DETUNING IMPACT OF COUPLE OF BLADES WITH FRICTION ELEMENT ON BENDING OSCILLATION

Jan Brůha*, Vladimír Zeman*

This presented work is concerned with a friction element impact and detuning mass impact on the bending oscillation of a couple of blades with a friction element embedded between the blade shrouds. Either of the blades is discretized by FEM using beam elements and continuously distributed weight is concentrated in nodal points. One of the blades is excited by harmonic varying force. The friction element, which is considered as a rigid body, is pulled using constant tension force into a wedge gap between the blade shrouds. The detuning of this system is caused by an additional mass mounting on one of the blade shrouds. Numerical simulation results are compared with results of the equivalent linearization method. The effects of the friction and detuning on the blades vibration suppression are analyzed.

Keywords : *blade oscillation, dry friction, numerical simulation, equivalent linearization method*

1. Introduction

The performance of steam turbines for nuclear and coal-fired power plants is still growing. The output range of these turbines currently achieves 1000–1200 MW at the speed 3000 rpm, and inlet saturated steam pressure 4–7 MPa. Development departments of major producers focus on enhancing the cycle efficiency, increasing the inlet temperature and steam pressure, using of new materials and improving operational reliability. As a result of these requirements, demands on turbine blades rise. Blade vibrations as a side effect the operation of these machines can be very dangerous because of the high cycle fatigue failure. Dissipation of energy, due to dry friction between blade shroud surfaces and friction element surfaces, or using detuning masses mounting on certain blade shrouds belong to basic methods of bladed disks vibration suppression. Detail investigation of influences of a friction (mainly the friction phenomenon) on a dynamical response of a beam can be found in [1]. Some publications deal with the friction induced by means of friction elements inserted between the blades. A method for the calculation of a static balance supposing an in-plane motion of the wedge friction element is discussed in [2]. An analytical approach is described in [3] and comparison of numerical simulation results with the results obtained by linearization is shown in [4]. The equivalent linearization method for the evaluation of friction effects can be used as the first approximation by means of so-called equivalent viscous damping [5]. This method has been experimentally verified in [6] and applied in the Department of Mechanics of University of West Bohemia in Pilsen to the harmonic forced vibration of two rotating blades with friction damping [7]. The modal analysis of the bladed disk with sixty blades and friction elements has been presented in [8]. The vibration problem of bladed disks with

* J. Brůha, prof. Ing. V. Zeman, DrSc., Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia in Pilsen, Univerzitní 22, 306 14 Plzeň

friction elements placed between the blade shrouds is in the long term experimentally [9–10] or theoretically by using analytical and numerical solutions [11] investigated in the Institute of Thermomechanics of the Academy of Sciences of the Czech Republic (IT AS CR). The experiments with a couple of blades have been a motivation to the analysis of the effects of friction and detuning on suppression of the undesirable vibration in this paper. The main effort is especially to stifle the blade vibrations in the neighbourhood of eigenfrequencies that are near to excitation frequencies. The characteristic of dry friction is in its simplified form described by Coulomb's law, but in particular applications this first approximation can proves unsatisfactory and we have to include the micro-slip effect at the very low velocity and also the friction force is often non-constant at higher velocity.

The aim of this work has been to prepare a discretized mathematical model of a couple of blades with a friction element and a detuning mass mounted on one of the blade shrouds which simulates experimentally explored blade vibrations in IT AS CR [9–10]. The numerical simulation results have been compared with experimentally obtained values. The next part of this work has been to apply an equivalent linearization method on couple of blades, to determine if this method is acceptable to this strongly nonlinear case, and compare the results of the equivalent linearization method with numerical simulations results. These results in the form of amplitude-frequency responses of transversal blade shrouds displacements in the neighbourhood of the lowest eigenfrequency will be document the influences of the friction element inserted between the blade shrouds and detuning of the blades on the vibration suppression in the resonance state.

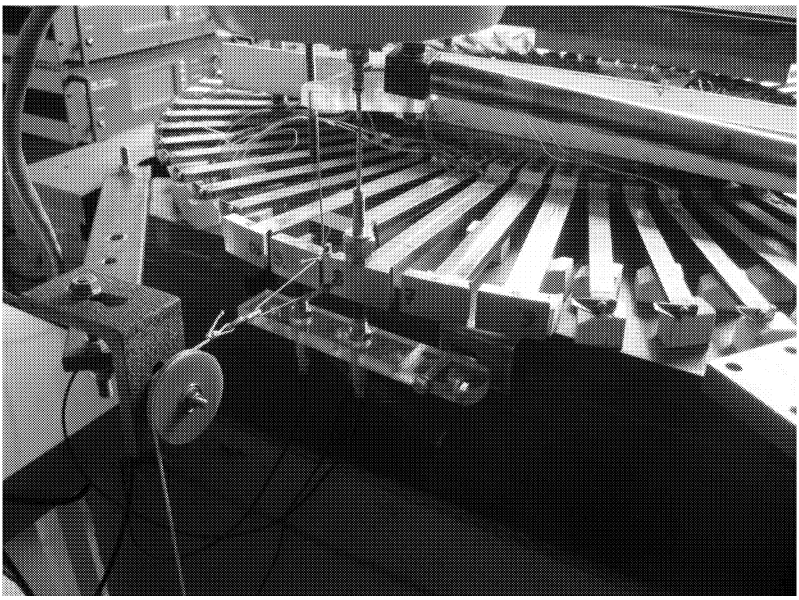

Fig.1: Experimental arrangement tested in IT AS CR

2. Computational and mathematical model

The experimental arrangement making use of a fixed bladed disk is shown in Figure 1 taken over from the research report [9]. The friction element is pulled into a wedge gap between shrouds of the blades number 8 and 9 by means of a weight hanged on a fibre wrapped across a pulley.

