SENSITIVITY ANALYSIS OF A BLADE COUPLE UNDER ROTATION

Miroslav Byrtus, Michal Hajžman, Vladimír Zeman*

The aim of the paper is to show the influence of design parameters on the suppression of rotating blade vibration. The derivation of nonlinear mathematical model of a blade couple mutually connected by a friction element is summarized. After that, the linearization of the mathematical model is performed by harmonic balance method. The linearized model is used for sensitivity analysis of real and imaginary parts of eigenvalues in the course of rotation and sensitivity analysis of steady-state response with respect to the design parameters of internal friction coupling.

Keywords: *sensitivity analysis, friction damper, blades, bladed disk, harmonic balance method*

1. Introduction

Stresses produced by resonant or forced vibration may significantly affect the life of turbine blades. In order to suppress blade vibration, different damping mechanisms are employed (i.e. underplatform dampers, shrouds, root joint, etc.). The damping asserts usually in contacts which are characterized by high contact pressures and a small amplitude of relative displacement. To predict the influence of friction contacts, accurate models have to be used. In many cases, the Coulomb's friction model is used along with Hertz or Hertz-Mindlin contact theory for spherical bodies [1]. Next and probably more important fact is the design of particular damping mechanism. Here, the design of the friction damping, which is realized by a friction element placed in between the shroud of rotating blades, will be studied [3]. Sensitivity analysis will be performed to investigate the influence of chosen design parameters on eigenvalues and steady-state response to harmonic excitation with nozzle frequency, which is given by a product of rotational frequency of bladed disk and the number of stator blades [2].

2. Mathematical model of rotating blade couple with friction element

Let us recall the mathematical model of rotating blade couple with a friction element which was presented in detail in [3]. Flexible blades are discretized by FEM using 1D Rayleigh beam elements. To linearize friction forces in contact surfaces between the blade shroud and the friction element, harmonic balance method is applied.

Let us consider a system of two blades fixed with rigid disk rotating with angular velocity ω. A friction element with inclined planar contact surfaces a and b is wedged in between the blade shroud (Fig. 1). As a simplification, the contacts of the friction element

* Ing. M. Byrtus, Ph.D., Ing. M. Hajžman, Ph.D., prof. Ing. V. Zeman, DrSc., Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia, Univerzitní 8, 306 14 Plzeň

and the blade shrouds are concentrated to point B in plane $b \equiv \widehat{\xi_\mathrm{B}\eta_\mathrm{B}}$ and to point A in plane $a \equiv \widehat{\xi_\mathrm{A}\eta_\mathrm{A}}$, respectively.

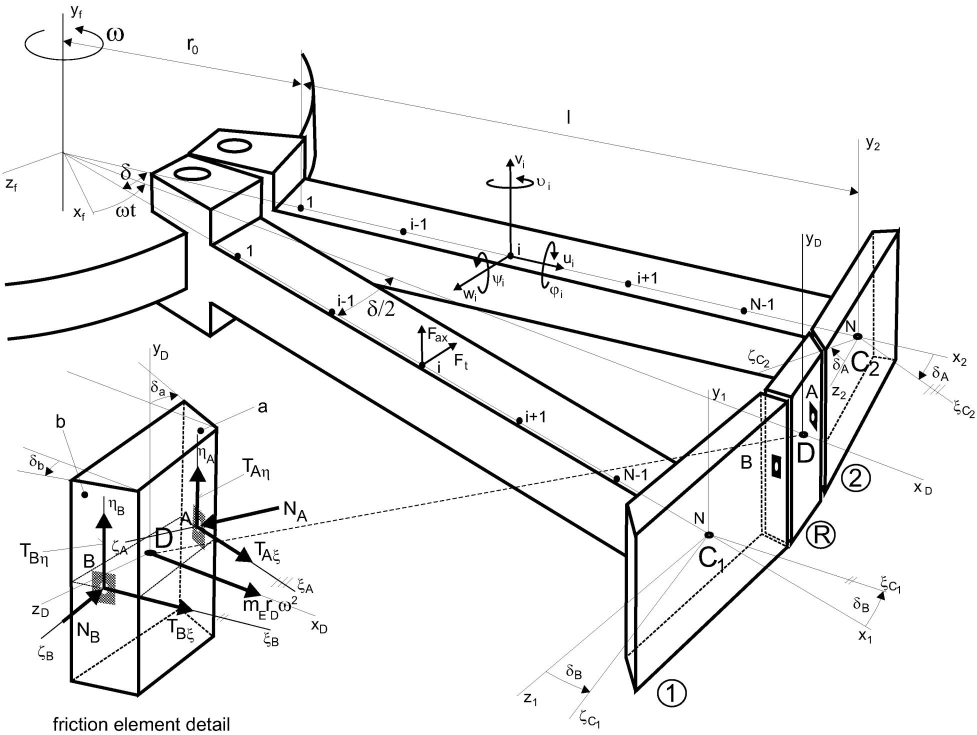

Fig.1: Two rotating blades with friction element

Blades are modelled as 1D continuum discretized by Rayleigh beam elements with uniformly distributed nodes along axes of the blades. End nodes C_1 and C_2 of the blades are fixed with rigid blade shroud. As the blades rotate, the centrifugal force $m_\mathrm{D}\, r_\mathrm{D}\, \omega^2$ pushes the friction element towards contact surfaces a and b of the adjacent blade shroud. The friction element acts on blades by normal N_A and N_B friction forces $\vec{T}_\mathrm{A}(T_{\mathrm{A}\xi}, T_{\mathrm{A}\eta})$ and $\vec{T}_\mathrm{B}(T_{\mathrm{B}\xi}, T_{\mathrm{B}\eta})$. Let us suppose, the blades are excited harmonically with frequencies $\omega_k = k\,\omega$ in tangential and axial (parallel to axis of rotation) directions. Excitation forces are uniformly concentrated in nodes along the blades.

