

## GENERATOR OF COMMAND SIGNALS FOR TESTING SERVOMECHANISMS OF PAN AND TILT DEVICES

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*The pan and tilt devices (P&TD) are often used for mounting of camera and antenna systems which must track moving targets precisely and speedily in many applications. For the basic testing of the adjustment and the quality of their positioning servomechanisms, the unit step functions of position or velocity are used as command signals. We have developed the program SNBP (Simulator of Random Excitation Processes) for the complex testing. The algorithms description of its foregoer EFG (Excitation Functions Generator) was published in the conference Engineering Mechanics 2004. As time goes on, it has shown the necessity to develop a connecting link – the generator of only basic command signals necessary in the middle phase during servos testing. We have utilized the traditional model of a target movement, i.e. the hypothesis about its uniform straight-line motion. This model is not able to generate a correct command signal for the elevation motion control in the range greater than  $\pm 90^\circ$ . At present, P&TDs are made with substantially greater elevation ranges. Therefore we have remade completely the model. The simulation model, which we present now, is able to generate the command signal for the unlimited traverse motion and for the elevation motion, too.*

**Keywords:** pan and tilt device (P&TD), positional servomechanism, basic command signals generator, passive optoelectronic rangefinder (POERF)

### 1. Motion model of the target

A basic clarification of the simulation target movement model is in Fig. 1–5. A pan and tilt device (P&TD) is placed in the point  $B$  (Fig. 1, 4, 5). Its traverse (pan) axis is perpendicular to the horizontal plane (tilts  $\delta_1, \delta_2 = 0$ , thus  $\varphi = \varepsilon_S, \psi = \psi_a$  – Fig. 1, 2) and it intersects the elevation (tilt) axis just in the point  $B$  (Fig. 1). Due to simplicity, we assume that the Line-of-Sight (LOS) of the camera lens passes through the same point and that the LOS is directed precisely to the target point  $T$ , which represents the target. Consequently, the target point  $T$  is identical to the aiming point. The non-simplified description of the configuration was published in [2] – Fig. 2.

The target is moving uniformly rectilinearly and so its trajectory is determined explicitly by the ground speed vector  $\mathbf{v}_T = (v_T, \alpha_T, \lambda_T)$ . The movement proceeds in the vertical target course over ground plane (track plane) (Fig. 3 – a set of  $T$ -points, specially  $T_0, T_H, T_A, T$ ). The target course over ground (track) is given by the unit vector of its speed, consequently by the angles  $(\alpha_T, \lambda_T)$ , where  $\alpha_T$  is the actual track bearing (azimuth) – Fig. 5,  $\lambda_T$  is the angle of course pitch over ground ( $\lambda_T = 0$  – ‘constant altitude’,  $\lambda_T > 0$  – pitching angle,  $\lambda_T < 0$  – diving angle) – Fig. 3, 4.

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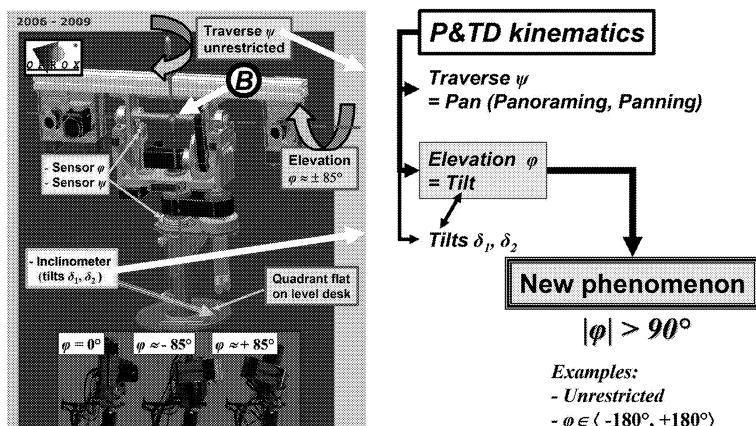


Fig.1: The example of P&T device – a subsystem of the Passive Optoelectronic Rangefinder (POERF), demonstration model 2009 [3] – and its kinematics