The computational model of a couple of steel blades clamped into a horizontal situated non-rotating disk is shown in Figure 2. Ends of the blades are fixed in end nodal points to the blade shrouds that are modeled as rigid bodies. The friction element, which is also considered a rigid body, is pulled using constant tension force F_C into a wedge gap between the shrouds. The second blade is excited in the centre of gravity of its shroud in vertical direction by harmonic varying force $F_0 \sin\omega t$. Detuning of the couple of blades is achieved through additional mass Δm mounted on the shroud of the unexcited blade.

In the next step either of the blades has been discretized by using the finite element method [12–13] by five identical beam elements with three degrees of freedom in every node: transversal displacement v, flexural displacement ψ and torsional displacement φ. Continuously distributed weight has been concentrated in particular nodal points.

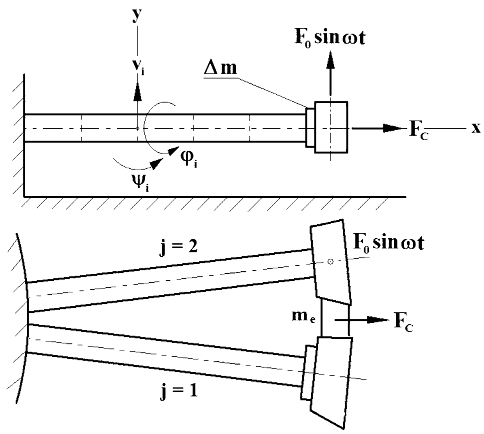

Fig.2: Scheme of the couple of blades with the friction element and additional mass mounted on one of them

Mass and stiffness matrixes of the beam element have been derived from kinetic and potential energy of that element in the configuration space

$$\mathbf{q}_\mathrm{e} = [\, v_i \quad \psi_i \quad v_{i+1} \quad \psi_{i+1} \quad \varphi_i \quad \varphi_{i+1} \,]^\mathrm{T} \tag{1}$$

and they have the form [14]

$$\begin{aligned} \mathbf{M}_\mathrm{e} &= \begin{bmatrix} \mathbf{S}_1^{-\mathrm{T}} (\mathbf{I}_{1\mathrm{A}} + \mathbf{I}_{2\mathrm{Z}}) \, \mathbf{S}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_3^{-\mathrm{T}} \, \mathbf{I}_{4\mathrm{P}} \, \mathbf{S}_3^{-1} \end{bmatrix} , \\ \mathbf{K}_\mathrm{e} &= \begin{bmatrix} \mathbf{S}_1^{-\mathrm{T}} \, \mathbf{I}_{3\mathrm{Z}} \, \mathbf{S}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_3^{-\mathrm{T}} \, \mathbf{I}_{5\mathrm{P}} \, \mathbf{S}_3^{-1} \end{bmatrix} . \end{aligned} \tag{2}$$

Matrixes $\mathbf{S}_1$ and $\mathbf{S}_3$ depend on length of element l

$$\mathbf{S}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & l & l^2 & l^3 \\ 0 & 1 & 2\,l & 3\,l^2 \end{bmatrix} , \qquad \mathbf{S}_3 = \begin{bmatrix} 1 & 0 \\ 1 & l \end{bmatrix} \tag{3}$$

and auxiliary integral matrixes $\mathbf{I}_{1\mathrm{A}}$, $\mathbf{I}_{2\mathrm{Z}}$, $\mathbf{I}_{3\mathrm{Z}}$, $\mathbf{I}_{4\mathrm{P}}$ and $\mathbf{I}_{5\mathrm{P}}$ have the form

$$\mathbf{I}_{1\mathrm{A}} = \varrho\,A\,l \begin{bmatrix} 1 & l/2 & l^2/3 & l^3/4 \\ & l^2/3 & l^3/4 & l^4/5 \\ & & l^4/5 & l^5/6 \\ \mathrm{sym.} & & & l^6/7 \end{bmatrix} , \qquad \mathbf{I}_{2\mathrm{Z}} = \varrho\,J_\mathrm{z}\,l \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & l & l^2 \\ & & 4\,l^2/3 & 3\,l^3/2 \\ \mathrm{sym.} & & & 9\,l^4/5 \end{bmatrix} ,$$

$$\mathbf{I}_{3\mathrm{Z}} = E\,J_\mathrm{z}\,l \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 6\,l \\ 0 & 0 & 6\,l & 12\,l^2 \end{bmatrix} , \tag{4}$$

$$\mathbf{I}_{4\mathrm{P}} = \varrho\,J_\mathrm{p}\,l \begin{bmatrix} 1 & l/2 \\ l/2 & l^2/3 \end{bmatrix} , \qquad \mathbf{I}_{5\mathrm{P}} = G\,J_\mathrm{k}\,l \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} ,$$

where ϱ is element density, A is square cross-section area with sides h and b, $J_\mathrm{z} = b\,h^3/12$ is the second moment of area of cross-section about the axis z, $J_\mathrm{p} = J_\mathrm{y} + J_\mathrm{z}$ is the polar second moment of area, $J_\mathrm{y} = h\,b^3/12$ is the second moment of area of cross-section about the axis y, $J_\mathrm{k} = 0.229\,b\,h^3$ is the torsion constant [15], E is Young's modulus and G is shear modulus.