Equations of motion of blades with shroud and friction element can be expressed in rotating local coordinate systems x_j, y_j, z_j, $j = 1, 2$ (blades) and $x_\mathrm{D}, y_\mathrm{D}, z_\mathrm{D}$ (friction element), where x_j and x_D are identified with axis of the blades and with radial of friction element. Axes y_j, y_D are parallel with fixed axis of disk rotation y_f (Fig. 1). Equations of motion can be then expressed in generalized coordinates

$$\mathbf{q}_j = [\ldots u_i, v_i, w_i, \varphi_i, \vartheta_i, \psi_i, \ldots]_j^\mathrm{T} \;, \quad j = 1, 2 \;. \tag{1}$$

defining displacements in axis direction and angular displacements about them in nodes $i = 1, \ldots, N$ of the blade with shroud considering excluded friction element, in following form [4]

$$\mathbf{M}_\mathrm{B}\,\ddot{\mathbf{q}}_j + (\omega\,\mathbf{G}_\mathrm{B} + \mathbf{B}_\mathrm{B})\,\dot{\mathbf{q}}_j + (\mathbf{K}_{\mathrm{s,B}} - \omega^2\,\mathbf{K}_{\mathrm{d,B}} + \omega^2\,\mathbf{K}_{\omega,\mathrm{B}})\,\mathbf{q}_j = \mathbf{f}_{\omega,\mathrm{B}} + \mathbf{f}_\mathrm{B}(t) \;, \tag{2}$$

where symmetric matrices $\mathbf{M}_\mathrm{B}$, $\mathbf{B}_\mathrm{B}$, $\mathbf{K}_{\mathrm{s,B}}$, $\mathbf{K}_{\mathrm{d,B}}$, $\mathbf{K}_{\omega,\mathrm{B}}$ are mass, material damping, static stiffness, softening under rotation and bending stiffening under rotation, respectively. Matrix $\omega\,\mathbf{G}_\mathrm{B}$ is skew symmetrical matrix of gyroscopic effects. Constant centrifugal forces expressed by vector $\mathbf{f}_{\omega,\mathrm{B}}$ and hydrodynamical forces caused by vapour flow through fixed nozzles. Based on the analysis of vapour pressure field [5], hydrodynamic forces can be approximately expressed in blade model (2) as a superposition of vectors of constant mean forces $\mathbf{f}_{\mathrm{B},0}$ and harmonic variable components with nozzle frequency $\omega_k = k\,\omega$ as follows

$$\mathbf{f}_\mathrm{B}(t) = \mathbf{f}_{\mathrm{B},0} + \mathbf{f}_\mathrm{B}\,\cos\omega_k\left(t + \frac{\delta_j}{\omega}\right) , \quad j = 1,2 , \quad \delta_1 = 0 , \quad \delta_2 = \delta \tag{3}$$

where δ represents pitch angle of blades.

Similarly, the equations of motion of still isolated rigid friction element can be written in generalized coordinates

$$\mathbf{q}_\mathrm{D} = [u, v, w, \varphi, \vartheta, \psi]^\mathrm{T} , \tag{4}$$

in matrix form analogous to the blade model [3]

$$\mathbf{M}_\mathrm{D}\,\ddot{\mathbf{q}}_\mathrm{D} + \omega\,\mathbf{G}_\mathrm{D}\,\dot{\mathbf{q}}_\mathrm{D} - \omega^2\,\mathbf{K}_{\mathrm{d,D}}\,\mathbf{q}_\mathrm{D} = \mathbf{f}_{\omega,\mathrm{D}} . \tag{5}$$

After placing the friction element in between the blade shroud, acting of contact elastic and friction forces is concentrated into contact points A and B. Linearized model of blades connected by means of friction element will be further expressed by using perturbance displacements, which define blade and friction element displacement from static equilibrium given by centrifugal forces and by mean values of hydrodynamical forces. Contact viscous-elastic and friction forces are then replaced by forces transmitted by springs and dampers with equivalent viscous damping, which are calculated under assumption of constant normal forces $N_{X,0}$. These forces are calculated from static equilibrium condition of friction element under rotation

$$N_{X,0} = m_\mathrm{D}\,r_\mathrm{D}\,\omega^2\,\frac{\cos\delta_X}{\sin(\delta_\mathrm{a}+\delta_\mathrm{b})} , \quad X = \mathrm{A}, \mathrm{B} . \tag{6}$$

Angles of contact surfaces skewing between blade shroud and friction element are displayed in Fig. 1. In configuration space of perturbed generalized coordinates defined by vector

$$\mathbf{q} = [\,\mathbf{q}_1^\mathrm{T}, \mathbf{q}_D^\mathrm{T}, \mathbf{q}_2^\mathrm{T}\,]^\mathrm{T} , \tag{7}$$

equations of motion of the system are then written in the form [3]

$$\mathbf{M}\,\ddot{\mathbf{q}} + (\omega\,\mathbf{G} + \mathbf{B} + \mathbf{B}_\mathrm{C})\,\dot{\mathbf{q}} + (\mathbf{K}_\mathrm{s} - \omega^2\,\mathbf{K}_\mathrm{d} + \omega^2\,\mathbf{K}_\omega + \mathbf{K}_\mathrm{C})\,\mathbf{q} + \mathbf{h}(\dot{\mathbf{q}}) = \mathbf{f}(t) . \tag{8}$$

