FURTHER APPLICATION OF PARAMETRIC ANTI-RESONANCE

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This contribution supplements the series of work oriented on active reduction of unacceptable vibrations by means of parametric excitation fulfilling certain conditions. Paper deals with the suppression of undesirable vibration using special parametric excitation fulfilling certain conditions. It is shown that this phenomenon can be used not only for suppressing the self-excited vibrations or parametric resonances arising outside the region of main resonances in the vicinity of eigenfrequencies as are subharmonic resonances.

Keywords: *parametric excitation, subharmonic resonance, anti-resonance effect, vibration quenching, time history, Poincare mapping, beats*

1. Introduction

In several publications (see e.g. [2–15]) it was shown that the parametric excitation in mechanical systems doesn't need always result in dangerous vibrations due to the stability loss and initiation of parametric resonances as it is commonly stated in publications dealing with parametrically excited systems, but it can also be used for suppressing dangerous vibrations not only externally excited but also for self-excited vibrations. In the recently published paper [15] this phenomenon (parametric anti- resonance) was further analyzed and his practical application was shown on the case where parametric resonance of the first kind can be fully suppressed by an additional parametric excitation fulfilling certain conditions. In this paper it is also mentioned that using appropriate additional parametric excitation at certain conditions the subharmonic resonance vibrations could be suppressed.

This contribution deals with the suppression problem of the subharmonic vibration link up paper [15] and the whole set of publications using this anti-resonance phenomenon for limiting or even for full suppressing undesirables vibrations. This active means is characterized by additional parametric excitation fulfilling certain conditions. For example, if additional parametric excitation is due to the periodic variation of the elastic element stiffness, then the frequency of this stiffness variation is equal to the difference of two eigenfrequencies (see Appendix in [15]). This phenomenon can be used not only for suppressing self-excited vibrations but also for suppressing parametric resonances. A very good survey on parametrically excited systems inclusive the parametric anti-resonance is in [16].

The stability analysis of externally excited vibrations leads to differential equations with periodic variable coefficients and so to the certain instability intervals, which leads to initiation of nonlinear resonances as e.g. subharmonic resonance. These resonances, which have not been considered in the design of certain equipment, can be dangerous for it's save

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run. In this contribution the application of the mentioned phenomenon for reducing such a subharmonic resonance will be presented.

2. Basic analysis

Let us consider a system with n degrees of freedom excited by external harmonic force. Let us suppose that coefficients of expressions in differential equations of motion representing damping and nonlinearities are small. So, transforming the differential equations of motion into the quasi-normal form, the following equations are obtained:

$$\ddot{y}_s + \Omega_s^2\, y_s = a_s\, \cos\omega t + \varepsilon\, [F_s(\dot{y}_1, \dots, \dot{y}_n) + \Phi_s(y_1, \dots, y_n)]\ , \quad (s = 1, 2, \dots, n)\ , \tag{1}$$

where Ω_s are the eigenfrequencies of the abbreviated system (for $\varepsilon = 0$), ε is a small parameter, the functions F_s express the damping and Ψ_s the system nonlinearities. The steady solution outside the main resonances in the first approximation (for $\varepsilon = 0$) reads:

$$y_{s0} = \frac{a_s}{\Omega_s^2 - \omega^2}\, \cos\omega t\ . \tag{2}$$

The stability of this solution can be analyzed when inserting in equations (1)

$$y_s = y_{s0} + x_s\ . \tag{3}$$

The linearized differential equations of the disturbed motion have the form:

$$\ddot{x}_s + \Omega_s^2\, x_s + \varepsilon \sum_{k=1}^{n} (\Theta_{sk}\, \dot{x}_k + Q_{sk}\, x_k) = 0\ , \quad (s = 1, 2, \dots, n)\ , \tag{4}$$

where Θ_{sk}, Q_{sk} are periodic functions with frequency ω. Take a notice that for $\omega = 2\,\Omega_1$ the solution of the differential equations of the disturbed motion will be unstable. It means that for the system governed by equations (1) subharmonic resonance occurs. When adding the additional parametric excitation with frequency $\Omega_k - \Omega_1$ to the original system governed by equations (1), where Ω_k is the eigenfrequency of sufficiently damped vibration mode, the subharmonic resonance can be suppressed. For simplicity let us show the effect of the additional parametric excitation on the system with two degrees of freedom.