Our requirement is to transform the matrixes of the beam element into a configuration space

$$\tilde{\mathbf{q}}_\mathrm{e} = [\, v_i \quad \psi_i \quad \varphi_i \quad v_{i+1} \quad \psi_{i+1} \quad \varphi_{i+1} \,]^\mathrm{T} , \tag{5}$$

to do it, it is necessary to derive a transformation matrix $\mathbf{T}$ for this purpose

$$\mathbf{q}_\mathrm{e} = \begin{bmatrix} v_i \\ \psi_i \\ v_{i+1} \\ \psi_{i+1} \\ \varphi_i \\ \varphi_{i+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \psi_i \\ \varphi_i \\ v_{i+1} \\ \psi_{i+1} \\ \varphi_{i+1} \end{bmatrix} = \mathbf{T}\,\tilde{\mathbf{q}}_\mathrm{e} . \tag{6}$$

Transformed mass and stiffness matrixes of the beam element are defined by equations

$$\tilde{\mathbf{M}}_\mathrm{e} = \mathbf{T}^\mathrm{T}\,\mathbf{M}_\mathrm{e}\,\mathbf{T} , \qquad \tilde{\mathbf{K}}_\mathrm{e} = \mathbf{T}^\mathrm{T}\,\mathbf{K}_\mathrm{e}\,\mathbf{T} . \tag{7}$$

Obviously, the blade's generalized coordinate vector has the structure

$$\mathbf{q}_j = [\dots \quad v_i \quad \psi_i \quad \varphi_i \quad \dots]_j^\mathrm{T} \in \mathbb{R}^{18,1} , \quad j = 1, 2 \text{ (blades)} . \tag{8}$$

Transformed matrixes of all beam elements are embedded in mass and stiffness matrixes of the blades and they overlap themselves in common nodes. The number of degrees of freedom is 15 for each of the blades, because we assume that the blades are clamped into a non-rotating disk,

$$\mathbf{M}_j, \mathbf{K}_j \in \mathbb{R}^{15,15} , \quad j = 1, 2 . \tag{9}$$

The friction element is considered a rigid body suspended on a thin elastic fibre with vertical stiffness $k_\mathrm{e} = 10\,\mathrm{N\,m^{-1}}$. In this case we respect only vertical displacement v_e of the friction element in a wedge gap.

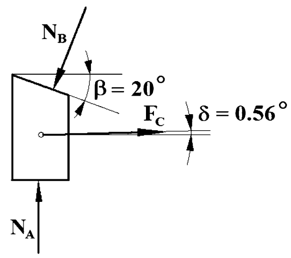

Fig.3: Friction element pulled into a wedge gap between the blade shrouds

The pulling force F_{C} represents centrifugal force, but during the experiment carried out in IT AS CR and making use of the non-rotating bladed disk it was realized by using the weight hanged on a fibre wrapped across a pulley. According to the couple of blades geometry (Figure 2), δ is the angle between the radius vector of the friction element centre of gravity and the radial contact area between the shroud of the first blade and friction element (Figure 3). The normal forces N_{A}, N_{B}, acting in contact areas between the friction element and blade shrouds, are determined by friction element geometry and results from equations of equilibrium in horizontal plane in the form

$$N_{\mathrm{A}} = F_{\mathrm{C}} \, \frac{\cos(\beta+\delta)}{\sin\beta} \ , \tag{10}$$

$$N_{\mathrm{B}} = F_{\mathrm{C}} \, \frac{\cos\delta}{\sin\beta} \ . \tag{11}$$

The frictional forces in contact areas are approximated by the continuous function tgh with the argument expressed slip velocity v_{s} of the central contact point of the blade shroud relative to the friction element multiplied by κ constant, because we want to respect the micro-slips at the very low slip velocities and almost constant friction value at higher slip velocities [11]. The κ parameter can be generally expressed as $\kappa = \mathrm{tg}(\alpha)/(f N)$, where α is the inclination angle of the friction force tangent for the slip velocity $v_{\mathrm{s}} = 0$ (Figure 4) and f is a coefficient of friction. For $\kappa \to \infty$ the characteristic of dry friction changes to the Coulomb's law.

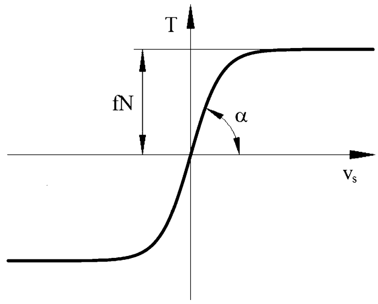

Fig.4: Characteristic of frictional forces in contact points in relation to slip velocity

Force vectors of the frictional effects in the contact points A, B of the blade shrouds can be expressed in the form

$$\mathbf{f}_{\mathrm{d1}}(\dot{\mathbf{q}}_1, \dot{v}_{\mathrm{e}}) = f \, N_{\mathrm{A}} \, \mathbf{p}_1 \ \mathrm{tgh}[\kappa \, (\dot{v}_6^{(1)} + r_{\mathrm{A}} \, \dot{\varphi}_6^{(1)} - \dot{v}_{\mathrm{e}})] \ , \tag{12}$$

$$\mathbf{f}_{\mathrm{d2}}(\dot{\mathbf{q}}_2, \dot{v}_{\mathrm{e}}) = f \, N_{\mathrm{B}} \, \mathbf{p}_2 \ \mathrm{tgh}[\kappa \, (\dot{v}_6^{(2)} - r_{\mathrm{B}} \, \dot{\varphi}_6^{(2)} - \dot{v}_{\mathrm{e}})] \ , \tag{13}$$

where f is a coefficient of friction in contact areas, auxiliary vectors $\mathbf{p}_1$, $\mathbf{p}_2$, corresponding to the blades' generalized coordinates (8), are

$$\mathbf{p}_1 = [0 \quad 0 \quad 0 \quad \dots \quad 1 \quad 0 \quad r_\mathrm{A}]^\mathrm{T} \in \mathbb{R}^{15,1} \ , \tag{14}$$

$$\mathbf{p}_2 = [0 \quad 0 \quad 0 \quad \dots \quad 1 \quad 0 \quad -r_\mathrm{B}]^\mathrm{T} \in \mathbb{R}^{15,1} \ , \tag{15}$$

and constants r_A, r_B are perpendicular distances between the contact points and the axis of the blades. The frictional forces in points A, B are approximately (for $\delta \ll \beta$) expressed in the form

$$T_\mathrm{A} \approx f\,\frac{F_\mathrm{C}}{\mathrm{tg}\,\beta} \ , \qquad T_\mathrm{B} \approx f\,\frac{F_\mathrm{C}}{\sin\beta} \ . \tag{16}$$