In accordance with equations of motion (2) and (5), below presented matrices have block-diagonal structure

$$\begin{gathered}\mathbf{M} = \mathrm{diag}\,(\mathbf{M}_\mathrm{B}, \mathbf{M}_\mathrm{D}, \mathbf{M}_\mathrm{B}) , \qquad \mathbf{G} = \mathrm{diag}\,(\mathbf{G}_\mathrm{B}, \mathbf{G}_\mathrm{D}, \mathbf{G}_\mathrm{B}) , \\ \mathbf{B} = \mathrm{diag}\,(\mathbf{B}_\mathrm{B}, \mathbf{0}, \mathbf{B}_\mathrm{B}) , \qquad \mathbf{K}_\mathrm{s} = \mathrm{diag}\,(\mathbf{K}_{\mathrm{s,B}}, \mathbf{0}, \mathbf{K}_{\mathrm{s,B}}) , \\ \mathbf{K}_\mathrm{d} = \mathrm{diag}\,(\mathbf{K}_{\mathrm{d,B}}, \mathbf{K}_{\mathrm{d,D}}, \mathbf{K}_{\mathrm{d,B}}) , \qquad \mathbf{K}_\omega = \mathrm{diag}\,(\mathbf{K}_{\omega,\mathrm{B}}, \mathbf{0}, \mathbf{K}_{\omega,\mathrm{B}}) .\end{gathered} \tag{9}$$

Nonlinear friction terms are included in vector $\mathbf{h}(\dot{\mathbf{q}})$ and excitation vector

$$\mathbf{f}(t) = \left[\mathbf{f}_\mathrm{B}^\mathrm{T}\,\cos\omega_k t,\, \mathbf{0},\, +\mathbf{f}_\mathrm{B}^\mathrm{T}\cos\omega_k\left(t + \frac{\delta}{\omega}\right)\right]^\mathrm{T} \tag{10}$$

is defined by vector of amplitudes of harmonic variable components of hydrodynamic forces

$$\mathbf{f}_{\mathrm{B}} = [\ldots, 0,\, F_{\mathrm{ax}},\, -F_{\mathrm{t}},\, 0,\, 0,\, 0 \ldots]^{\mathrm{T}} \tag{11}$$

acting at each blade node in axial and tangential direction (Fig. 1). The influence of contact viscous-elastic and friction forces is described by stiffness coupling matrix $\mathbf{K}_{\mathrm{C}}$, damping matrix proportional to contact stiffness matrix $\mathbf{B}_{\mathrm{C}} = \beta_{\mathrm{C}}\,\mathbf{K}_{\mathrm{C}}$ comprising the influence of contact damping in contact surfaces.

Now, let us deal with the nonconservative part of coupling forces defined by vector $\mathbf{h}(\dot{\mathbf{q}})$. The friction forces acting on the friction element concentrated into central contact points B_1 and A_2 where superscript 1 corresponds to the first blade and superscript 2 to second blade. These forces are nonlinear and can be expressed as

$$\vec{T}_{\mathrm{B}_1} = f_{\mathrm{b}}\, N_{\mathrm{B}_1} \frac{\vec{v}_{\mathrm{s,B}_1}}{|\vec{v}_{\mathrm{s,B}_1}|}\ , \qquad \vec{T}_{\mathrm{A}_2} = f_{\mathrm{a}}\, N_{\mathrm{A}_2} \frac{\vec{v}_{\mathrm{s,A}_2}}{|\vec{v}_{\mathrm{s,A}_2}|}\ , \tag{12}$$

where f_{b} (f_{a}) is the friction coefficient of friction surface b (a) and $\vec{v}_{\mathrm{s,B}_1}$ ($\vec{v}_{\mathrm{s,A}_2}$) is a slip velocity of blade shroud '1' ('2') with respect to the friction element in point B_1 (A_2) and is expressed in $\xi_{\mathrm{B}}\,\eta_{\mathrm{B}}$ ($\xi_{\mathrm{A}}\,\eta_{\mathrm{A}}$) plane. The friction forces acting on the blade shroud have opposite direction.

To linearize the nonlinear friction forces, the harmonic balance method is used. The aim of the linearization technique is to replace the original nonlinear system with a linear one. The harmonic balance method is derived under following assumptions:

- Both nonlinear friction torques and forces acting on a friction element interact mutually very weak, therefore equivalent damping coefficients can be considered independently.
- The slip motion of friction surfaces can be simply considered as two degree of freedom elliptical motion in the friction surface.
- Excitation is supposed to be a periodic function as well as the steady-state response.
- The friction and excitation forces are expandable into a Fourier series.

