MODELING AND CONTACT ANALYSIS OF CROWNED SPUR GEAR TEETH

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This paper provides a novel method to model lead crowned spur gears. The teeth of circular and involute crowned external spur gears are modeled for the same crowning magnitude. Based on the theory of gearing, mathematical model of tooth generation and meshing are presented. Effect of major performance characteristics of uncrowned spur gear teeth are studied at the pitch point and compared with longitudinally modified spur gear teeth. The results of three dimensional FEM analyses from ANSYS are presented. Contact ellipse patterns and other contact parameters are also studied to investigate the crowning effects.

Keywords: *profile modification, crowning, tooth contact analysis, stress analysis, contact ellipse, tooth generation*

1. Introduction

The main purpose of gear mechanisms is to transmit rotation and torque between axes. The gear wheel is a machine element that has intrigued many engineers because of numerous technological problems arises in a complete mesh cycle. In order to achieve the need for high load carrying capacity with reduced weight of gear drives but with increased strength in gear transmission, design, gear tooth stress analysis, tooth modifications and optimum design of gear drives are becoming major research area.

Gears with involute teeth have widely been used in industry because of the low cost of manufacturing. Transmission error occurs when a traditional non-modified gear drive is operated under assembly errors. Transmission error is the rotation delay between driving and driven gear caused by the disturbances of inevitable random noise factors such as elastic deformation, manufacturing error, alignment error in assembly. It leads to very serious tooth impact at the tooth replacing point, which causes a high level of gear vibration and noise. At the same time, edge contact often happens, which induces a significant concentration of stress at the tooth edge and reduces the life time of a gear drive. Many researchers have proposed modified shapes for traditional gears to localize the bearing contact thereby avoiding edge contact. In the present work, involute spur gear teeth for the selected module, gear ratio, centre distance and number of teeth is taken for analysis. Two more sets of differently crowned involute spur gear teeth are also considered for stress and tooth contact analysis. The procedure followed to create the 3D models of teeth in mesh is described. Proportions of contact ellipses are determined. Their performance behavior is studied by assuming loading at pitch point under static load and frictionless hypothesis. In order to handle critical profile variations, the analyses are done in three dimensions with face contact

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model. Surface Contact Stress (SCS), Root Bending Stress (RBS), and Tooth Deflection (TD) calculations of the pair of spur gears with and without lead crowning are carried out through FEM. Comparison of these parameters are also carried out and interpreted suitably.

2. Literature review

It is necessary to choose a quicker way of evaluation of the performance of gear teeth, regardless of the material of construction and manufacturing process adopted. Performance parameters like tooth bending, surface distress and tooth deflection are the basic modes of fatigue failure of any toothed gearing [1]. Kramberger et al. [2] applied finite element method and boundary element method for analyzing the bending fatigue life of thin-rim spur gear. A computational model for determining the service life of gear teeth flanks with regard to surface pitting was presented by Glodez et al. [3] through which the probable service life period of contacting surfaces are also estimated for surface curvatures and loading that are most commonly encountered in engineering practice. Akata et al. [4] determined geometrical location of HPSTC from the basic gear geometry and compared bending fatigue behavior of the gear teeth by loading them from the HPSTC, was chosen as a rapid way of evaluation of performance of gear teeth produced by different manufacturing methods. Zeping Wei [5] used three dimensional finite element method to conduct surface contact stress and root bending stress calculations of a pair of spur gears with machining errors, assembly errors and tooth modifications.

Traditional non-modified gear drive is operated under assembly errors, which leads to very serious tooth impact at the tooth replacing point, which causes a high level of gear vibration and noise. Also, ideal spur gears with line contact are very sensitive to axial misalignment and manufacturing errors which cause the edge contact of gear drive. Concentration of contact load at one end of a gear tooth produces bending stress concentration and reduces life time of a gear drive. Tooth breakage at one end of a gear associated with misalignment. The light scuff marks towards the end of the teeth also indicate misalignment. The easiest way to avoid the edge contact is to localize the bearing contact by crowning gear tooth surfaces [6]. Further crowning modification is still required to transform the meshing from line contact to point contact. The most common way of crowning is the so-called lead crowning. The lead crowning of spur gear can be considered as the longitudinal deviation from a straight line along the tooth width. This modification prevents excessive loading at the ends due to misalignment, tooth inaccuracies, deflection, or heat treatment. As the load increases, a smooth spreading of the contact occurs until the entire flank is loaded.

There are different forms that can be chosen for the modified tooth profile including linear and parabolic variation. Nowadays, in order to pursue the development of a higher quality of transmission, the former method of using parabolic functions need improvement. The parabolic function demands a shrinking of the tooth face inwards near the top land and fillet, which results in a decrease in bending strength of the tooth fillet. But, for a fourth order polynomial function shrinking near the tooth fillet is smaller and thus the tooth fillet is stronger [7].

An increased magnitude of crowning weakens the pitting durability, and too small crowning decreases the ability to stabilize the bearing contact. Although crowning is a very important manufacturing technique in gearing field, a sufficient analysis for the proper amount of crowning has not been provided [8]. It is preferable to crown the pinion tooth surface than the gear tooth surface since the number of pinion teeth is smaller. The conjugation of crowned pinion tooth surface and an ordinary gear tooth surface should be a subject of special investigation directed at minimization of transmission errors and favourable location of bearing contact [9].