According to the experiment in IT AS CR [9–10], we consider the coefficient of friction in the range $f \in \langle 0.2; 0.5\rangle$, pulling force $F_\mathrm{C} = 0.5\,\mathrm{N}$, amplitude of exciting force $F_0 = 1\,\mathrm{N}$ and chamfer angle of friction element $\beta = 20°$. In agreement with equations (16), we get the ratio of friction forces at the contact points to amplitude of exciting force as

$$0.274 < \frac{T_\mathrm{A}}{F_0} < 0.687 \ , \qquad 0.292 < \frac{T_\mathrm{B}}{F_0} < 0.731 \ . \tag{17}$$

The mathematical model of the whole system of a couple of blades with the friction element disposes of 31 degrees of freedom and consists of three subsystems – one for either of blades and one for the friction element

$$\begin{aligned}
&\begin{bmatrix} \mathbf{M}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & m_\mathrm{e} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_2 \end{bmatrix}
\begin{bmatrix} \ddot{\mathbf{q}}_1 \\ \ddot{v}_\mathrm{e} \\ \ddot{\mathbf{q}}_2 \end{bmatrix} +
\begin{bmatrix} \mathbf{B}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_2 \end{bmatrix}
\begin{bmatrix} \dot{\mathbf{q}}_1 \\ \dot{v}_\mathrm{e} \\ \dot{\mathbf{q}}_2 \end{bmatrix} +
\begin{bmatrix} \mathbf{K}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & k_\mathrm{e} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_2 \end{bmatrix}
\begin{bmatrix} \mathbf{q}_1 \\ v_\mathrm{e} \\ \mathbf{q}_2 \end{bmatrix} = \\
&= -\begin{bmatrix} \mathbf{f}_{1\mathrm{g}} \\ m_\mathrm{e}\, g \\ \mathbf{f}_{2\mathrm{g}} \end{bmatrix} +
\begin{bmatrix} \mathbf{0} \\ 0 \\ \mathbf{f}_2 \sin\omega t \end{bmatrix} +
\begin{bmatrix} -\mathbf{f}_{\mathrm{d}1}(\dot{\mathbf{q}}_1, \dot{v}_\mathrm{e}) \\ f_{\mathrm{d}1_{13}}(\dot{\mathbf{q}}_1, \dot{v}_\mathrm{e}) + f_{\mathrm{d}2_{13}}(\dot{\mathbf{q}}_2, \dot{v}_\mathrm{e}) \\ -\mathbf{f}_{\mathrm{d}2}(\dot{\mathbf{q}}_2, \dot{v}_\mathrm{e}) \end{bmatrix} \ ,
\end{aligned} \tag{18}$$

where $\mathbf{M}_j$, $\mathbf{B}_j$, $\mathbf{K}_j$, $j = 1, 2$, are symmetrical mass, damping and stiffness matrixes of blades with a shroud, m_e is the mass of the friction element and k_e is the vertical stiffness of the friction element suspension fiber. Vectors $\mathbf{f}_{j\mathrm{g}}$, $j = 1, 2$, express the blade's weight concentrated in nodal points. The harmonic excitation of the second blade is described by vector $\mathbf{f}_2 \sin\omega t$ with non-zero 13^th coordinate $F_0 \sin\omega t$.

3. Application of equivalent linearization method

The next purpose of this work has been to apply the equivalent linearization method on our strongly non-linear system and decide whether this method is acceptable. The friction in contact areas using Coulomb's friction model has been replaced with viscous dampers with the coefficients of equivalent damping [5]

$$b_X(a_X, \omega) = \frac{4\, f\, N_X}{\pi\, a_X\, \omega} \ , \quad X = \mathrm{A}, \mathrm{B} \ , \tag{19}$$

where a_A and a_B are amplitudes of the slip displacements in contact areas

$$a_\mathrm{A} = v_6^{(1)} + r_\mathrm{A}\, \varphi_6^{(1)} - v_\mathrm{e} \ , \tag{20}$$

$$a_\mathrm{B} = v_6^{(2)} - r_\mathrm{B}\, \varphi_6^{(2)} - v_\mathrm{e} \ . \tag{21}$$

The coefficients of the equivalent damping have been derived in terms of the simplification from the Coulomb's friction characteristic, because owing to the large κ parameter the friction characteristics using the function tgh approximate to the Coulomb's friction characteristic and also we are primarily interested in behavior of the couple of blades in the neighbourhood of the lowest resonance frequency where the slip velocities are higher.