Based on this, the term for determination of equivalent damping coefficient for k-th harmonic component with angular frequency ω_k can be derived assuming Coulomb friction law in following form [3]

$$b_{\mathrm{e}}(a_k, \omega_k) = \frac{4\,T}{\pi\, a_k\, \omega_k}\ , \tag{13}$$

where T is the magnitude of friction force, a_k is the amplitude of steady slip motion and ω_k is excitation angular frequency. According to known experimental observations, the term (13) does not fit real, measured amplitudes of slip motion. In [8], a modification of (13) is suggested

$$b_{\mathrm{e}}(a_k, \omega_k) = \frac{4\,T}{\pi\, (a_k\, \omega_k)^{1.112}}\ . \tag{14}$$

Physically, the term $a_k\,\omega_k$ represents the amplitude of corresponding harmonic component of the slip velocity. Using the modified equivalent damping coefficient (14), each harmonic component of nonlinear friction forces (12) can be linearized and expressed by the coefficient of equivalent viscous translational and rotational damping (e.g. for the point B_1)

$$b_{\mathrm{e}}^{(\mathrm{t})}(a_{\mathrm{B},k}^{(\mathrm{t})}, \omega_k) = \frac{4\,T}{\pi\, (a_{\mathrm{B},k}^{(\mathrm{t})}\, \omega_k)^{1.112}}\ , \qquad b_{\mathrm{e}}^{(\mathrm{r})}(a_{\mathrm{B},k}^{(\mathrm{r})}, \omega_k) = \frac{4\,M}{\pi\, (a_{\mathrm{B},k}^{(\mathrm{r})} \omega_k)^{1.112}}\ . \tag{15}$$

Variables $a^{(t)}_{B,k}$ and $a^{(r)}_{B,k}$ constitute translational and rotational slip amplitudes excited by k-th harmonic component, respectively, magnitude M of friction torque is $M = 2\, f_b\, r_{ef}\, N_{B,0}/3$. Finally, the nonlinear mathematical model (8) can be equivalently replaced by linearized one for each excitation harmonic component with frequency ω_k

$$\mathbf{M}\,\ddot{\mathbf{q}} + (\omega\,\mathbf{G} + \mathbf{B} + \mathbf{B}_C + \mathbf{B}_e(\mathbf{a}_k,\omega_k))\,\dot{\mathbf{q}} + (\mathbf{K}_s - \omega^2\,\mathbf{K}_d + \omega^2\,\mathbf{K}_\omega + \mathbf{K}_C)\,\mathbf{q} = \\ = \mathbf{f}_\omega + \mathbf{f}(\omega_k)\,e^{i\omega_0 t}\ . \tag{16}$$

Friction torques and forces are represented by equivalent damping matrix $\mathbf{B}_e(\mathbf{a}_k,\omega_k))$, where $\mathbf{a}_k = [\,a^{(t)}_{A,k},\, a^{(r)}_{A,k},\, a^{(t)}_{B,k},\, a^{(r)}_{B,k}\,]^T$ is a vector containing steady slip amplitudes. Vector $\mathbf{f}(\omega_k)$ of complex amplitudes represents external harmonic excitation with frequency equal to ω_k.

3. Sensitivity analysis – computational aspects

The methodology of the modelling presented above is used for sensitivity analysis of a real blade couple. The blades are fixed to a rigid disk rotating with constant angular velocity. Detail geometrical description of the blades was gained from [8]. Using the in-house software for computational blade modelling, each blade was divided by six nodal points into five finite beam elements. Final computational model has 78 DOF (two blades and one friction element) [3]. The linearized model (16) serves as the first approximation of the nonlinear model of the blade couple and is used for sensitivity analysis calculation. Here, we are dealing with determination of sensitivity of eigenvalues and steady-state response of the blade couple. To determine the sensitivity, one can derive analytical formulas or use numerical calculations to gain the desired sensitivity results [7].

Although the mathematical model of a blade couple is linearized, the usage of analytical formulas for sensitivity of nonconservative mathematical model with equivalent damping matrix $\mathbf{B}_e(\mathbf{a}_k,\omega_k)$ is complicated, because the particular expression of this matrix depends on external excitation. That is why we decided to use the numerical calculations of sensitivity using finite difference formulas. Let us have monitored quantity $q = q(\mathbf{p})$. We desire to express its partial derivative with respect to a vector of S selected parameters of the system $\mathbf{p} = [p_1, p_2, \ldots, p_S]^T$. Small change Δq of the monitored quantity q can be expressed with a small change $\Delta\mathbf{p}$ of the initial parameters vector $\mathbf{p}_0$. If the conditions of continuity of derivatives of the monitored quantity q are fulfilled, two terms of Taylor's formula can be used

$$\Delta q = q(\mathbf{p}_0 + \Delta\mathbf{p}) - q(\mathbf{p}_0) = \sum_{j=1}^{S} \frac{\partial q(\mathbf{p}_0)}{\partial p_j}\,\Delta p_j\ . \tag{17}$$

After modification of relation (17), one can obtain

$$\frac{\Delta q}{q(\mathbf{p}_0)} = \sum_{j=1}^{S} \frac{\partial q(\mathbf{p}_0)}{\partial p_j}\,\frac{p_{j0}}{q(\mathbf{p}_0)}\,\frac{\Delta p_j}{p_{j0}}\ . \tag{18}$$

From relation (18) we can get relative sensitivity $\Delta\overline{q}_j$ of quantity q with respect to change of parameter p_j

$$\Delta\overline{q}_j = \frac{\partial q(\mathbf{p}_0)}{\partial p_j}\,\frac{p_{j0}}{q(\mathbf{p}_0)}\ . \tag{19}$$