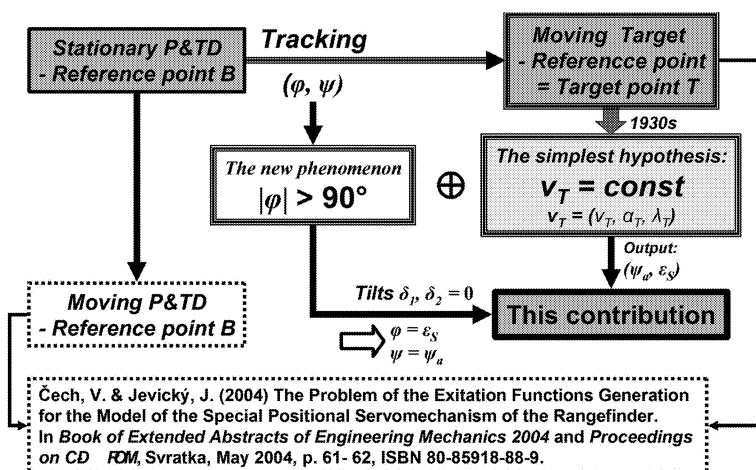


Fig.2: Delimitation of the content of this contribution [2, 4].

The shortest horizontal range  $d_{TB}$  from the point  $B$  to the track plane (the line segment  $BP_C$ ) is denoted as (azimuthal) course (track) parameter  $|p_A| = \min d_{TB}$  ( $d_{TB} \geq 0$ ). If  $p_A = 0$ , then it is called as ‘coming course (track)’; if  $p_A \neq 0$ , then it is called as ‘crossing course (track)’ – Fig. 4. It is presumed traditionally, that  $p_A \geq 0$ . For simplification of calculations, we will assume that  $p_A$  is a real number – Fig. 5.

Vertically over the point  $P_C$  there is lying so-called midpoint  $T_A$  of the Course (Track). Actual path  $s_A$  is contractually equal to zero in this point, i.e.  $s_A(T_A) = 0$  – Fig. 3. It is valid contractually that  $s_A = v_T t_A$ , where  $t_A$  is contractual time of the target motion. The half-line is denoted as ‘approaching leg’ for  $t_A < 0$  and as ‘receding leg’ for  $t_A > 0$  – Fig. 3. In calculations there is used the actual horizontal path  $x_A = s_A \cos \lambda_T$ , which lies in the target horizontal track. The horizontal increments of topographical coordinates of the system UTM relative to the point  $B$  are (Fig. 5)

$$\begin{aligned} \Delta E_T &= x_A \sin \alpha_T + p_A \sin(\alpha_T + 270^\circ), \\ \Delta N_T &= x_A \cos \alpha_T + p_A \cos(\alpha_T + 270^\circ). \end{aligned} \quad (1)$$



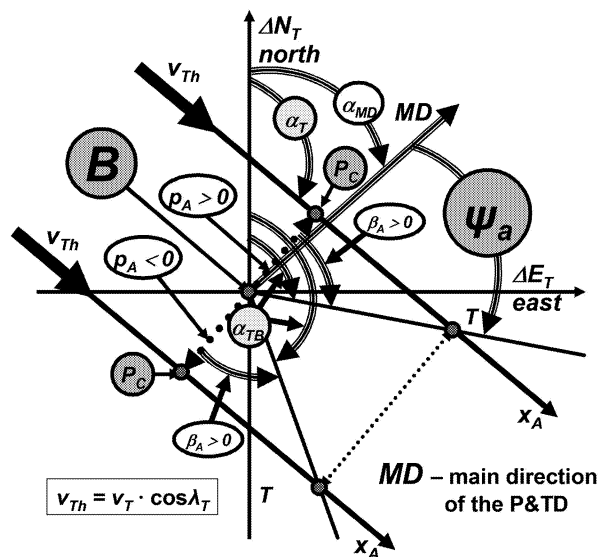
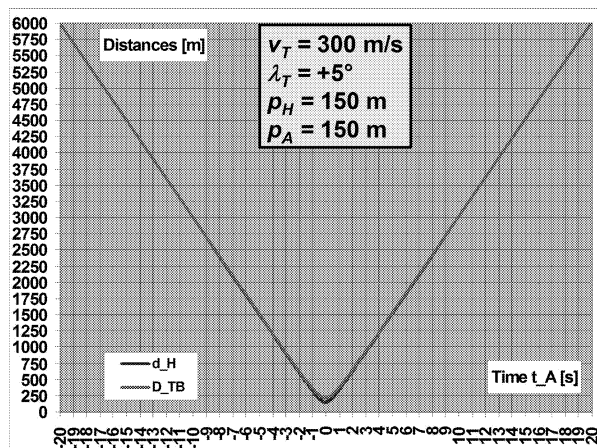