The force vector of the friction effects on the right side of the motion equations (18) can be replaced by damping forces of fictive viscous dampers with the coefficients of equivalent damping (19) vertically placed in the central contact points between the shrouds and friction element. The damping forces of these dampers can be written in the form

$$\mathbf{f}_{\mathrm{d}}(\mathbf{q},\dot{\mathbf{q}},\omega) = -\begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_{\mathrm{A}}\,(\dot{v}_6^{(1)} + r_{\mathrm{A}}\,\dot{\varphi}_6^{(1)} - \dot{v}_{\mathrm{e}}) \\ 0 \\ r_{\mathrm{A}}\,b_{\mathrm{A}}\,(\dot{v}_6^{(1)} + r_{\mathrm{A}}\,\dot{\varphi}_6^{(1)} - \dot{v}_{\mathrm{e}}) \\ -b_{\mathrm{A}}\,(\dot{v}_6^{(1)} + r_{\mathrm{A}}\,\dot{\varphi}_6^{(1)} - \dot{v}_{\mathrm{e}}) - b_{\mathrm{B}}\,(\dot{v}_6^{(2)} - r_{\mathrm{B}}\,\dot{\varphi}_6^{(2)} - \dot{v}_{\mathrm{e}}) \\ 0 \\ \vdots \\ 0 \\ b_{\mathrm{B}}\,(\dot{v}_6^{(2)} - r_{\mathrm{B}}\,\dot{\varphi}_6^{(2)} - \dot{v}_{\mathrm{e}}) \\ 0 \\ -r_{\mathrm{B}}\,b_{\mathrm{B}}\,(\dot{v}_6^{(2)} - r_{\mathrm{B}}\,\dot{\varphi}_6^{(2)} - \dot{v}_{\mathrm{e}}) \end{bmatrix} = -\mathbf{B}_{\mathrm{e}}(\mathbf{q},\omega)\,\dot{\mathbf{q}}\ , \qquad (22)$$

where $\mathbf{B}_{\mathrm{e}}(\mathbf{q},\omega)$ is the square matrix of equivalent damping and $\dot{\mathbf{q}}$ is a derivate global vector of the generalized coordinates of the couple of blades with a friction element in the form

$$\mathbf{q} = [\,\mathbf{q}_1 \quad v_{\mathrm{e}} \quad \mathbf{q}_2\,]^{\mathrm{T}} \in \mathbb{R}^{31,1}\ . \qquad (23)$$

The mathematical model of the linearized system of the couple of blades with a friction element and equivalent viscous dampers is

$$\begin{aligned} &\begin{bmatrix} \mathbf{M}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & m_{\mathrm{e}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_2 \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_1 \\ \ddot{v}_{\mathrm{e}} \\ \ddot{\mathbf{q}}_2 \end{bmatrix} + \left(\begin{bmatrix} \mathbf{B}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_2 \end{bmatrix} + \mathbf{B}_{\mathrm{e}}(\mathbf{q},\omega) \right) \begin{bmatrix} \dot{\mathbf{q}}_1 \\ \dot{v}_{\mathrm{e}} \\ \dot{\mathbf{q}}_2 \end{bmatrix} + \\ &+ \begin{bmatrix} \mathbf{K}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & k_{\mathrm{e}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_2 \end{bmatrix} \begin{bmatrix} \mathbf{q}_1 \\ v_{\mathrm{e}} \\ \mathbf{q}_2 \end{bmatrix} = -\begin{bmatrix} \mathbf{f}_{1\mathrm{g}} \\ m_{\mathrm{e}}\,g \\ \mathbf{f}_{2\mathrm{g}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ 0 \\ \mathbf{f}_2\,\sin\omega t \end{bmatrix}\ . \end{aligned} \qquad (24)$$

In the first stage we will consider the displacements from the static equilibrium position. The motion equations (24) to the calculation of the steady state harmonic excited vibration can be written in the complex form

$$\mathbf{M}\,\ddot{\tilde{\mathbf{q}}}(t) + (\mathbf{B} + \mathbf{B}_{\mathrm{e}})\,\dot{\tilde{\mathbf{q}}}(t) + \mathbf{K}\,\tilde{\mathbf{q}}(t) = \mathbf{f}\,\mathrm{e}^{\mathrm{i}\omega t}\ , \qquad (25)$$

where $\tilde{\mathbf{q}}(t)$ is the complex vector of the generalized coordinates, $\mathbf{M} = \mathrm{diag}(\mathbf{M}_1, m_{\mathrm{e}}, \mathbf{M}_2)$, $\mathbf{B} = \mathrm{diag}(\mathbf{B}_1, 0, \mathbf{B}_2)$, $\mathbf{K} = \mathrm{diag}(\mathbf{K}_1, k_{\mathrm{e}}, \mathbf{K}_2)$ are the global symmetrical mass, damping and stiffness matrixes of the couple of blades with a friction element and $\mathbf{f} = [\,\mathbf{0}\ \ 0\ \ \mathbf{f}_2\,]^{\mathrm{T}}$ is the global vector of amplitudes of excitation. Using particular solution in the form $\tilde{\mathbf{q}}(t) = \tilde{\mathbf{q}}\,\mathrm{e}^{\mathrm{i}\omega t}$ we are able to isolate the vector of complex amplitudes of the generalized coordinates $\tilde{\mathbf{q}}$,

$$\tilde{\mathbf{q}} = [-\omega^2\,\mathbf{M} + \mathrm{i}\,\omega\,(\mathbf{B} + \mathbf{B}_\mathrm{e}) + \mathbf{K}]^{-1}\,\mathbf{f}\ . \tag{26}$$

Now an iterative process can start. First of all we choose the matrix $\mathbf{B}_\mathrm{e} = \mathbf{0}$ and compute the amplitudes of slip displacements

$$\begin{aligned} a_\mathrm{A} &= |\tilde{v}_6^{(1)} + r_\mathrm{A}\,\tilde{\varphi}_6^{(1)} - \tilde{v}_\mathrm{e}|\ , \\ a_\mathrm{B} &= |\tilde{v}_6^{(2)} - r_\mathrm{B}\,\tilde{\varphi}_6^{(2)} - \tilde{v}_\mathrm{e}|\ , \end{aligned} \tag{27}$$

where the values marked with a tilde are the corresponding components of the vector $\tilde{\mathbf{q}}$. After that we specify the matrix $\mathbf{B}_\mathrm{e} = \mathbf{B}_\mathrm{e}(a_\mathrm{A}, a_\mathrm{B}, \omega)$ and repeat all this process. The vector of amplitudes of the dynamic displacements from the static equilibrium position is

$$\mathbf{q}_\mathrm{d} = |\tilde{\mathbf{q}}|\ . \tag{28}$$

In the second stage we have to add to the vector $\mathbf{q}_\mathrm{d}$ the vector of static displacements caused by the weights of the blades and the friction element in the form

$$\mathbf{q}_\mathrm{st} = \mathbf{K}^{-1}\,\mathbf{f}_\mathrm{st}\ , \tag{29}$$

where the vector $\mathbf{f}_\mathrm{st} = -\,[\,\mathbf{f}_{1\mathrm{g}}\ \ m_\mathrm{e}\,g\ \ \mathbf{f}_{2\mathrm{g}}\,]^\mathrm{T}$ expresses the blades' and friction element's weights concentrated in the nodal points of the blades and the centre of gravity of the friction element.