3. Methodology

The major design parameters like center distance, module and number of teeth on pinion [10] are directly taken for analysis and tooth modifications for a given pinion speed, power and gear ratio. This paper represents longitudinal circular crowned gears in which the tooth surfaces as the surface of revolution that is generated by rotation of the involute curve about a fixed axis. Further, an attempt is made to analyze longitudinal involute crowning. In the proposed involute crowning the amount of tooth face shrinking is reduced. Crowning magnitude in both the cases are taken as equal. Crowned and uncrowned tooth surfaces are generated and transferred into commercial finite element analysis software [11] for determining stresses and deflections with proper element mesh. Loading is applied as input torques at the gear centres. Tab. 1 provides the design and operating parameters of the standard spur gear considered for contact analysis and crowning modifications.

Description	**Pinion**	**Wheel**
Material	40Ni2Cr1Mo28 (En24) high strength alloy steel	
Ultimate tensile strength, MPa	1550	
Yield strength, MPa	1300	
Modulus of elasticity, MPa	2.07×10^5	
Density, kg/m^3	7840	
Pitch circle diameter, mm	36	180
Root diameter, mm	31	175
Outside diameter, mm	40	184
Base diameter, mm	33.83	169.15
Number of teeth	18	90
Radius of curvature at pitch point, mm	6.1563	30.782
Fillet radius, mm	0.3module	
Top land thickness, mm	0.4module	
Face width, mm	30	
Addendum, mm	1module	
Dedendum, mm	1.25module	
Poisson's Ratio	0.25	
Pressure angle, deg.	20	

Tab.1: Design and operating parameters of standard spur gears

3.1. Tooth generation

The idea of using the involute is that it generates a straight contact path when turning the gear wheels which is important since a non-straight contact path or line of action leads to vibration at higher rpm. In order to avoid high contact pressures in the outermost regions root relief, tip relief and tip rounding are employed. All these tooth profile modifications towards involute curve can of course, if not properly designed, lead to a non-straight contact path, which causes noise and vibration.

It is more suitable to use 3D modeling, because of tooth height, tooth thickness and tooth face width of a gear tooth in three mutually perpendicular directions are comparable with each other. Further, an elliptical contact area formed when two three-dimensional bodies, each described locally with orthogonal radii of curvature, come into contact. In order to consider the effect of adjacent teeth, three tooth model was taken into account, which can also be used to analyze simultaneous contact of single and double pair teeth. For the selected pitch diameter and number of teeth, tooth profile models have been developed by Pro/ENGINEER and then transferred to ANSYS for static stresses and deflection analysis. The gears are provided with involute profile based on the following geometric Eq. (1), described in parametric form and it is highly flexible in terms of gear size and face width. The innermost cord of involute is used in profile generation.

$$
\begin{aligned}
x &= r_\mathrm{b} \sin\theta - r_\mathrm{b}\,\theta \cos\theta \ , \\
y &= r_\mathrm{b} \cos\theta + r_\mathrm{b}\,\theta \sin\theta \ , \\
z &= 0 \ .
\end{aligned}
\tag{1}
$$

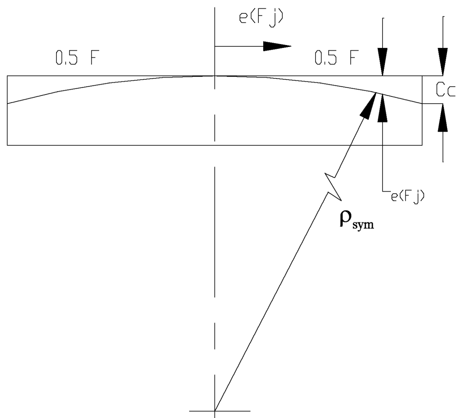

Fig.1: Radius of curvature – circular crowning

Crowning towards the face width is another feature taken for analysis. Crowning is the advancement of end relief. Edge of the end relieved gear results in edge contact again. To avoid such problems the end relief is made in the form of curved surfaces adopting any standard curve profiles. The point contact of tooth surfaces is achieved by the crowning of the pinion and wheel tooth faces in the longitudinal direction. Two types of lead crowning are carried out in which the profile generating curve is an exact involute curve, which is the innermost portion of the involute curve generated by the base circle of respective pinion and wheel. In the first case, lead circular crowning with a magnitude derived from the outermost portion of an arc of involute of wheel is done and the radius of rotation is given accordingly. The magnitude is found in such a way that outermost portion of the arc of the involute is positioned in the top land of the gear teeth giving equal magnitude on both the ends. This will be the only minimum magnitude and used in both of these models and avoids the problem of giving magnitude. The radius of curvature of the symmetrical circular crowning ϱ_sym and amount of modification $e(F_j)$ referring to Figure 1 are calculated using Eqs. (2) and (3) [12] .

$$
\varrho_\mathrm{sym} = \frac{4\,C_\mathrm{c}^2 + F^2}{8\,C_\mathrm{c}} \ , \tag{2}
$$

$$
e(F_j) = \varrho_\mathrm{sym} - \sqrt{\varrho_\mathrm{sym}^2 - F_j^2} \ , \qquad -\frac{F}{2} \le F_j \le \frac{F}{2} \ . \tag{3}
$$

Further, to overcome the effects shrinkage of tooth face and root stress concentration, involute crowning towards face width is attempted. Care is taken because improper crowning which could move the load path from the centre of face width towards the heel or toe regions. Excessive wear caused by this load shift could result in premature surface failure or breakage.