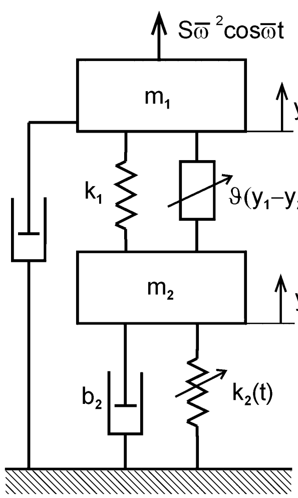

Fig.1: Model of 2 DOF system with external and parametric excitations

Let us consider a Double Degree of Freedom (DDOF) system (see Fig. 1), where mass m_1 is harmonically excited and the amplitude of excitation $F_0 = S\,\bar{\omega}^2$ depends on quadrate of excitation frequency $\bar{\omega}$ (in order to increase the subharmonic resonance excitation possibility). This mass is elastically mounted by a nonlinear spring having unsymmetrical characteristics $\vartheta\,(y_1 - y_2)^2$ to mass m_2, which is elastically mounted, by another spring having stiffness k_2. The additional parametric excitation can be realized by periodic variation of the stiffness $k_2(t) = k_{20}\,(1 + \alpha\,\cos\nu t)$. Denoting masses deflections as y_1, y_2 then the system is governed by the following equations:

$$\begin{aligned} &m_1\,\ddot{y}_1 + b_1\,\dot{y}_1 + k_1\,(y_1 - y_2) + \vartheta\,(y_1 - y_2)^2 = S\,\bar{\omega}^2 \cos\bar{\omega}t\ , \\ &m_2\,\ddot{y}_2 + b_2\,\dot{y}_2 + k_{20}\,(1 + \alpha\,\cos\nu t)\,y_2 - k_1\,(y_1 - y_2) - \vartheta\,(y_1 - y_2)^2 = 0\ . \end{aligned} \tag{5}$$

S is the static moment of the exciting unbalance, α indicates the depth of the parametric modulation of the stiffness.

After rearrangement and time transformation $\omega_1\,t = \tau$, $\omega_1 = \sqrt{k_1/m_1}$ equations (5) get the form:

$$\begin{aligned} &y_1'' + y_1 - y_2 + \varepsilon\left[-\alpha_1\,\omega^2 \cos\omega\tau + \kappa_1\,y_1' + \gamma\,(y_1 - y_2)^2\right] = 0\ , \\ &y_2'' - M\,(y_1 - y_2) + q^2\,y_2 + \varepsilon\left[\kappa_2\,y_2' + \alpha_2\,y_2\,\cos\eta\tau - \gamma\,(y_1 - y_2)^2\right] = 0\ , \end{aligned} \tag{6}$$

where ε is small parameter and

$$\begin{gathered} M = \frac{m_1}{m_2}\ ,\quad \omega_2^2 = \frac{k_{20}}{m_2}\ ,\quad q^2 = \frac{\omega_2^2}{\omega_1^2}\ ,\quad \omega = \frac{\bar{\omega}}{\omega_1}\ ,\quad \eta = \frac{\nu}{\omega_1}\ ,\quad \varepsilon\,\gamma = \frac{\vartheta}{k_1}\ , \\ \varepsilon\,\kappa_1 = \frac{b_1}{m_1\,\omega_1}\ ,\quad \varepsilon\,\kappa_2 = \frac{b_2}{m_2\,\omega_1}\ ,\quad \varepsilon\,\alpha_1 = \frac{S}{m_1}\ ,\quad \varepsilon\,\alpha_2 = q^2\,\alpha\ . \end{gathered} \tag{6a}$$

For $\varepsilon = 0$ the abbreviated linear system related with equations (6) is obtained:

$$\begin{aligned} &y_1'' + y_1 - y_2 = 0\ , \\ &y_2'' - M\,(y_1 - y_2) + q^2\,y_2 = 0\ . \end{aligned} \tag{7}$$