The total vector of amplitudes of the generalized displacements is

$$\mathbf{q} = \mathbf{q}_\mathrm{d} + \mathbf{q}_\mathrm{st}\ . \tag{30}$$

4. Numerical simulation results

The numerical simulation conditions have been chosen according to the experiment realized in IT AS CR. The frictional characteristics have been approximated in the form $T = f\,N\ \mathrm{tgh}(\kappa\,v_\mathrm{s})$ for $\kappa = 100$ because of achieving the top micro-slip velocity about $0.01\,\mathrm{m\,s^{-1}}$. The amplitude of the exciting force in vertical direction has been set to $F_0 = 1\,\mathrm{N}$, the friction element is pulled into a wedge gap in radial direction by the pulling force $F_\mathrm{C} = 0.5\,\mathrm{N}$, the coefficient of a friction in the contact points has been selected in the interval $f \in \langle 0.2, 0.5\rangle$ and the mass $\Delta m = 0.0468\,\mathrm{kg}$ of the detuning body has been mounted on the shroud of an unexcited blade. The frequency of exciting force has been changing in the neighbourhood of the lowest resonance frequency corresponding to the bending vibration ($f \in \langle 120, 140\rangle\,\mathrm{Hz}$). Provided that we have tested the computational model in this interval of exciting frequencies and for relatively small ratios of friction forces to exciting force the torsional displacements of the blade shrouds are very small and we can consider that the vertical slip displacements in the contact areas between the blade shrouds and friction element are continuous.

We have dealt with an ascertaining of amplitude-frequency responses of the transversal blade shroud displacements in blade shroud centres of gravity. The numerical simulation results are represented on Figures 5–8 and they have been compared with the results of

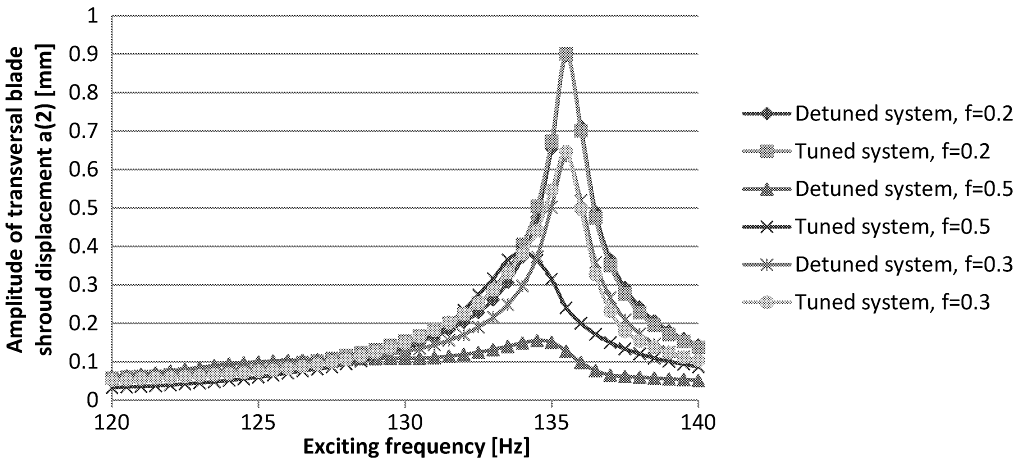

Fig.5: Amplitude-frequency responses of excited blade transversal displacements

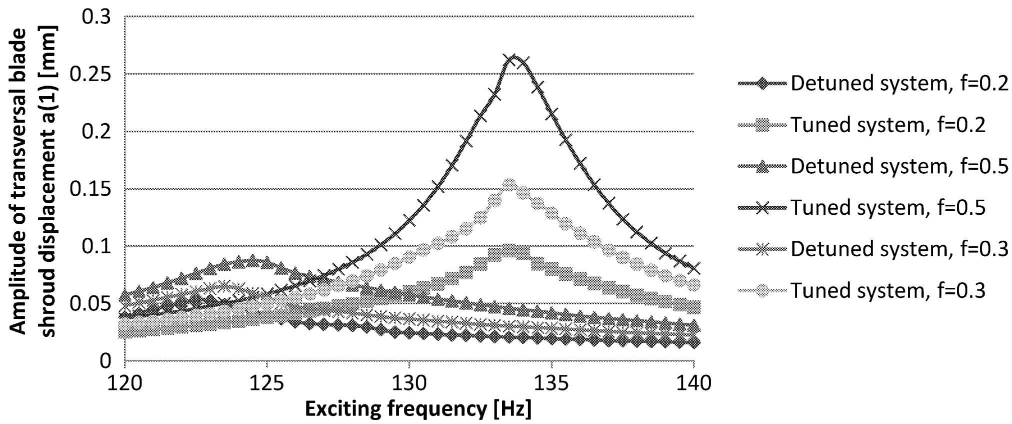

Fig.6: Amplitude-frequency responses of unexcited blade transversal displacements