For the given face width, the outermost portion of the involute curve, as shown in Figure 2, is generated by the selected base circle radius of wheel is used as trajectory to sweep the involute profile for the crowning modification of pinion and gear wheel.

$$\text{Total length of the cord} = 2\,\pi^2\,R_\mathrm{b}\ . \tag{4}$$

The outermost portion used for providing crowning for any selected face width is calculated by the following expression obtained using Figure 2.

$$F = \left\{\left[\frac{2\,\pi\,\theta_3\,R_\mathrm{b}}{360} - R_\mathrm{b}\,\tan\alpha - \left(\frac{2\,\pi\,R_\mathrm{b}\,(\theta_3 - 2\,\alpha)}{360} + R_\mathrm{b}\,\tan\alpha\right)\cos 2\alpha\right]^2 + \right.$$
$$\left. + \left[\left(\frac{2\,\pi\,R_\mathrm{b}\,(\theta_3 - 2\,\alpha)}{360} + R_\mathrm{b}\,\tan\alpha\right)\sin 2\alpha\right]^2\right\}^{\frac{1}{2}}\ ,$$
$$\frac{F}{R_\mathrm{b}} = \left\{\left[\frac{2\,\pi\,\theta_3}{360} - \tan\alpha - \left(\frac{2\,\pi\,(\theta_3 - 2\,\alpha)}{360} + \tan\alpha\right)\cos 2\alpha\right]^2 + \right.$$
$$\left. + \left[\left(\frac{2\,\pi\,(\theta_3 - 2\,\alpha)}{360} + \tan\alpha\right)\sin 2\alpha\right]^2\right\}^{\frac{1}{2}}\ , \tag{5}$$

The outermost portion is so positioned as to give minimum crowning. So, the cross section of tooth surfaces of the gear along both the planes is involute. Referring to Figure 2, the geometrically determined magnitude of involute crowning C_c is given in Eq. (6).

$$C_\mathrm{c} = \overline{\mathrm{CI}} = \frac{2\,\pi\,R_\mathrm{b}\,\theta_\mathrm{M}}{360} - \sqrt{\overline{\mathrm{AO}}^2 - \overline{\mathrm{AD}}^2}\ . \tag{6}$$

Referring to Fig. 3, radius of curvature increases continuously, ultimately it flattens and its magnitude at point of contact, ϱ_inv is given in Eq. (7)

$$\varrho_\mathrm{inv} = \overline{\mathrm{EI}} = \frac{2\,\pi\,R_\mathrm{b}\,\theta_\mathrm{M}}{360}\ . \tag{7}$$

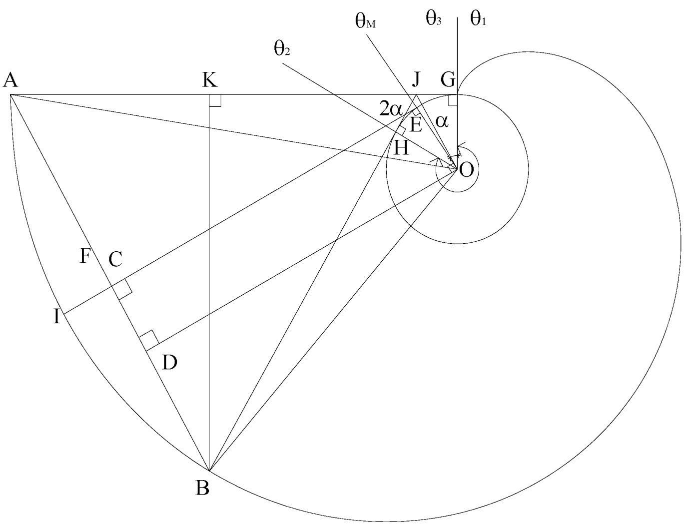

Fig.2: Selection of crowning segment from the involute profile

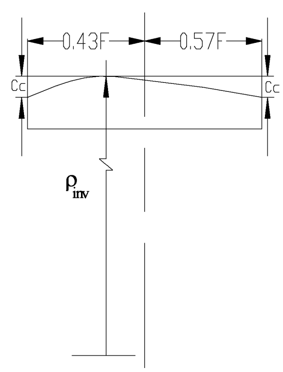

Fig.3: Radius of curvature – involute crowning

The magnitude and radius of curvature at the contact points are then used in tooth contact analysis. Figure 4 presents the solid modeling of assembly models of standard gear, circular crowned and involute crowned gears from Pro/ENGINEER.