Partial derivative in (19) is approximated using the finite difference

$$\frac{\partial q(\mathbf{p}_0)}{\partial p_j} = \frac{q(\mathbf{p}_0 + \Delta \mathbf{p}_j) - q(\mathbf{p}_0)}{\Delta p_j} \,, \tag{20}$$

where vector $\Delta \mathbf{p}_j = [0, \ldots, 0, \Delta p_j, 0, \ldots, 0]^{\mathrm{T}}$ contains one nonzero element on j-th position. The differential relation for calculation of relative sensitivity $\Delta \overline{q}_j$ of quantity q to a change of parameter p_j can be written using (19) and (20) in final form

$$\Delta \overline{q}_j = \frac{q(\mathbf{p}_0 + \Delta \mathbf{p}_j) - q(\mathbf{p}_0)}{\Delta p_j} \, \frac{p_{j0}}{q(\mathbf{p}_0)} \,. \tag{21}$$

In the next, we will use the relative sensitivity for particular sensitivity calculations.

4. Sensitivity analysis of a blade couple on a test rig

The proposed damping mechanism through the friction element targets the vibration suppression. Therefore, the sensitivity analysis should reveal whether there exist some design parameters which can significantly positively influence the blade vibration, i.e. which of their values can suppress steady-state vibration of the blade couple. To find out that, it is also necessary to choose suitable design parameters which will be ordered in following vector

$$\mathbf{p} = [f \; k \; \delta_{\mathrm{a}} \; \delta_{\mathrm{A}} \; \delta_{\mathrm{b}} \; \delta_{\mathrm{B}}]^{\mathrm{T}} \,, \tag{22}$$

where f stands for friction coefficient, k for multiple of fundamental excitation frequency ω (i.e. angular speed of the bladed disk) and the angles δ_{a}, δ_{A}, δ_{b}, δ_{B} are defined in Fig. 1. Practically, the sensitivity calculations are performed for different initial parameters $\mathbf{p}_0$ (e.g. for different initial value of friction coefficient f).

According to the harmonic balance method, the steady-state response has to be used for contact slip motion calculation. At first, the steady-state response to external excitation is determined neglecting friction effects. The response is then used to determine the slip motion between shrouds and friction element and based on that the equivalent damping

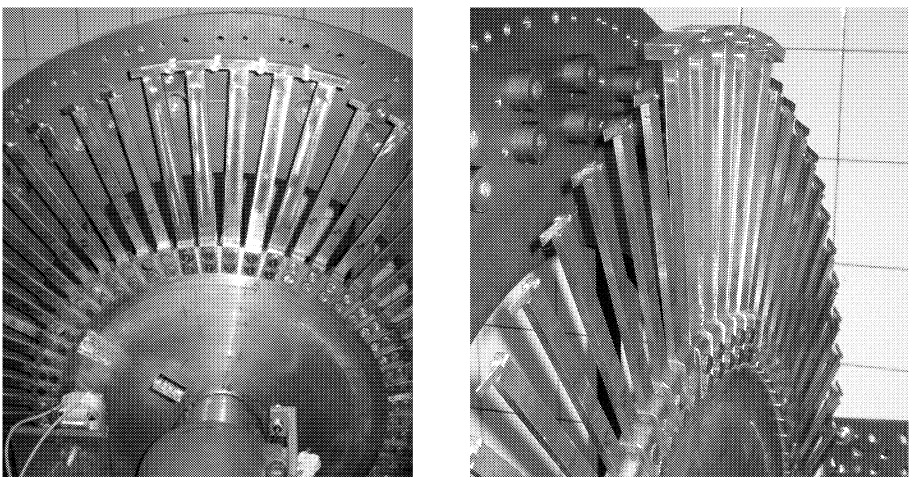

Fig.2: Test rig for measuring blade and disk vibration (Institute of Thermomechanics, Academy of Sciences of the Czech Republic) [6]

coefficients are calculated according to relations (13) to (15). Let us define the system design parameters (22) which primarily influence the blade motion. Slip properties of the contact surfaces are defined by friction coefficient $f_\mathrm{a} = f_\mathrm{b} = f_0$. The excitation defined in (10) is uniformly distributed along the blade, i.e. radial and tangential forces (11) act at each blade node. Amplitudes of the forces are supposed to be inverse proportional to the multiple k of basic excitation frequency ω. Particular values are as follows

$$f_0 = 0.1 \div 0.25 \ , \quad k = 1 \div 30 \ , \quad F_\mathrm{ax} = \frac{10}{k}\,[\mathrm{N}] \ , \quad F_\mathrm{t} = \frac{20}{k}\,[\mathrm{N}] \ , \quad \omega_k = k\,\omega\,[\mathrm{rad/s}] \ . \quad (23)$$

4.1. Sensitivity of eigenvalues

Because of the presence of damping in the mathematical model (16), the eigenvalues are complex. Hence, we deal with relative sensitivity calculations of corresponding real and imaginary parts. It has been already shown in [3] that imaginary parts of eigenvalues depend on rotational speed of the disk to which the blades are mounted and are significantly influenced by contact coupling forces arising between shroud and friction element. Fig. 3 clearly demonstrates the mentioned dependence of eigenvalues with respect to rotational speed of the disk. As the rotational speed increases, an interesting phenomenon occurs. The two areas which are marked by grey colour, are important because furthermore a set of low eigenfrequencies arises. They are caused by the friction damping and correspond to