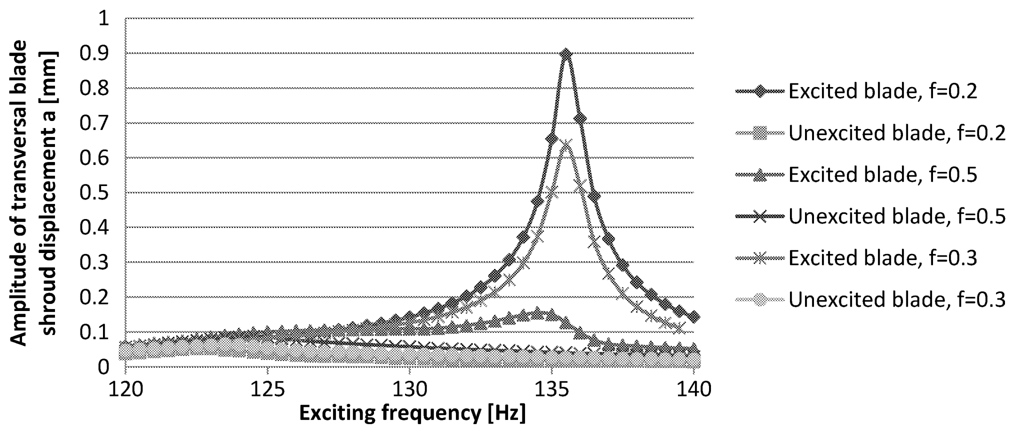

Fig.7: Amplitude-frequency responses of detuned system transversal displacements

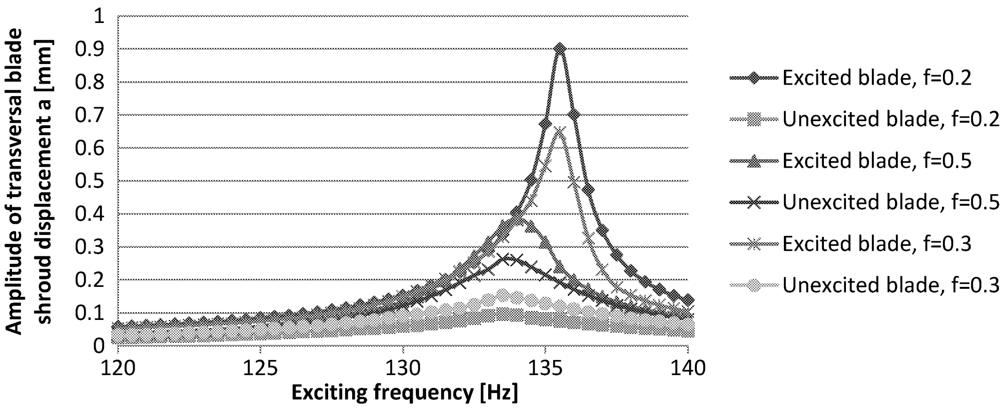

Fig.8: Amplitude-frequency responses of tuned system transversal displacements

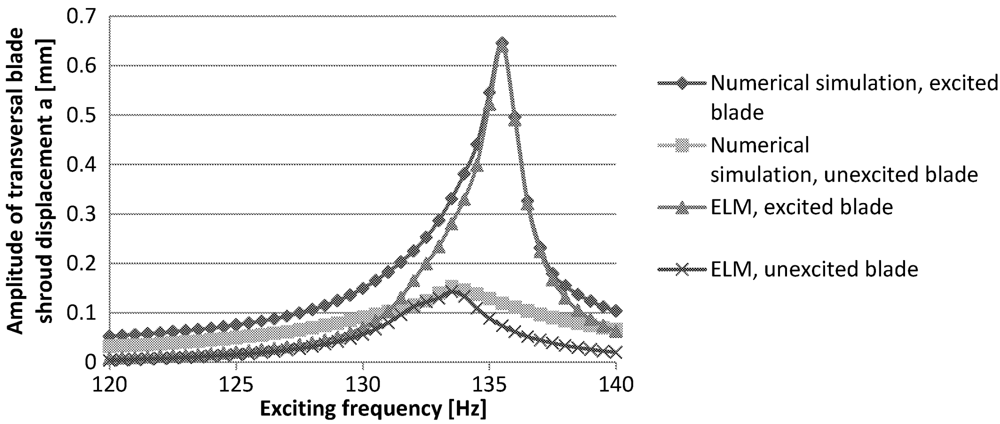

Fig.9: Comparison of amplitude-frequency responses of tuned system transversal displacements with coefficient of friction $f = 0.3$

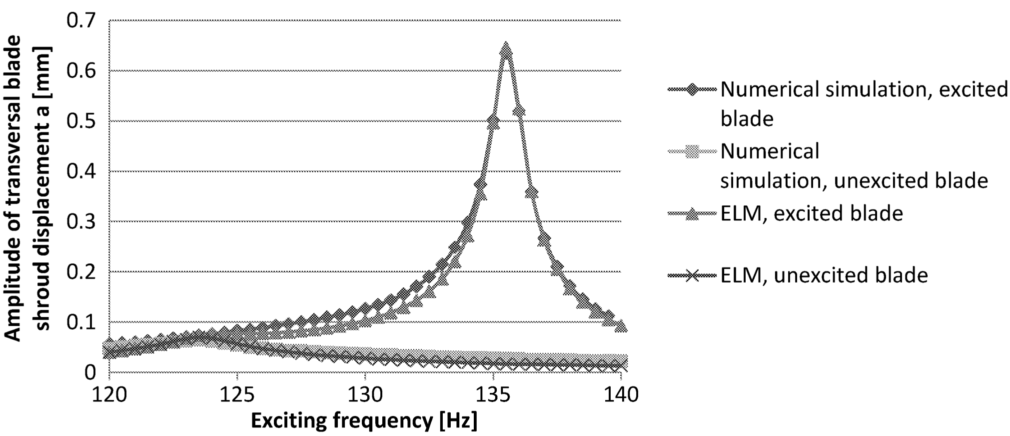

Fig.10: Comparison of amplitude-frequency responses of detuned system transversal displacements with coefficient of friction $f = 0.3$

equivalent linearization method (ELM) on Figures 9, 10. The computational program has been created in the MATLAB environment and the numerical simulation method uses the ODE45 numerical solver that is based on an explicit Runge-Kutta formula [16]. The amplitudes of transversal blade shroud displacements have been ascertained by the numerical integration in sufficiently long time interval, when the blade vibrations got steady.