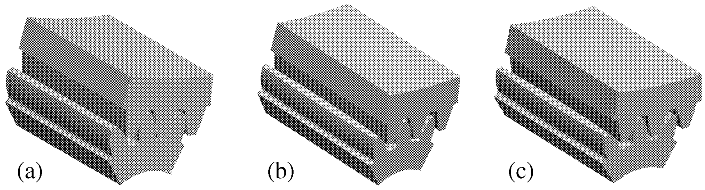

Fig.4: Assembly models: (a) standard gear, (b) circular crowned, (c) involute crowned

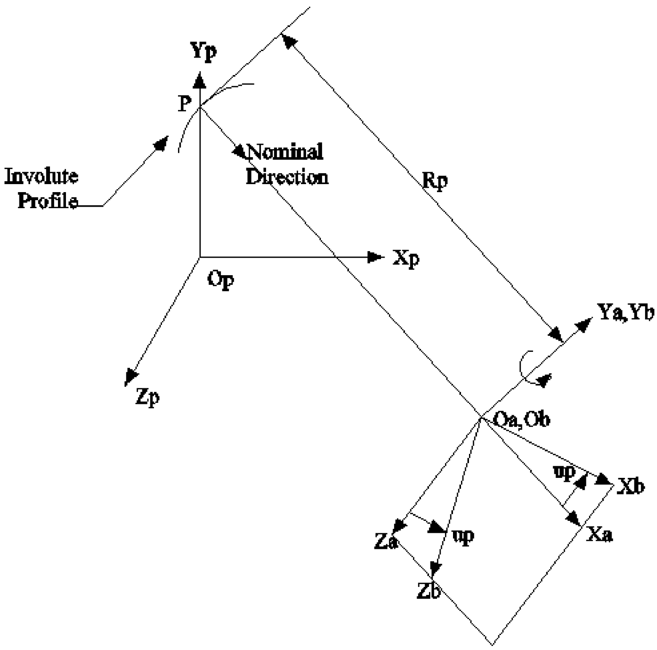

Fig.5: Coordinate system applied for generating crowned tooth surfaces

Based on the theory of gearing, mathematical model of tooth generation and meshing are developed. Tooth surface is represented as the surface of revolution that is generated by rotation of the involute curve about a fixed axis. First, the involute profile is represented in X_{p}-Y_{p} plane in one-parameter form as shown in Figure 5.

$$r_{\mathrm{p}} = r_{\mathrm{p}}(\theta_{\mathrm{p}}) \ .$$

In auxiliary coordinate system S_{a}, Y_{a} and X_{a} axes coincide with the tangential vector and normal vector at pitch point P, respectively. The origin O_{a} is positioned at the distance of R_{p} from P measured along the direction of the normal. The surface of revolution is generated in another auxiliary coordinate system S_{b} by rotating the involute profile about Y_{a} axis. Finally, after coordinate transformation, the crowned pinion tooth surface is represented in Σ_{p} as follows:

$$r_{\mathrm{p}}(u_{\mathrm{p}}, \theta_{\mathrm{p}}, R_{\mathrm{p}}) = M_{\mathrm{pb}}\, M_{\mathrm{ba}}\, M_{\mathrm{ap}}\, r_{\mathrm{p}}(\theta_{\mathrm{p}}) \ . \qquad (8)$$

Here, the radius of revolution R_p controls the amount of crowning. If R_p is given, the pinion tooth surface is represented in two-parameter form where parameters θ_p and u_p are independent.

Similar procedure can be applied for generation of gear tooth surface. The crowned gear tooth surface Σ_g is obtained as

$$r_\mathrm{g}(u_\mathrm{g}, \theta_\mathrm{g}, R_\mathrm{g}) \ .$$

For the second case, the crowned pinion tooth surface Σ_p is represented in S_p as follows:

$$r_\mathrm{p}(u_\mathrm{p}, \theta_\mathrm{p}, u_\mathrm{cp}, \theta_\mathrm{cp}, \varrho_\mathrm{cp}) = M_\mathrm{pb}\, M_\mathrm{ba}\, M_\mathrm{ap}\, r_\mathrm{p}(\theta_\mathrm{p}) \ . \tag{9}$$

Similarly, crowned gear tooth surface Σ_g is obtained as

$$r_\mathrm{g}(u_\mathrm{g}, \theta_\mathrm{g}, u_\mathrm{cg}, \theta_\mathrm{cg}, \varrho_\mathrm{cg}) \ .$$

3.2. Analysis of meshing

The contacting surfaces must be in continuous tangency and this can be obtained if their position vectors and normals coincide at any instant. The tooth normals and their unit normals are represented in a common fixed coordinate system S_f. The contact of interacting surfaces Σ_i are localized and they are in point tangency. The conditions of continuous tangency of surfaces are represented by the Eqs. (10) and (11).

$$r_\mathrm{f}^{(\mathrm{p})}(u_\mathrm{p}, \theta_\mathrm{p}, \phi_\mathrm{p}) - r_\mathrm{f}^{(\mathrm{g})}(u_\mathrm{g}, \theta_\mathrm{g}, \phi_\mathrm{g}) = 0 \tag{10}$$

$$n_\mathrm{f}^{(\mathrm{p})}(u_\mathrm{p}, \theta_\mathrm{p}, \phi_\mathrm{p}) - n_\mathrm{f}^{(\mathrm{g})}(u_\mathrm{g}, \theta_\mathrm{g}, \phi_\mathrm{g}) = 0 \tag{11}$$

where, u_i, θ_i, $(i = \mathrm{p}, \mathrm{g})$ are the surface parameters, ϕ_i are the parameter of motion, $r_\mathrm{f}^{(\mathrm{i})}$ and $n_\mathrm{f}^{(\mathrm{i})}$ are the position vector and the surface unit normal of the contact point on surface Σ_i. Vector Eqs. (10) and (11) yield five independent scalar equations in six unknowns, since $|n_\mathrm{f}^{(\mathrm{p})}| = |n_\mathrm{f}^{(\mathrm{g})}| = 1$. Here

$$f_i(u_\mathrm{p}, \theta_\mathrm{p}, \phi_\mathrm{p}, u_\mathrm{g}, \theta_\mathrm{g}, \phi_\mathrm{g}) = 0 \ , \qquad f_i \in C^1 \ , \qquad i = 1,2,3,4,5 \ . \tag{12}$$