Fig.5: The third diagram for explanation of geometric relations

Fig.6: The example showing the change of the distances  $d_H$  and  $D_{TB}$  during simulation (a variant)

An instantaneous position of the target point  $T$  is determined by the pair of angles  $\beta_A \in \langle -90^\circ, +90^\circ \rangle$ ,  $\beta_H \in \langle -90^\circ, +90^\circ \rangle$  (Fig. 3, 4, 5). Their sizes can be calculated easily from values  $(v_T, \lambda_T, \rho_H, \rho_A)$ , which determine simulated motion of the target, and from the chosen time  $t_A$ . It is valid for  $t_A \in (-\infty, +\infty)$

$$s_A = v_T t_A, \quad s_H = s_A + \Delta s_{HA}, \quad \Delta s_{HA} = \rho_H \tan \lambda_T, \quad (4)$$

$$\beta_H = \arctan \frac{s_H}{|\rho_H|}, \quad \beta_A = \arctan \frac{x_A}{|\rho_A|} \quad \text{for } x_A = s_A \cos \lambda_T, \quad (5)$$

$$\gamma = \arctan \frac{|\rho_A|}{d_H}, \quad d_{TB} = \frac{|\rho_A|}{\cos \beta_A} \quad \text{for } \rho_A \neq 0. \quad (6)$$

## 2. Usage of model of target movement for generating command signals $\varepsilon_S$ , $\psi_a$

It can be determined consequently with the use of angles  $\beta_A$ ,  $\beta_H$ :

- a) The angular height of the target point  $T$  angle detected by an elevation angle sensor (Fig. 1, 7)

$$\varepsilon_S = \begin{cases} [\beta_H + (90^\circ - \lambda_T)] \operatorname{sgn}(p_H) & \text{if } p_A = 0, \\ \arctan \frac{\Delta H_T}{d_{TB}} & \text{otherwise,} \end{cases} \quad \varepsilon_S \in \langle -180^\circ, +180^\circ \rangle. \quad (7)$$

- b) Traditional angular height of the target point  $T$  (Fig. 4, 7)

$$\varepsilon_{TB} = \begin{cases} \arcsin(\sin \varepsilon_S) & \text{if } p_A = 0, \\ \varepsilon_S & \text{otherwise,} \end{cases} \quad \varepsilon_{TB} \in \langle -90^\circ, +90^\circ \rangle. \quad (8)$$

Henceforth, we will consider the basic variant for simulations, which is given by the condition  $p_A \neq 0$ . It always holds under this condition that  $\varepsilon_{TB} = \varepsilon_S$  and  $|\varepsilon_S| < 90^\circ$ . The target is tracked simultaneously in the elevation  $\varphi$  and the traverse  $\psi$ . We add yet contractually two other variants to the basic one, which are determined for the same parameters, but for  $p_A = 0$ . We will use the new denotation  $(\varepsilon_{TB0}, \varepsilon_{S0})$  instead of the common  $(\varepsilon_{TB}, \varepsilon_S)$  – Fig. 7.

The first variant corresponds with the traditional model, i.e. the elevation is considered with limitation just on  $\varepsilon_{TB} = \varepsilon_S \in \langle -90^\circ, +90^\circ \rangle$ , and the target is tracked again simultaneously in the elevation  $\varphi$  and the traverse  $\psi$ . We will denote this variant as  $(p_A = 0, \psi_a = \text{var resp. } |\varphi| \leq 90^\circ)$ .

The second variant presupposes that the elevation is unlimited ( $\varepsilon_S$  is commonly unlimited). The target is tracked with the use of the elevation movement  $\varphi$  only, whereas the traverse  $\psi$  is not varying ( $\psi = \text{const}$ ; the traverse movement control is ‘off’). We will denote this variant as  $(p_A = 0, \psi_a = \text{const resp. } |\varphi| > 90^\circ)$ .

Under real conditions, the second variant can be used also in situations, where  $p_A \neq 0$ . The uncompensated control deviation  $e = \gamma \operatorname{sgn}(p_A)$  then arises, where  $\gamma$  is its absolute value (see relations (6) and Fig. 4, 7a). The given procedure can be used, if the target tracking is done with the use of a camera whose lens supports sufficiently large angle of view. The angle of view must be of such size, that the target occurs in the field of view during all observed action and the automatic algorithm can evaluate control deviations  $e$  sufficiently accurately. The size of the control deviations is consequently exploited in relations for estimates of the UTM coordinates of the target. More detailed analysis of this problem exceeds this article.