The characteristic determinant

$$D = \begin{vmatrix} 1 - \Omega^2 & -1 \\ -M & M + q^2 - \Omega^2 \end{vmatrix} = \Omega^4 - (M + q^2 + 1)\,\Omega^2 + q^2 \tag{8}$$

provides the eigenfrequencies:

$$\Omega_{1,2}^2 = \frac{1}{2}\,(M + q^2 + 1) \pm \sqrt{\frac{(M + q^2 + 1)^2}{4} - q^2}\ . \tag{9}$$

The numerical results can be also be expressed by quasi-normal coordinates

$$y_1 = v_1 + v_2\ ,\qquad y_2 = a_1\,v_1 + a_2\,v_2\ , \tag{10}$$

i.e. using expressions:

$$v_1 = \frac{1}{a_1 - a_2}\,(y_2 - a_2\,y_1)\ ,\qquad v_2 = \frac{1}{a_1 - a_2}\,(a_1\,y_1 - y_2)\ , \tag{11}$$

where

$$a_{1,2} = -\frac{1}{2}\,(M + q^2 - 1) \pm \left[\frac{1}{4}\,(M + q^2 - 1)^2 + 1\right]^{1/2}\ . \tag{11a}$$

For the case of parameters $M = q = 1$ the corresponding modal quantities are:

$$\Omega_1 = 0.618\ , \quad \Omega_2 = 1.618\ , \qquad a_1 = 0.618\ , \quad a_2 = -1.618\ . \tag{12}$$

3. Solution of selected examples

The theoretical analysis of dynamic system with external and parametric excitation gives general qualitative conclusion, but does not provide answer concerning the quantitative behavior of such systems as: How strong does the auxiliary parametric excitation have to be in order to suppress the subharmonic oscillations, either completely or only to a prescribed rate; what type is the resulting oscillation when both excitation forces act on system, how precisely should be maintained the ratio of external and parametric frequencies for quenching of subharmonic vibration; how the small mistuning of mentioned frequencies influences the degree of subharmonic vibrations quenching, etc. These answers can be gained by numerical experiments, i.e. by numerical solution of differential equations (5) or (6) for given parameters.

4. Examples

Submitted theory of suppressing subharmonic resonance by means additional parametric excitation let us promote by numerical solution of two masses system from Fig. 1 described by equations (6) with the numerical values given at the end of chapter 2: $M = q = 1$, $\Omega_1 = 0.618$, $\Omega_2 = 1.618$ and for further parameters

$$\eta = \Omega_2 - \Omega_1 = 1\ , \quad \varepsilon\,\kappa_2 = 0.05\ , \quad \varepsilon\,\kappa_2 = 0.05\ , \quad \varepsilon\,\alpha_1 = 1\ , \quad \omega = 2\,\Omega_1 = 1.236\ .$$

The various levels of auxiliary parametric excitation are: $\varepsilon\,\alpha_2 = 0; 0.3; 0.6; 0.9; 1$.

Time history of motion $y_1(\tau)$ of upper mass m_1 is plotted in Fig. 2 together with the course of excitation force $F(t) = \alpha_1\,\omega^2\,\cos\omega\tau$, avoiding parametric excitation, i.e. $\alpha_2 = 0$. In order to eliminate the influence of transient free oscillations due to the initial conditions in $\tau = 0$, the motion $y_1(\tau)$ is in Fig. 2 recorded after sufficiently distant time from origin i.e. in dimensionless time interval $\tau \in (1900, 2000)$. The pure subharmonic course of vibration is obvious from the twice longer period of motion $y_1(\tau)$ against period of external excitation $\varepsilon\,\alpha_1\,\omega^2\,\cos\omega\tau$.

Further Fig. 3 shows the course of motion $y_2(\tau)$ of the bottom mass m_2 in the same time interval $\tau \in (1900, 2000)$. Ratio of frequency of vibration y_2 to the frequency of excitation