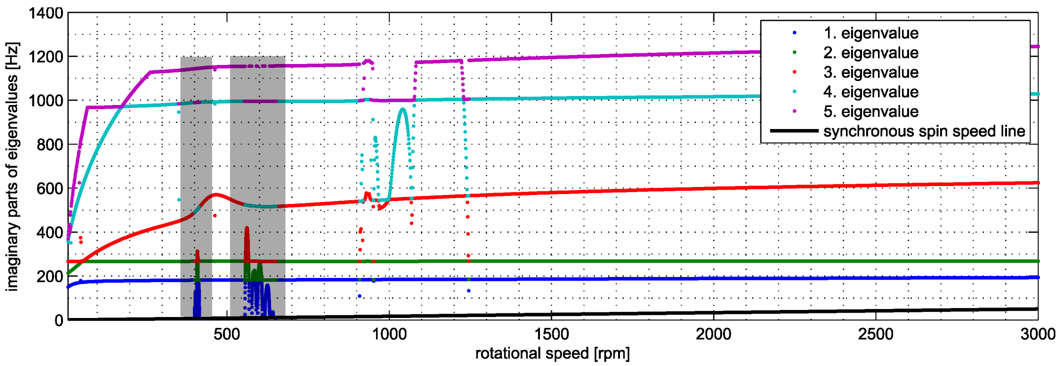

Fig.3: Imaginary parts of chosen eigenvalues in dependence on rotational speed of the bladed disk ($f_0 = 0.1$, $k = 1$)

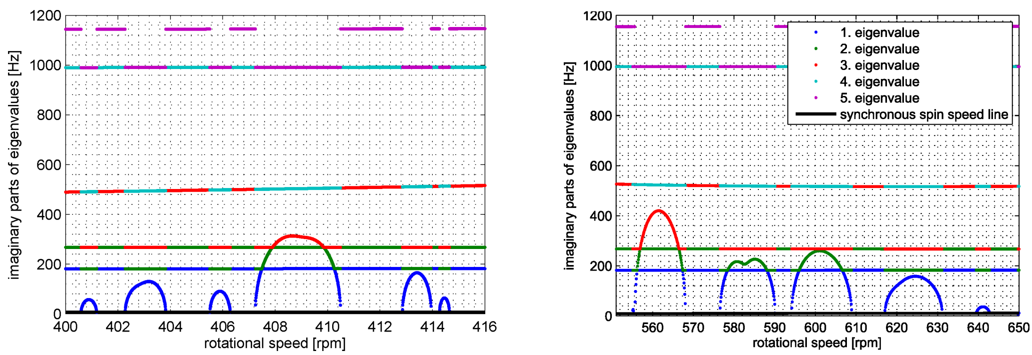

Fig.4: Zoomed grey areas from the previous figure – imaginary parts of chosen eigenvalues in dependence on rotational speed of the bladed disk ($f_0 = 0.1$, $k = 1$)

such operational state when the blade motion does not affect the friction element motion and the friction element then behaves like unattached and moves like there were no damping in the coupling. The origin of this phenomenon consists in the fact that the friction coupling is supposed to be viscous only. Fig. 4 shows a detail of the course of newly arising eigenfrequencies which are intersected by the synchronous spin speed line corresponding to excitation with rotational frequency and therefore the resonant state occurs. This effect is supposed to be very sensitive to coupling friction coefficient and that is the reason why the friction coefficient has been chosen as a design parameter. Let us further note that the coupling damping is in general strongly dependent on the external excitation because the harmonic balance method is used for determination of the coupling damping.

As an illustration, the following figures below (Fig. 5 – Fig. 8) show the relative sensitivity of imaginary and real parts of first five eigenvalues for different initial friction coefficient f_0 and first harmonic excitation. Let us mention that the damping of the friction coupling causes that different number of real and complex eigenvalues appears as the rotational speed increases. Therefore, after each modal analysis calculation the reordering of calculated eigenvalues has to be performed. The eigenvalues are ordered ascending with respect to the imaginary part, firstly with positive and after with negative imaginary part. The real eigenvalues are placed at the end.

During the plotting of sensitivity, there are 'color jumps' because of the change of number of complex and real eigenvalues. The red peaks around 300–400 rpm in Figs. 5 and 7

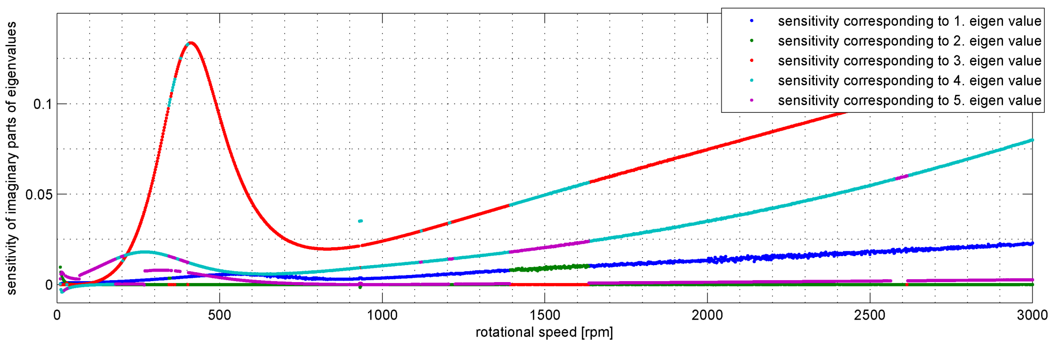

Fig.5: Sensitivity of imaginary parts of first five eigenvalues with respect to change of friction coefficient ($f_0 = 0.1$, $k = 1$)