Considering ϕ_p as the known input parameter, Eq. (12) can be solved with functions of $\{u_\mathrm{p}(\phi_\mathrm{p}), \theta_\mathrm{p}(\phi_\mathrm{p}), u_\mathrm{g}(\phi_\mathrm{p}), \theta_\mathrm{g}(\phi_\mathrm{p}), \phi_\mathrm{g}(\phi_\mathrm{p})\}$. The Jacobian of the scalar equations will differ from zero as the precondition of point tangency of meshing surfaces. Therefore,

$$\frac{\mathrm{D}(f_1, f_2, f_3, f_4, f_5)}{\mathrm{D}(u_\mathrm{p}, \theta_\mathrm{p}, u_\mathrm{g}, \theta_\mathrm{g}, \phi_\mathrm{g})} = \begin{vmatrix} \dfrac{\partial f_1}{\partial u_\mathrm{p}} & \dfrac{\partial f_1}{\partial \theta_\mathrm{p}} & \dfrac{\partial f_1}{\partial u_\mathrm{g}} & \dfrac{\partial f_1}{\partial \theta_\mathrm{g}} & \dfrac{\partial f_1}{\partial \phi_\mathrm{g}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dfrac{\partial f_5}{\partial u_\mathrm{p}} & \dfrac{\partial f_5}{\partial \theta_\mathrm{p}} & \dfrac{\partial f_5}{\partial u_\mathrm{g}} & \dfrac{\partial f_5}{\partial \theta_\mathrm{g}} & \dfrac{\partial f_5}{\partial \phi_\mathrm{g}} \end{vmatrix} \neq 0 \ . \tag{13}$$

The computational procedure provides the paths of contact on pinion and gear tooth surfaces and function of transmission errors. Function $\phi_\mathrm{g}(\phi_\mathrm{p})\}$ represents the relation between the angles of gear rotation (the law of motion). Functions $r_\mathrm{p}(u_\mathrm{p}, \theta_\mathrm{p})$, $u_\mathrm{p}(\phi_\mathrm{p})$, $\theta_\mathrm{p}(\phi_\mathrm{p})$

determine the path of contact points on surface Σ_p. Similarly, functions $r_g(u_g, \theta_g)$, $u_g(\phi_p)$, $\theta_g(\phi_p)$ determine the path of contact points on surface Σ_g. The function of transmission errors is represented in Eq. (14)

$$\Delta\phi_g(\phi_p) = \phi_g(\phi_p) - \frac{N_p}{N_g}\,\phi_p \ . \tag{14}$$

Aligned ideal gear drives are in line contact. Due to misalignment, the line contact of meshing surfaces is turned into point contact resulting in shifting of bearing contact and transmission errors. The simulation of meshing and analysis of contact for such drives is a difficult problem since Jacobian becomes equal to zero.

When the path of contact on the gear tooth surface for one cycle of meshing exceeds the dimensions of the tooth, edge contact will occur. It is a curve-to-surface tangency and is represented by the Eqs. (15) and (16)

$$r_f^{(p)}(u_p, \theta_p, \phi_p) = r_f^{(g)}(u_g, \theta_g, \phi_g) \ , \tag{15}$$

$$\frac{\partial r_f^{(p)}}{\partial \theta_1}\, n_f^{(p)} = 0 \ , \tag{16}$$

where $r_f^{(p)}(u_p(\theta_p), \theta_p, \phi_p)$ represents the edge E_p on pinion surface and $\partial r_f^{(p)}/\partial\theta_1$ is the tangent to E_p.

4. Tooth contact analysis

The stresses on or beneath the surface of contact are the major causes of failure. These contact stresses are created when surfaces of two bodies are pressed together by external loads. H. Hertz replaced the complicated gear pair geometry with an equivalent model of two cylinders and obtained only principal stresses on the area of contact. The Hertz formula can be applied to spur gears by considering the contact conditions of each gear to be equivalent to those of cylinders that have the same radius of curvature at the point of contact as the gears have. This is an approximation because the radius of curvature of an involute tooth will change while going across the contact zone [13]. The contact stress, by the Hertz equation is a nominal stress and assumes uniform distribution of load [14]. But, the pitting durability of gear pair depends on the contact stress. With crowned gears this uniform distribution will not be the case. The contact area becomes an ellipse, or in most cases, the centre portion of an ellipse, with highest contact stress at the centre of the face width and progressively lower contact stress towards ends. Instantaneous contact ellipse determines the amount of contact stress. The determination of dimensions and orientation of instantaneous contact ellipse needs the details of (i) principal directions and curvatures of meshing surfaces, and (ii) the elastic approach. Proportions of contact ellipses correspond to the pitch point of the crowned gears are calculated empirically [14, 15]. Radius of curvature of the selected gear drive along the lead for circular crowning from the Eq. (2)is 535.82 mm. The radius of curvature along the lead for involute crowning is geometrically determined using Eq. (7) and the value is 528.99 mm. Also, the crowning magnitude for both the crowning models is taken as equal. The value of the crowning magnitude for a given face width is determined from the Eq. (2) for the circular crowning where as for involute crowning, it is determined geometrically using Eq. (6) and the value of crowning magnitude is 0.21 mm. Geometry factor related to radius of curvature at the pitch point, $B = 0.09746$. Geometry factor related to radius of curvature along the lead of the gears for circular crowning, $A = 1.866\times10^{-3}$. Geometry factor related to radius of curvature along the lead of the gears for involute crowning, $A = 1.8904\times10^{-3}$. Semi contact width of contact cylinder with line contact is given by the Eq. (17) and is computed as 0.1152 mm.