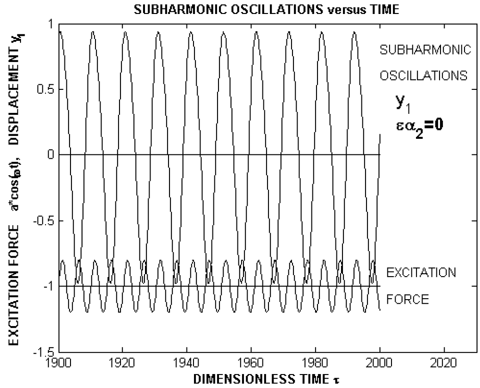

Fig.2

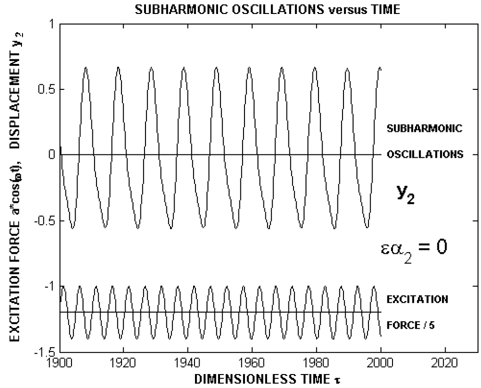

Fig.3

force is again 1:2. Amplitude of mass m_2 is approx. 0.6 amplitude of mass m_1, what corresponds to the mode of vibration of investigated DDOF abbreviated system in the first resonance.

If the stiffness k_2 of the bottom spring changes periodically with time $k_2(t) = k_{20}\,(1 + \alpha\,\cos\omega t)$, then both motions $y_1(\tau)$ and $y_2(\tau)$ change their courses. These motions are shown in Fig. 4 and 5 for comparatively low stiffness modulation $\varepsilon\,\alpha_2 = 0.3$ and again after sufficient long time $\tau = 1900$ from origin $\tau = 0$.

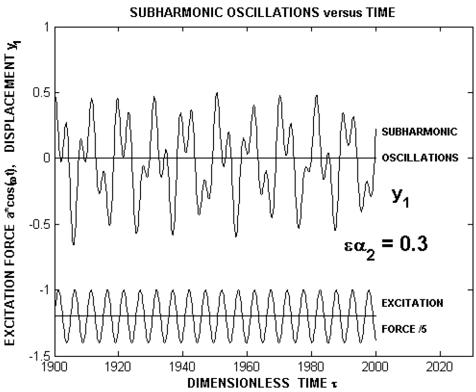

Fig.4

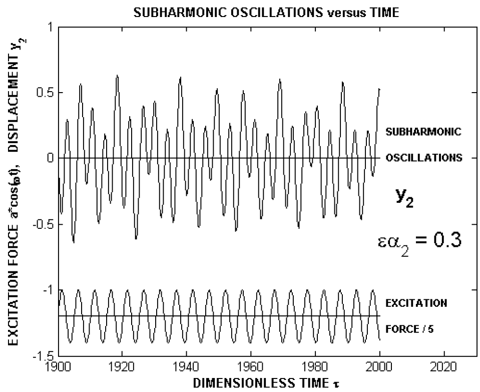

Fig.5

Decrease of component amplitude of y_1 of upper mass m_1 to approximately 50 % of subharmonic oscillations initial amplitude and marked change of oscillation form is shown in Fig. 4. A higher harmonic component with frequency close to the second eigenfrequency Ω_2 of the abbreviated system ($\varepsilon = 0$) appears in the time history $y_1(\tau)$.

Motion y_2 of the bottom mass m_2 has the similar course as well, see Fig. 5. Maximum displacements of y_2 did not change by addition of parametric excitation $\varepsilon\,\alpha_2 = 0.3$ against the case $\varepsilon\,\alpha_2 = 0$, but due to the fluctuation the average effective value drops. A motion component corresponding to the second eigenfrequency Ω_2 appears in this motion, too.

Further increase of level of auxiliary parametric excitation to $\varepsilon\,\alpha_2 = 0.6$ does not bring, with exception of small lowering, any essential change in the course of subharmonic vibrations $y_1(\tau)$ and $y_2(\tau)$, as shown in Fig. 6 and 7.

