of states of the system, in which the i-th object is not affected. Similarly, let $S_1^i = \{\omega \in \Omega : \omega_i = 1\}$ consists of the states, in which the i-th object is affected. The sets S_0^i and S_1^i are disjoint and $S_0^i = \Omega - S_1^i$ stays.

First, let us consider the model 1. Reordering states in both of these sets in ascending order according to $|\omega|$, we can write the transition intensity matrix in the following block form:

$$\begin{array}{cc} & \begin{array}{cc} S_0^i & S_1^i \end{array} \\ \begin{array}{c} S_0^i \\ S_1^i \end{array} & \begin{pmatrix} A & C \\ O & B \end{pmatrix} \end{array}$$

It can be easily shown that the matrices $\mathbf{A}$ and $\mathbf{B}$ are upper triangular square matrices of the same type as in the proposition 1, $\mathbf{O}$ is the null matrix and $\mathbf{A}$ is a square matrix of size $(2^{n-1}-1) \times (2^{n-1}-1)$, whereas $\mathbf{B}$ is of size $2^{n-1} \times 2^{n-1}$. In the following, let us denote (2^{n-1})-dimensional vector $\boldsymbol{\pi}^i = (\pi_1, \dots, \pi_{i-1}, \pi_{i+1}, \dots, \pi_{n-1}, 0, \dots, 0, \pi_i)$.

Proposition 5. *The first affect time T_i for an object i in the system is the random variable with the phase type probability distribution with representation $(\boldsymbol{\pi}^i, \mathbf{A})$.*

The situation is slightly more complicated in the model 2. After reordering of states we obtain the transition intensity matrix

$$\begin{array}{cc} & \begin{array}{cc} S_0^i & S_1^i \end{array} \\ \begin{array}{c} S_0^i \\ S_1^i \end{array} & \begin{pmatrix} A & C \\ G & B \end{pmatrix} \end{array}$$

where $\mathbf{G}$ is not null matrix.

In the example above, let us consider the 5th object, i.e. aqueduct. The matrix $\mathbf{A}$ has the form of

$$\begin{pmatrix} -(a+b) & a & b & 0 & 0 & 0 \\ 0 & -c & 0 & c & 0 & 0 \\ 0 & 0 & -(d+e) & d & e & 0 \\ 0 & 0 & 0 & -f & 0 & f \\ 0 & 0 & 0 & 0 & -(g+h) & g \\ 0 & 0 & 0 & 0 & 0 & -i \end{pmatrix} .$$

Using the formula for first moment of PH-distribution, we obtain the mean time to attack this object as

$$E(\tau) = -\boldsymbol{\pi}\mathbf{A}^{-1}\mathbf{e}' .$$

and variation

$$\mathrm{var}(\tau) = 2\,\boldsymbol{\pi}\,\mathbf{A}^{-2}\,\mathbf{e}' - (\boldsymbol{\pi}\,\mathbf{A}^{-1}\,\mathbf{e}')^2 .$$

5. Renewal period

Under the assumptions of model 2, the process can be understood as a renewal process. The renewal occurs when the process reaches the state ω^0 and the renewal period covers the time which the process needs to return to the state ω^0 first, after it started in ω^0. In what follows, we try to describe the distribution of renewal period using PH-distribution.

For this purpose, we shall consider the state ω^0 in two manners: first, as a starting state, what means that the initial distribution $\boldsymbol{\pi}^0$ will be assumed to be equal to $(1, 0, \dots, 0)$ and, second, as an absorption state after the process will move to it form another state.

Proposition 6. *In the model 2, the distribution of renewal period can be described by PH-distribution with the representation* $(\boldsymbol{\pi}^0, \mathbf{W})$*, where*

$$\mathbf{W} = \begin{pmatrix} \mathbf{D}_{0,0} & \mathbf{P}_{0,1} & \mathbf{O}_{0,2} & \mathbf{O}_{0,3} & \cdots & \mathbf{0} \\ \mathbf{O}_{1,0} & \mathbf{D}_{1,1} & \mathbf{P}_{1,2} & \mathbf{O}_{1,3} & \cdots & \mathbf{O}_{1,n} \\ \mathbf{O}_{2,0} & \mathbf{R}_{2,1} & \mathbf{D}_{2,2} & \mathbf{P}_{2,3} & \cdots & \mathbf{O}_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{O}_{n,1} & \mathbf{O}_{n,2} & \mathbf{O}_{n,3} & \cdots & \mathbf{D}_{n,n} \end{pmatrix} .$$

6. Conclusion remark

In this paper a Markovian model of disaster propagation was presented. The model provides the probability distribution of the *first affect time*, especially, of the life time of the system or of the lenght of renewal period. Two cases are taken into consideration. In the first one, we consider all objects as nonrepairable. This represents 'one-way' model, in which the only disastrous event is spreading from one object to to another, until it affects all objects in the system. The second model allows repairs of objects. It means that the state of an object in the system can be changed not only from 0 (unaffected) to 1 (affected) as in the first model, but also in the opposite direction from 1 (affected) to 0 (repaired 'like unaffected').

Both of these models can be described by a continuous time Markovian proces, the states of which represent 'global states' of the whole system. The most frequent question in such cases is: how much time we have until affection of some object? What is the time after which we must evacuate people, material and technique? In the presented model, we describe this time as a random variable with a phase type probability distribution. It allows us to compute all its moments and interval estimators.

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