- c) Hereafter, it is valid for  $p_A = 0$  (Fig. 7):

$$\dot{\beta}_H = \dot{\beta}_{Hm} \cos^2 \beta_H, \quad \dot{\beta}_{Hm} = \frac{v_T}{|p_H|}, \quad (9)$$

$$\begin{aligned} \ddot{\beta}_H &= -3.079202 \ddot{\beta}_{Hm} \cos^3 \beta_H \sin \beta_H, \\ \ddot{\beta}_{Hm} &= 0.649519 \left( \frac{v_T}{|p_H|} \right)^2 \quad \text{for } \beta_H = \pm 30^\circ. \end{aligned} \quad (10)$$

It holds for the mechanical power of the elevation traction

$$\begin{aligned} P_{\beta_H} &= J_\varphi \dot{\beta}_H \ddot{\beta}_H, \quad |P_{\beta_H}| \leq P_{Hm}, \\ P_{Hm} &= 0.517608 J_\varphi \left| \frac{v_T}{p_H} \right|^3 \quad \text{for } \beta_H = \pm 24^\circ 5' 41.4'', \end{aligned} \quad (11)$$

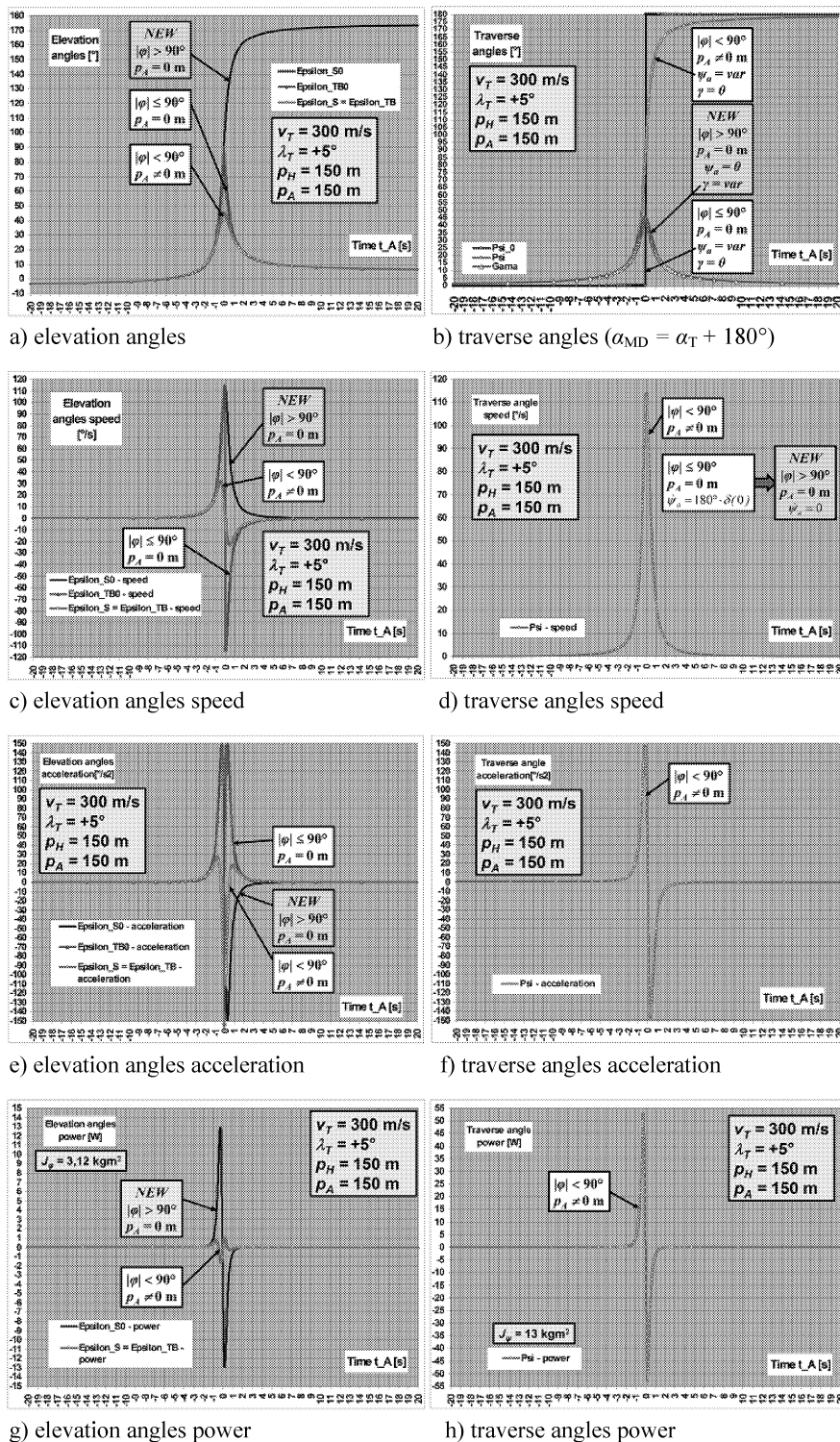


Fig.7: Example – results of simulation calculations of command signals for testing P&TD