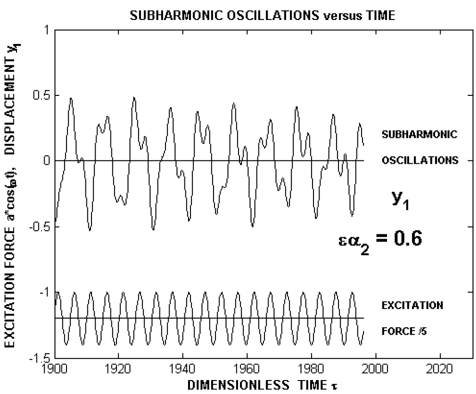

Fig.6

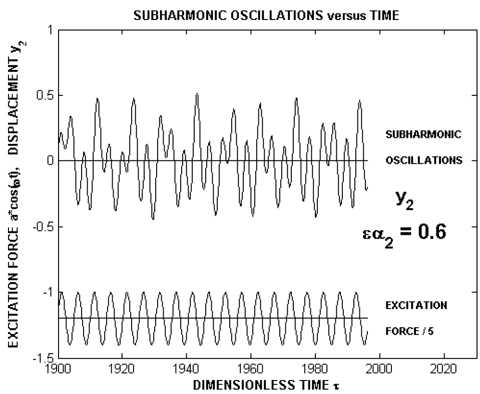

Fig.7

Another property of these records should be noticed. It concern the higher frequency component. The level of this component noticeable increases in both records during the 20 periods of excitation force in the short time $\tau \in (1900, 2000)$.

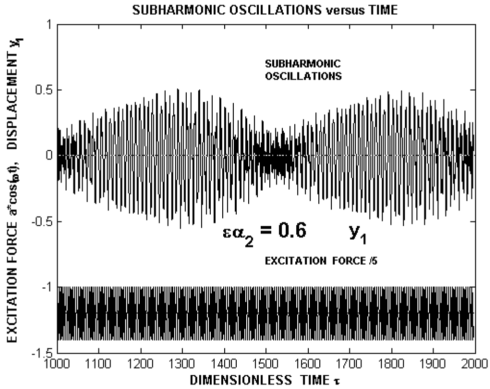

Fig.8

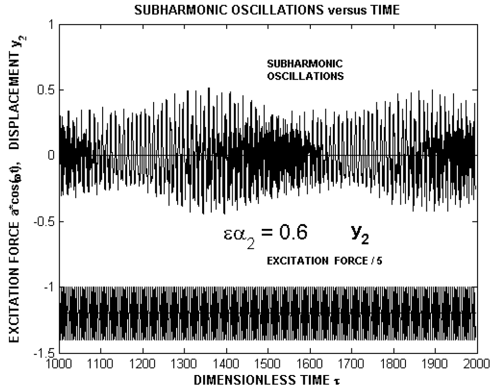

Fig.9

In order to explain this phenomenon, the 10 times longer course of vibrations $y_1(\tau)$ and $y_2(\tau)$ in $\tau \in (1000, 2000)$ was recorded for $\varepsilon\,\alpha_2 = 0.6$ and the results are shown in Fig. 8 for the upper mass m_1 and in Fig. 9 for the bottom mass m_2. The longer records have shown that the initial conditions do not influence the investigated motion, but this one transforms and settled on beat motion type, where the levels of different components periodically fluctuate.

Period of beats for given $\varepsilon\,\alpha_2 = 0.6$ is $T_{0.6} \cong 500$. Energy of vibrations flows during these beats from the subharmonic motion into vibrations with higher frequency and back.