In the first two illustrations, the detuning impact on excited blade (Figure 5) and unexcited blade (Figure 6) is analysed for different coefficients of friction. In the next two pictures, the amplitude-frequency responses of excited and unexcited blade of detuned system (Figure 7) and tuned system (Figure 8) are compared for different coefficients of friction. In the last two pictures, the amplitude-frequency responses of tuned system (Figure 9) and detuned system (Figure 10) ascertained by the equivalent linearization method are compared with the numerical simulations.

5. Conclusions

This paper deals with detuning and friction impact of a couple of blades with a friction element on bending vibrations. The solution of the problem has been solved by two different methods. The numerical simulation using the numerical solver ODE45 in the MATLAB environment is a suitable instrument for a vibration analysis of the blades with friction elements modeled by discrete nonlinear models with small degrees of freedom number. This method is not limited by simplifying assumptions for the friction characteristics and it can be employed in steady and transient cases. The equivalent linearization method is suitable for the linearization of originally nonlinear motion equations on conditions simplified friction characteristics and harmonic excitation.

It has been shown that the detuning of this system has a positive effect in the suppression of bending vibrations providing higher friction, especially in an unexcited blade. Extreme values of amplitudes of the end of unexcited blade have been decreased by using additional detuning mass mounted on the blade shroud of unexcited blade by $30 \div 50\,\%$ and values of amplitudes of the end of excited blade by $40\,\%$, but only providing the highest friction. These conclusions are relevant in case the ratios of the friction forces in contact points to amplitude of exciting force are in range $0.2 \div 1$. Friction forces depend on the pulling force F_{C} effecting the friction element, the slope angle of the wedge gap and the coefficient of friction in the contact areas. Using the equivalent linearization method has proven good conformity with the numerical simulation in the neighbourhood of the lowest eigenfrequency of blades corresponding to bending vibration. Results in the form of amplitude-frequency responses of transversal blade shrouds displacements have been compared with an experiment performed in IT AS CR [9] and they have shown to be very similar.

Acknowledgement

This work has been elaborated in a frame of the grant project GA CR 101/109/1166 and was supported by the research project MSM 4977751303 of the Ministry of Education, Youth and Sports of the Czech Republic.

References

[1] Cigeroglu E., An N., Menq C.H.: A microslip friction model with normal load variation induced by normal motion, Nonlinear Dynamics, 50 (2007) 609–626

[2] Firrone C.M., Zucca S.: Underplatform dampers for turbine blades: The effect of damper static balance on the blade dynamics, Mechanics Research Communications, 36 (2009) 515–522

[3] Borrajo J.M., Zucca S., Gola M.M.: Analytical formulation of the Jacobian matrix for non-linear calculation of forced response of turbine blade assemblies with wedge friction dampers, International Journal of Non-linear Mechanics, 41 (2006) 1118–1127

[4] Csaba G.: Forced response analysis in time and frequency domains of a tuned bladed disk with friction dampers, Journal of Sound and Vibrations, 214 (1998) 395–412

[5] Brepta R., Půst L., Turek F.: Mechanical vibration, Sobotáles, Praha, 1994 (in Czech)

[6] Sextro W.: Dynamical Contact Problems with Friction, Springer, Berlin, Heidelberg, 2007

[7] Zeman V., Byrtus M., Hajžman M.: Harmonic forced vibration of two rotating blades with friction damping, Engineering Mechanics, 17 (2010) 187

[8] Kellner J., Zeman V.: Modelling of bladed disk vibration with damping elements in blade shroud, Applied and Computational Mechanics, 4 (1) (2010) 37-48

[9] Půst L., Pešek L., Veselý J., Radolfová A.: Detuning impact of a beam with frictional damper on resonance tops heights, Research report Z-1441/09, IT AS CR, v.v.i, 2009 (in Czech)

[10] Půst L., Horáček J., Radolfová A.: Vibrations of beams with frictional surfaces on perpendicular shrouds, Research report Z-1421/08, IT AS CR, 2008 (in Czech)

[11] Půst L., Pešek L.: Non-proportional nonlinear damping in experimental bladed disk, Engineering Mechanics, 17 (2010) 237

[12] Brůha J., Zeman V.: Bending vibration of a blade with friction contact surface, Extended abstracts of Student Scientific Conference 2010, ZČU, Plzeň, 2010 (in Czech)

[13] Brůha J., Zeman V.: Detuning impact of couple of blades with friction element on bending oscillation, Extended abstracts of Computational mechanics 2010, ZČU, Plzeň, 2010

[14] Slavík J., Stejskal V., Zeman V.: Basics of machines dynamics, ČVUT, Praha, 1997 (in Czech)

[15] Trebuňa F., Jurica V., Šimčák F.: Elasticity and strength II, Vienala Košice, 2000 (in Slovak)

[16] Míka S., Brandner M.: Numerical methods I, ZČU, Plzeň, 2000 (in Czech)

Received in editor's office: April 13, 2011
Approved for publishing: May 31, 2011

Note: This paper is an extended version of the contribution presented at the national colloquium with international participation *Dynamics of Machines 2011* in Prague.