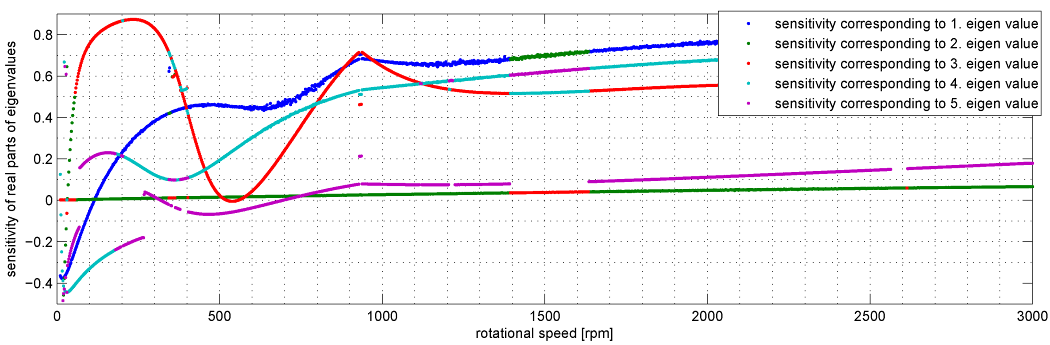

Fig.6: Relative sensitivity of real parts of first five eigenvalues with respect to change of friction coefficient ($f_0 = 0.1$, $k = 1$)

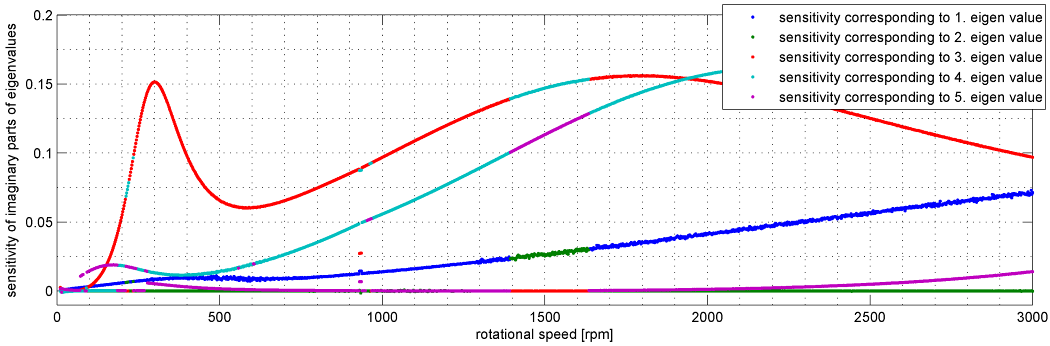

Fig.7: Sensitivity of imaginary parts of first five eigenvalues with respect to change of friction coefficient ($f_0 = 0.2$, $k = 1$)

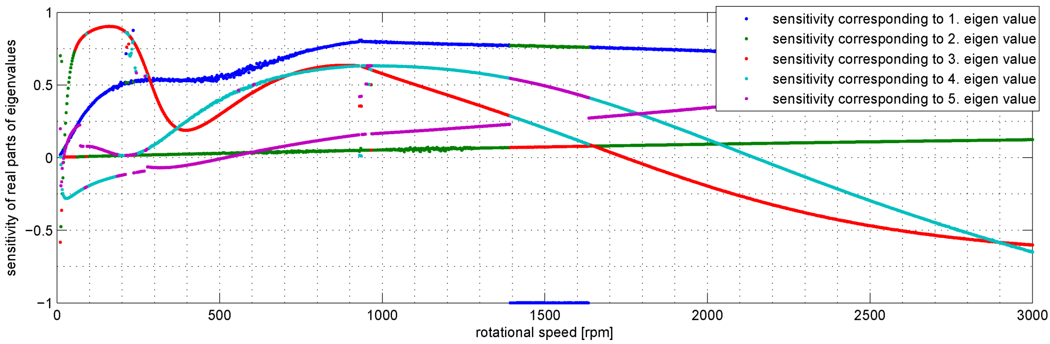

Fig.8: Relative sensitivity of absolute values of real parts of first five eigenvalues with respect to change of friction coefficient ($f_0 = 0.2$, $k = 1$)

correspond to resonant state of imaginary parts of newly arising eigenvalues. Their sensitivity shifts to the left because as the friction coefficient increases, the damping increases too and the resonant states shift to the left. More important, with respect to this particular application, is the sensitivity of the absolute value of the eigenvalues real parts. According to the results plotted in Figs. 6 and 8, one can see that sensitivities calculated for $f_0 = 0.1$ are more less positive. Practically, it means that a small increase of the friction coefficient causes more damped effect in the investigated operational area. On the contrary, sensitivity of absolute values of eigenvalues real parts for friction coefficient $f_0 = 0.2$ is negative in the upper operational area for the third and the fourth eigenvalue. This fact is given by the complex eigenmode shapes of these eigenvalues with regard to different phase shifts between the displacements during the eigenmode shape motion. Moreover, for higher rotational speed, the friction coupling locks up and the friction element loses the damping capability.