$$b = \sqrt{\frac{\dfrac{4\,P}{\pi\,F}\left[\dfrac{1-\mu_{\mathrm{P}}^2}{E_{\mathrm{P}}}+\dfrac{1-\mu_{\mathrm{G}}^2}{E_{\mathrm{G}}}\right]}{\dfrac{1}{r_{\mathrm{P}}}+\dfrac{1}{r_{\mathrm{G}}}}}\ . \tag{17}$$

The elastic approach for line contact is given by the Eq. (18) and is computed as 0.00823 mm.

$$\delta = \frac{2\,P}{\pi\,F}\left[\frac{1-\mu_{\mathrm{P}}^2}{E_{\mathrm{P}}}\left(\ln\frac{4\,r_{\mathrm{P}}}{b}-\frac{1}{2}\right)+\frac{1-\mu_{\mathrm{G}}^2}{E_{\mathrm{G}}}\left(\ln\frac{4\,r_{\mathrm{G}}}{b}-\frac{1}{2}\right)\right]\ . \tag{18}$$

Maximum pressure (Hertz stress)is given by the Eq. (19) and is computed as $1238\,\mathrm{N/mm^2}$.

$$p = \frac{2\,P}{\pi\,b} = \sqrt{\frac{P\,E_{\mathrm{c}}}{\pi\,R_{\mathrm{c}}}}\ , \tag{19}$$

effective curvature

$$\frac{1}{R_{\mathrm{c}}} = \frac{1}{R_{\mathrm{P}}}+\frac{1}{R_{\mathrm{G}}} \tag{20}$$

and contact modulus

$$\frac{1}{E_{\mathrm{c}}} = \frac{1-\mu_{\mathrm{P}}^2}{E_{\mathrm{P}}}+\frac{1-\mu_{\mathrm{G}}^2}{E_{\mathrm{G}}}\ . \tag{21}$$

4.1. Contact stress-elliptical contact

An elliptical contact area is formed when two spheres of different radius is in contact. With crowned gears contact area contracts towards the centre of face width and becomes an ellipse. The highest contact stress occurs at the centre of the face width and progressively lower contact stress towards the ends. At the point of contact, the contact spheres are taken as toroids. Hertz equation is used to calculate deflection and stress between two elastic toroids. The angular orientation between the axes of pinion and gear is assumed to be zero. Dimensions of equivalent sphere and toroid and other contact parameters are given in Eqs. (22) to (28).

Equivalent sphere in contact with a plane

$$R_{\mathrm{c}} = \sqrt{R_{\mathrm{a}}\,R_{\mathrm{b}}}\ .$$

Equivalent toroids in contact with a plane is given as

$$R_{\mathrm{a}} = \frac{1}{(A+B)-(B-A)} \qquad \text{and} \qquad R_{\mathrm{b}} = \frac{1}{(A+B)+(B-A)}\ ,$$

where

$$A = \frac{1}{2}\left(\frac{1}{R'_{\mathrm{P}}}+\frac{1}{R'_{\mathrm{G}}}\right) \qquad \text{and} \qquad B = \frac{1}{2}\left(\frac{1}{R_{\mathrm{P}}}+\frac{1}{R_{\mathrm{G}}}\right)\ .$$

Eccentricity of contact ellipse

$$e^2 = 1 - \left(\frac{b}{a}\right)^2 \cong 1 - \left(\frac{R_\mathrm{b}}{R_\mathrm{a}}\right)^{\frac{4}{3}} . \tag{22}$$

Equivalent radius of contact

$$c = \sqrt{a\,b} = \left(\frac{3\,P\,R_\mathrm{c}}{4\,E_\mathrm{c}}\right)^{\frac{1}{3}} F_1 . \tag{23}$$

Major and minor contact radii

$$a = c\,(1 - e^2)^{-\frac{1}{4}} , \qquad b = c\,(1 - e^2)^{\frac{1}{4}} . \tag{24}$$

Maximum pressure

$$p = \frac{3\,P}{2\,\pi\,c^2} = \frac{3\,P}{2\,\pi\,a\,b} . \tag{25}$$

Normal displacement

$$\delta = \frac{c^2}{R_\mathrm{c}}\,\frac{F_2}{F_1^2} = \frac{a\,b}{R_\mathrm{c}}\,\frac{F_2}{F_1^2} . \tag{26}$$

Normal stiffness

$$k = \frac{2\,E_\mathrm{c}\,c}{F_1\,F_2} . \tag{27}$$

$$F_1 \cong 1 - \left[\left(\frac{R_\mathrm{a}}{R_\mathrm{b}}\right)^{0.0602} - 1\right]^{1.456} , \qquad F_2 \cong 1 - \left[\left(\frac{R_\mathrm{a}}{R_\mathrm{b}}\right)^{0.0684} - 1\right]^{1.531} . \tag{28}$$

Considering same formulae as that of circular crowned gears, replacing radius of curvature of crown along the lead, the contact stress (involute crowned) and other contact parameters are found and tabulated. The dimensions of contact ellipses, induced contact stresses and contact parameters are presented in Tab. 2.