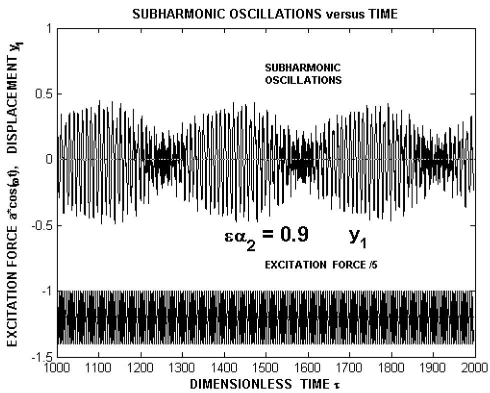

Fig.10

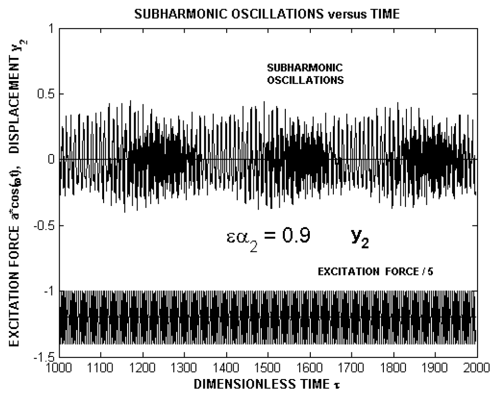

Fig.11

At further increase of parametric excitation level to $\varepsilon\,\alpha_2 = 0.9$, the maximum of amplitudes decreases a little bit, approx. on the 45 % of initial value at $\varepsilon\,\alpha_2 = 0$, the beat form of oscillation was preserved and namely with the shorter period $T_{0.9} \cong 330$, as it can be seen from Fig. 10 and Fig. 11.

Responses on the additional parametric excitation with the amplitude $\varepsilon\,\alpha_2 = 1$ are shown in Fig. 12 and 13. Small increase of $\varepsilon\,\alpha_2$ manifests itself by a moderate decrease of maximum

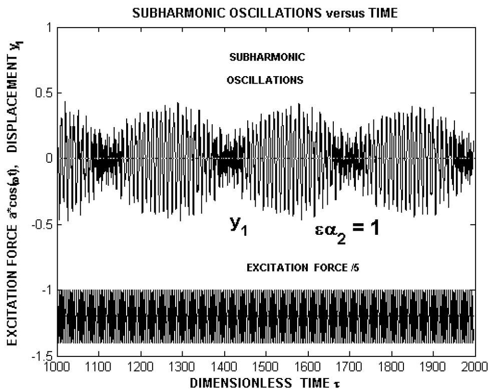

Fig.12

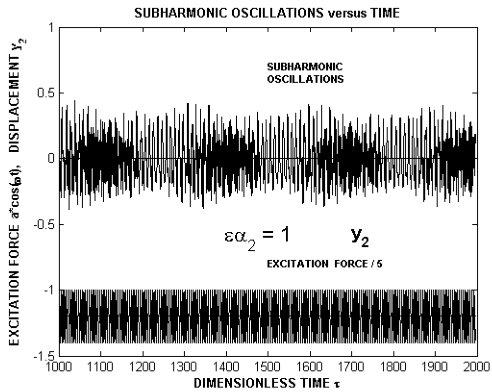

Fig.13

vibrations up to 40 % of initial amplitude value, but in the nodes of beats these values sink to approx. 20 %.

Period of beats is reduced again to $T_1 = 290$. Motions of both masses $y_1(\tau)$ and $y_2(\tau)$ are not pure beats, but they contain apart from two main harmonic components also further components, which are both periodical and chaotic.

Existence of chaotic properties of these external and parametric excited vibrations is evident from the records of Poincaré mappings shown in the following figures.

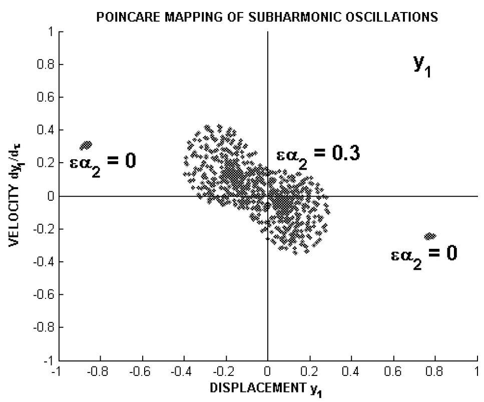

Fig.14

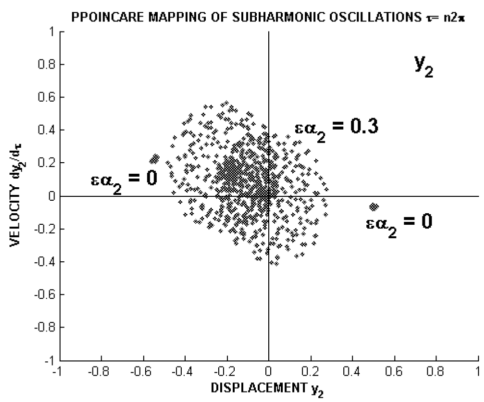

Fig.15

Points in Fig. 14 determine the dynamic state $(y_1, \dot{y}_1)$ of upper mass m_1 at the instants of time, when the excitation force reaches its prescribed state. Therefore two narrow clusters of points are recorded at pure subharmonic oscillation at $\varepsilon\,\alpha_2 = 0$. The central field of approx. 700 points corresponds to the motion of same system influenced by an auxiliary parametric excitation with modulation level $\varepsilon\,\alpha_2 = 0.3$. It is evident that due to the application of parametric excitation the subharmonic vibrations y_1 of first mass m_1 is reduced more than to a half.