where  $J_\varphi$  is the reduced moment of inertia of the mechanical system with respect to the elevation axis including influences of the mechanical efficiency [kg m<sup>2</sup>] – Fig. 7g.

Furthermore

$$\begin{aligned}\dot{\varepsilon}_S &= \dot{\beta}_H \operatorname{sgn}(p_H) , & \dot{\varepsilon}_{TB} &= -\dot{\varepsilon}_S \operatorname{sgn}(\beta_H - \lambda_T) , \\ \ddot{\varepsilon}_S &= \ddot{\beta}_H \operatorname{sgn}(p_H) , & \ddot{\varepsilon}_{TB} &= \left[ |\ddot{\beta}_S| - 2 \dot{\beta}_H (\lambda_T) \delta(\beta_H - \lambda_T) \right] \operatorname{sgn}(p_H) ,\end{aligned}\quad (12)$$

where  $\delta(\beta_H - \lambda_T)$  is the Dirac delta function. It is evident from the relations (11), why the variant ( $p_A = 0$ ,  $\psi_a = \text{const}$ ) is more advantageous than the first ‘traditional’ variant (Fig. 7c, e).

d) Traditional value of the bearing (azimuth) of the target point  $T$  (Fig. 5)

$$\alpha_{TB} = \begin{cases} (\alpha_T + 180^\circ) + (\beta_{AE0} + 90^\circ) & \text{if } p_A = 0 , \\ (\alpha_T + 180^\circ) + (\beta_A + 90^\circ) \operatorname{sgn}(p_A) & \text{otherwise ,} \end{cases} \quad \alpha_{TB} \in (0, 360^\circ) , \quad (13)$$

where the definition of the angle  $\beta_A$  is extended for the case of  $p_A = 0$  by the value

$$\beta_{AE0} = \begin{cases} 90^\circ \operatorname{sgn}(\beta_H - \lambda_T) & \text{in the traditional model } (\psi_a = \text{var}) , \\ -90^\circ & \text{in the new model } (\psi_a = \text{const}) . \end{cases} \quad (14)$$

Then

$$\dot{\beta}_{AE0} = \begin{cases} 180^\circ \delta(\beta_H - \lambda_T) & \text{if } \psi_a = \text{var} , \\ 0 & \text{if } \psi_a = \text{const} . \end{cases} \quad (15)$$

It is evident from (14), why the variant ( $p_A = 0$ ,  $\psi_a = \text{const}$ ) is more advantageous than the first ‘traditional’ variant (Fig. 7c, e).

e) The absolute traverse angle (pan) is defined as

$$\psi_a = \alpha_{TB} - \alpha_{MD} , \quad (16)$$

where  $\alpha_{MD}$  is the bearing (azimuth) of main direction of pan and tilt device (its orientation is towards the north in the horizontal plane) – Fig. 5. The absolute traverse is usually detected in the range  $\psi_a \in (0^\circ, 360^\circ)$ , but it must be taken in the range  $\psi_a \in (-\infty, +\infty)$  for simulations.

f) For the basic variant ( $p_A \neq 0$ ), it holds

$$\dot{\varepsilon}_S = \dot{\varepsilon}_{TB} = \frac{1}{d_{TB}} (v_t \sin \lambda_T \operatorname{sgn}(p_H) - \dot{D}_{TB} \sin \varepsilon_{TB}) , \quad \dot{D}_{TB} = \frac{v_T |p_H|}{D_{TB}} \tan \beta_H , \quad (17)$$

$$\ddot{\varepsilon}_S = \ddot{\varepsilon}_{TB} = \frac{1}{d_{TB}} [(\dot{\varepsilon}_{TB}^2 D_{TB} - \ddot{D}_{TB}) \sin \varepsilon_{TB} - 2 \dot{\varepsilon}_{TB} \dot{D}_{TB} \cos \varepsilon_{TB}] , \quad (18)$$