Contact parameters	**Contact type**		
	Line contact	**Elliptical contact**	
		Circular crowned	**Involute crowned**
Semi-contact width or contact radius, mm	0.1152	semi-major axis $a = 3.4225$ semi-minor axis $b = 0.1152$	semi-major axis $a = 3.4$ semi-minor axis $b = 0.1152$
Contact area, mm^2	6.912	1.239	1.23
Max. contact pressure (Hertz stress), $\mathrm{N/mm}^2$	1238	7792	7851
Approaches of centres (Normal displacement), mm	0.0073	0.0134	0.0135
Normal stiffness, N/μm	887.794	311.407	314.843

Tab.2: Contact parameters of crowned spur gear teeth

4.2. Finite element analysis

The root bending stress, contact compressive stress and gear tooth deflection are calculated with load application at the pitch points location are presented in Table 1. The most powerful method of determining accurate stress and deflection information is the finite element method. Gear geometry is a finite element replica of the simple cantilever beam model according to Lewis beam strength equation where one tooth is considered as a base and the root is fixed. The loads acting on the mating gears will act along the pressure line, which can be resolved into both tangential and radial components using the pressure angle. The radial component of force is neglected and the effect of only tangential component is considered as uniformly distributed load at the line of contact. In the case of crowned gears, entire load is assumed to take place at the point of contact. Initially, it is assumed that at any point of time only one pair of teeth is in contact and takes the total load. The gear tooth contact stresses, tooth bending stresses and gear tooth deflection of single tooth contact of spur gear are analyzed using FEM. A thick full rim with three teeth FE model is used for stress analysis. It is not recommended to have a fine mesh everywhere in the model, in order to reduce the computational requirements. The finer mesh size is selected to ensure line contact and uniform distribution of load over flank. The element is of the type SOLID45 eight-noded hexahedral is chosen. SOLID45 is used for the 3-D modeling of solid structures. The element is defined by eight nodes having three degrees of freedom at each node, viz. translations in the nodal x, y, and z directions. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities [16]. It has compatible displacement behaviour and well suited to model curved boundaries. The material is assumed to be isotropic and homogeneous. The type of contact was surface to surface. The contact pair consists of a target surface and a contact surface. The contact surface moves into the target surface. The target surface was chosen in the gear tooth and meshed by TARGE170, 3D target segment while the contact surface was chosen in the pinion tooth with CONTA174, 3D, 8-node surface-to-surface contact element. The contact element is defined as a connection between a node of a contact body and a target segment of a target body. Areas around the contacting surfaces have been meshed with larger density of finite elements mesh for results accuracy.

The wheel nodes placed on inner rim radius have been constrained in global Cartesian coordinate system in all directions. The movements in directions of both axes have been disabled. The pinion nodes placed on inner rim radius have been constrained. Rotation of the pinion nodes placed on inner rim radius around the centre of the global cylindrical coordinate system has been enabled i.e. the rotation about Z axis enabled. The torque is applied at Z axis in clockwise direction. Nodes at the side portions and the bottom portion of rim of the gear are fixed during crowned gear analysis to avoid misalignment and to localize the load distribution

The maximum principal stress at the root on the tensile side of tooth is used for evaluating the tooth bending strength of a gear. The Von Mises stress at the critical contact points was found out. Theoretical values of these stresses are compared with the FEM results and agree well. Further gear tooth deflection under load is of primary interest for proper operation of a gear set. Tooth tip deflection is also considered for analysis. Also, by considering the fact that the load is usually shared between two tooth pairs when the contact is away from the pitch point, the compression stress at the pitch line is used as the critical contact stress [13].

Tab. 3 presents the values of these performance parameters of the standard involute spur gear, circular crowned and involute crowned spur gears set at the pitch point by FEM analysis. Figures 6–9 represent the contact stress pattern, root bending stress pattern of standard, circular crowned and involute crowned gears through FEM using ANSYS.

Tooth modification	Contact stress MPa	Tooth bending stress at root, MPa	Gear tooth deflection mm
Non-modified	1021	393.38	0.0166
Circular crowned	6078	1025	0.0572
Involute crowned	5947	946	0.0577