Maximum amplitudes of bottom mass m_2 vibrations is not reduced to the same degree, but their mean value reaches this rate of reduction, as it follows from Fig. 15.

The greater reduction of maximum and mean value of subharmonic oscillations is achieved by increasing the auxiliary parametric excitation on $\varepsilon\,\alpha_2 = 0.6$. Fig. 16 shows

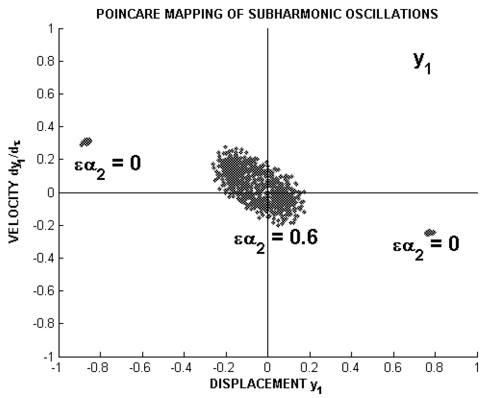

Fig.16

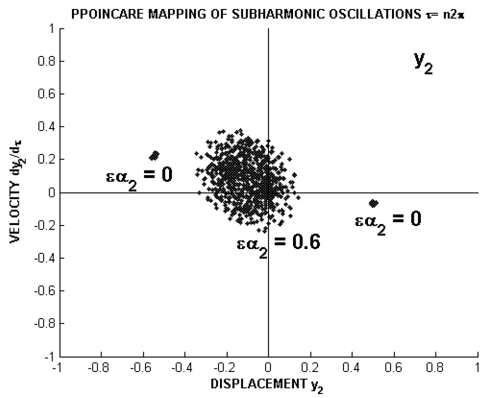

Fig.17

that maximal amplitude y_1 of upper mass m_1 is reduced on approx. 28 %, while the mean value on much lower level.

Reduction of mass m_2 vibration is not so high, maximum amplitude y_2 is 47 % of the amplitude without parametric excitation $\varepsilon\,\alpha_2 = 0$ as seen on Fig. 17.

If the modulation of spring stiffness k_2 is increased to $\varepsilon\,\alpha_2 = 1$, the reduction of vibration $y_1(\tau)$ drops to 22 % and reduction of vibration $y_2(\tau)$ to 35 %, as seen from the records of Poincaré mappings in Fig. 18 and Fig. 19.

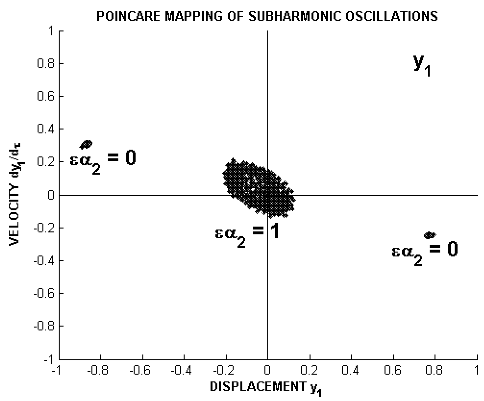

Fig.18

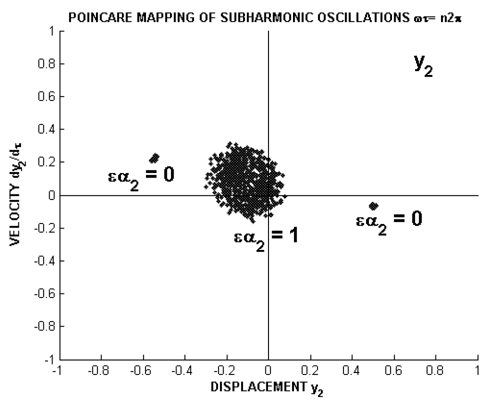

Fig.19

5. Conclusion

Harmonic excitation of a two masses system with a nonlinear spring characteristic, which contains a quadratic unsymmetrical part, in the range of subharmonic resonance generates highly dangerous subharmonic vibrations.