4.2. Sensitivity of steady-state response

Figs. 9 and 10 show the sensitivity of amplitude of steady-state response of three displacements in axial direction labelled 32 (shroud of 1st blade), 38 (friction element) and 74 (shroud of 2nd blade) to change of friction parameter f_0. The first figure corresponds to excitation caused by bladed disk unbalance with rotational frequency ω of the rotating disk and the latter figure corresponds to excitation by 30 fold multiple (number of stator nozzles) of fundamental excitation frequency caused by aerodynamic forces. Here, we can

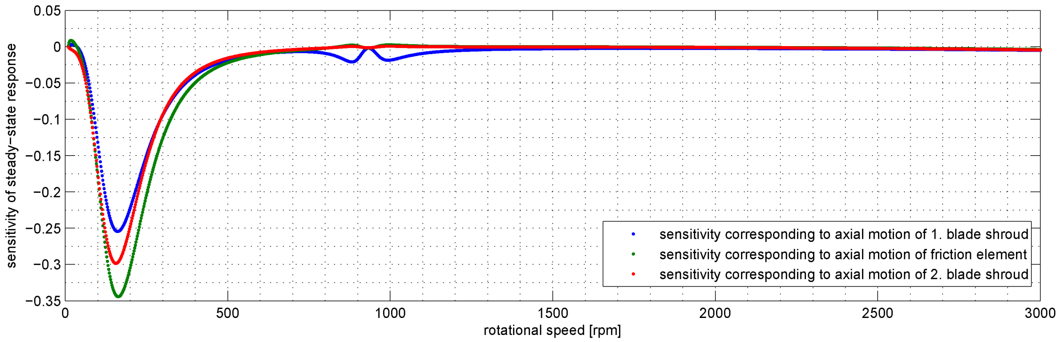

Fig.9: Relative sensitivity of steady-state response to change of friction coefficient ($f_0 = 0.25$, $k = 1$)

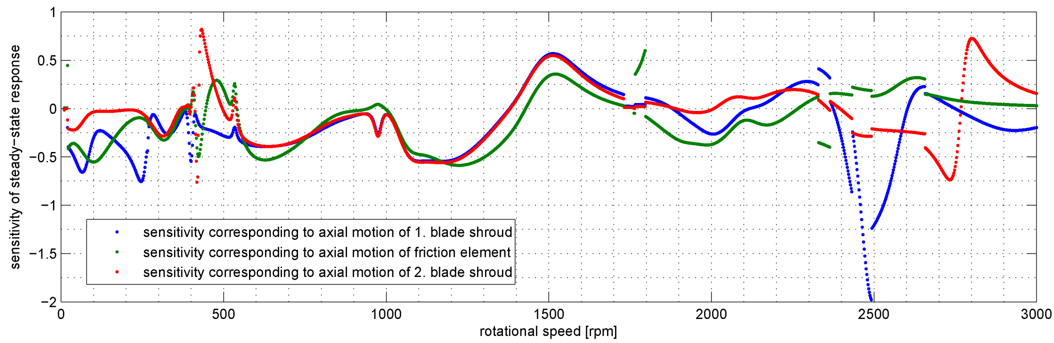

Fig.10: Relative sensitivity of steady-state response to change of friction coefficient ($f_0 = 0.25$, $k = 30$)

clearly see the strong dependence on excitation frequencies of the system. Higher excitation frequency excites more resonance peaks within the operational area 0–3000 rpm and therefore the sensitivity asserts around resonant peaks.

The sensitivity is dominant in the lower frequency range because as soon as the rotational speed crosses the value approximately of 1700 rpm, the friction element locks due to large centrifugal forces. That is the reason why mentioned design parameters have been chosen. In many applications, the bladed disks usually run with constant rotational speed. The sensitivity analysis can thus predict values of design parameters which can ensure that the friction element is not still locked and the vibration energy can be lost during the slip motion between friction element and the shroud. It can be clearly seen in Fig. 10 that increasing the friction coefficient has positive influence on suppression of steady-state blade vibration at approx. $n = 1100$ rpm. On the contrary, increasing the friction coefficient has negative influence on steady-state blade vibration at approx. $n = 1500$ rpm.

5. Conclusions

This paper presents a short overview of a blade couple with a friction element modelling which is based on the harmonic balance method for friction effects linearization. The derived mathematical model is further used for sensitivity calculations of complex eigenvalues and steady-state response of a blade couple. The sensitivity is calculated numerically using difference method instead of using analytical formulas because the equivalent damping matrix depends directly on excitation and hence to derive analytical formulas is very complicated. Performing the sensitivity analysis of dynamic steady-state response to high-frequency excitation (e.g. nozzle excitation) with respect to friction coefficient, the limit rotational speed of the bladed disk can be determined according to the damping effect of the friction element. After crossing that speed, the friction element is locked between the shroud and the energy cannot be dissipated by its slip motion anymore. Based on presented methods and formulas for sensitivity calculations, the in-house software in MATLAB was created and tested on a model of two rotating blades with friction element which has been developed for experimental research at UT AV CR within the solution of GA CR project No. 101/09/1166.

Acknowledgement

This work was supported by GA CR project No. 101/09/1166 'Research of dynamic behaviour and optimization of complex rotating systems with non-linear couplings and high damping materials' and by the research project MSM 4977751303 of the Ministry of Education, Youth and Sports of the Czech Republic.

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Received in editor's office: April 12, 2011
Approved for publishing: August 19, 2011

Note: This paper is an extended version of the contribution presented at the national colloquium with international participation *Dynamics of Machines 2011* in Prague.