Tab.3: Value of performance parameters of standard spur gear teeth by FEM

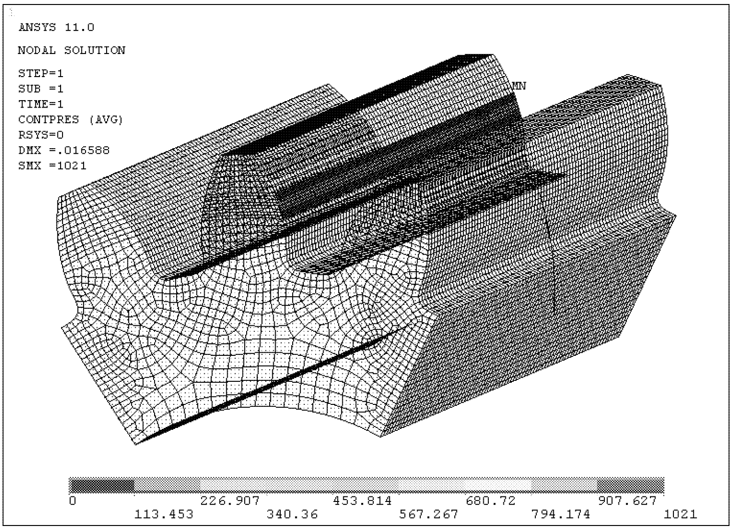

Fig.6: Contact stress pattern at the contact area of standard spur gear

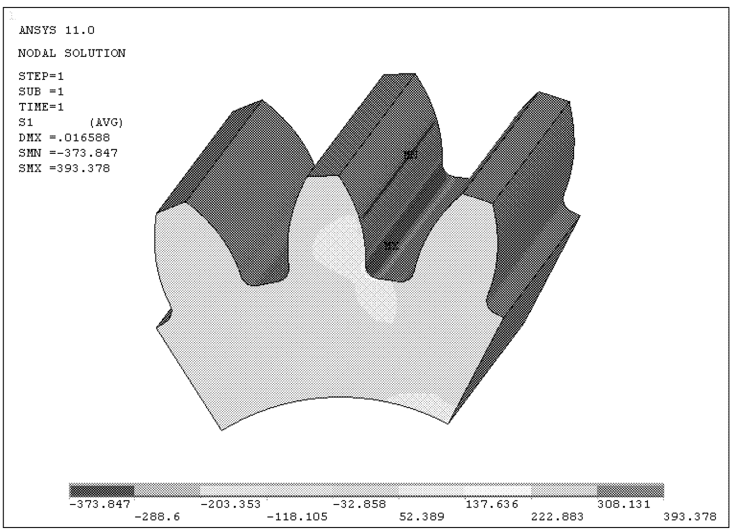

Fig.7: Root bending stress of standard spur gear

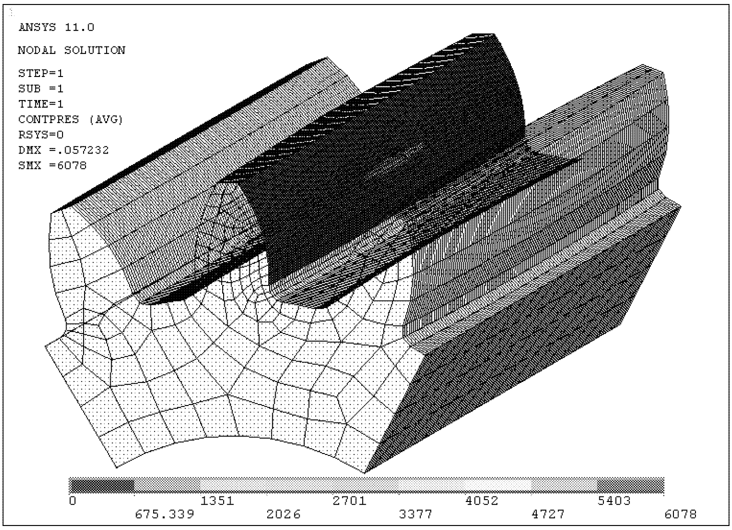

Fig.8: Contact stress pattern of circular crowned gears

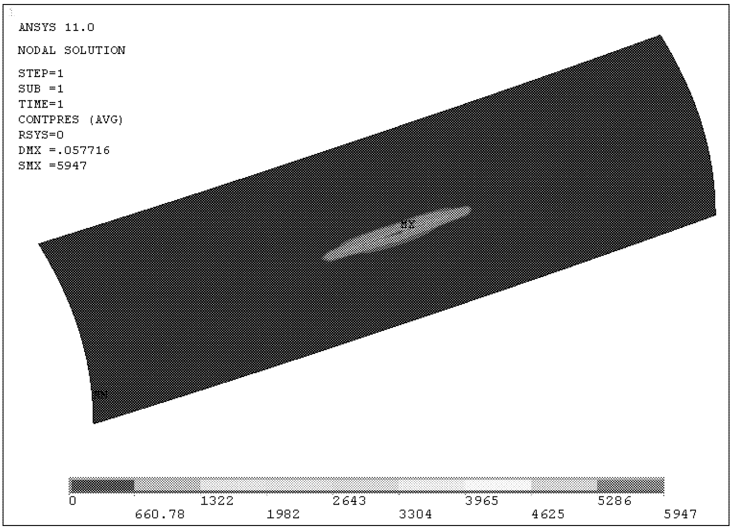

Fig.9: Contact stress pattern of involute crowned gears

5. Conclusions

Amount of crowning can be determined from the given face width or vice versa and it can be verified analytically and geometrically. Effect of crown radius in crowning is studied and reveals that the area of contact of involute crowned gears is slightly lower than the circular crowned gears which show the existence of more point contact compared to circular crowned gears. Also, the area of contact and contact pattern depend on the radius of curvature at the point of interest and profile of curves used for lead crowning generation. The proposed method that the portion of involute curve defining the crowning magnitude can be applied to other types of crowning modifications done with standard curve profiles. The crowned gears with elliptical contact are approximated as an equivalent sphere in contact for contact analysis. It is evident from the results that the transmission error of spur gear which has large changes in mesh stiffness can be reduced by applying the proposed crowning modification.

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