Theoretical and numerical analyses show the very positive effect of auxiliary additional parametric excitation to suppress undesirable subharmonic vibrations to a large extent. Selected examples demonstrate, that the additional parametric excitation besides the suppressing of subharmonic vibrations changes considerably the type of vibrations from the periodic oscillation to a beat oscillation with a large amount of chaotic components.

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References

[1] Tondl A.: On the Interaction between Self-Excited and Parametric Vibrations, Monographs and Memoranda of the National Research Institute for Machine Design, No. 25, Prague 1978

[2] Tondl A.: Metoda k určení intervalů nestability quasiharmonických kmitů (The method for the determination of instability intervals of quasiharmonic vibration systems), Aplikace matematiky, 4, 1959, No. 4, pp. 278–289

[3] Tondl A.: To the interaction of different types of excitation, Proc. Seminar Interaction and Feed Back '97, Institute of Thermomechanics, Czech Academy of Sciences, Prague 1997, pp. 111–118

[4] Tondl A.: To the problem of quenching self-excited vibrations, Acta Technica ČSAV, 43, 1998, pp. 109–116

[5] Ecker H.: Suppression of Self-Excited Vibrations in Mechanical Systems by Parametric Stiffness Excitation, Forschriftsberichte Simulation, Band 11, ARGESIM/ASIM-Verlag, Wien, 2003

[6] Tondl A., Ecker H.: Self-excited vibration quenching using parametric excitation: the effect of an additional external excitation, Proc. of the 6th International Conference on Vibration Problems ICOVP-2003, (6 pages), University of Liberec, Czech Republic, 2003

[7] Tondl A., Nabergoj R.: The effect of parametric excitation on a self-excited three-mass system, Proc.of International Journal of Non-Linear Mechanics 39, 2004, pp. 821–832

[8] Dohnal F., Ecker H.: Vibration control of self-excited oscillations by parametric stiffness excitation, Proc. of the 11th International Congress on Sound and Vibration, 5–8 July 2004, St. Petersburg, Russia, pp. 339–346

[9] Tondl A.: To the problem of self-excited vibration suppression, Egineering MECHANICS, Vol. 15, 2008, No. 4, p. 297–308

[10] Tondl A.: Passive and active means for self-excited vibration suppressing: Two-mass model, Engineering Mechanics, Vol. 15, 2008, No. 2, pp. 133–138

[11] Ecker H., Tondl A.: Stabilization of a rigid rotor by a time-varying stiffness of the bearing mounts, Proceedings of the 8th International Conference on Vibrations in Rotating Machinery, 7–9 September 2004, Univ. of Wales, Swansea, Professional Engineering Publishing Limited for The Institution of Mechanical Engineers, Berry St. Edmunds and London, UK 2004, pp. 45–54

[12] Ecker H., Tondl A.: Increasing the stability threshold of a rotor by open-loop control of the bearing mount stiffness, Proceedings of ISCORMA-3 (International Symposium on Stability Control of Rotating Machinery), Cleveland, Ohio, 19–23 September, 2005, pp. 351–361

[13] Ecker H., Tondl A.: Suppression of self-excited vibrations in valve systems: Active Means, Proc. Colloquium Dynamics of Machines 2007, Prague, February 6–7, 2007, pp. 55–62

[14] Dohnal F., Tondl A.: Suppressing flutter vibrations by parametric inertia excitation, Journal of Applied Mechanics, May 2009, Vol. 76, pp. 031006-1 – 031006-7

[15] Tondl A., Půst L.: To the parametric anti-resonance application, Engineering MECHANICS, 2010, Vol. 17, No. 2, pp. 135–144

[16] Ecker H: Exploring the use of parametric excitation, 10th Internat. Conf. on Recent Advances in Structural Dynamics, July 2010, Southampton, keynote lecture

[17] Tondl A., Půst L.: Další využití parametrické anti-rezonance, in: Dynamics of Machines 2011 (ed. L. Pešek), ÚT AV ČR, Praha 2011, pp. 97–104

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