$$\ddot{D}_{TB} = \frac{1}{D_{TB}} (v_T^2 - \dot{D}_{TB}^2) ,$$

$$\dot{\beta}_A = \dot{\beta}_{Am} \cos^2 \beta_A , \quad \dot{\beta}_{Am} = \frac{v_T \cos \lambda_T}{|p_A|} , \quad (19)$$

$$\ddot{\beta}_A = -3.079202 \ddot{\beta}_{Am} \cos^3 \beta_A \sin \beta_A ,$$

$$\ddot{\beta}_{Am} = 0.649519 \left( \frac{v_T \cos \lambda_T}{|p_A|} \right)^2 \quad \text{for } \beta_A = \pm 30^\circ . \quad (20)$$

Furthermore

$$\dot{\psi}_a = \dot{\alpha}_{TB} = \dot{\beta}_A \operatorname{sgn}(p_A) , \quad \ddot{\psi}_a = \ddot{\alpha}_{TB} = \ddot{\beta}_A \operatorname{sgn}(p_A) . \quad (21)$$

It holds for the mechanical power of the traverse traction

$$P_{\beta_A} = J_\psi \dot{\beta}_A \ddot{\beta}_A , \quad |P_{\beta_A}| \leq P_{Am} , \quad (22)$$

$$P_{Am} = 0.517608 J_\psi \left| \frac{v_T \cos \lambda_T}{p_A} \right|^3 \quad \text{for } \beta_A = \pm 24^\circ 5' 41.4'' ,$$

where  $J_\psi$  is the reduced moment of inertia of the mechanical system with respect to the traverse axis including influences of the mechanical efficiency [kg m<sup>2</sup>] – Fig. 7h.

### 3. Simulation of the change in aiming from one target to the another one in the start of the action

Let us suppose that it has been aimed to a previous stationary target  $(\varepsilon_{S1}, \varepsilon_{a1})$ . The system has received the command to track another target at the moment  $t_{A0}$ , when this target has been situated in the point  $T_0$  (Fig. 3, 4) with corresponding angles  $(\varepsilon_S(0) = \varepsilon_{TB}(0), \psi_a(0) \text{ for } p_A \neq 0)$ , resp.  $(\varepsilon_{S01}(0) = \varepsilon_{TB01}(0), \psi_{a01}(0) \text{ for } p_A = 0 \text{ and } \psi_a = \text{var})$  and  $(\varepsilon_{S02}(0), \psi_{a02}(0) \text{ for } p_A = 0 \text{ and } \psi_a = \text{const})$ .

We will introduce the relative simulation time  $t = t_A - t_{A0}$ , which value for the point  $T_0$  of the target trajectory is  $t = 0$ .

In many cases, it is convenient to respect the traditional assignment of command signals [1, 8], where it is claimed that for  $t < 0$  all command signals must be equal to zero. In our case, it will be valid e.g. for the basic variant ( $p_A \neq 0$ ), that

$$\Delta \varepsilon_{TB} = \begin{cases} [\varepsilon_{TB} - \varepsilon_{TB}(0)] + \Delta \varepsilon_{TB}(0) , & \Delta \varepsilon_{TB}(0) = \varepsilon_{TB}(0) - \varepsilon_{S1} \quad \text{if } t \geq 0 , \\ 0 & \text{otherwise ,} \end{cases} \quad (23)$$

$$\Delta \psi_a = \begin{cases} [\psi_a - \psi_a(0)] + \Delta \psi_a(0) , & \Delta \psi_a(0) = \psi_a(0) - \psi_{a1} \quad \text{if } t \geq 0 , \\ 0 & \text{otherwise ,} \end{cases} \quad (24)$$

where the next inputs to simulations are the time  $t_{A0}$  and arbitrarily chosen sizes of jumps  $\Delta \varepsilon_{TB}(0)$  and  $\Delta \psi_a(0)$  in the time  $t = 0$ . Analogously for remaining two variants with  $p_A = 0$ , too.

The choice of the point  $T_0$  (resp. the time  $t_{A0}$ ) depends usually on demands on sizes of respectively elevation and traverse speeds and accelerations in this point ('jumps' at speed and acceleration in the time  $t = 0$ ).

### 4. Tracking of the target from a moving P&TD

Let us suppose that P&TD is placed on a moving platform, which vector of speed  $\mathbf{v}_B$  is a constant, i.e.  $\mathbf{v}_B = (v_B, \alpha_B, \lambda_B) = \mathbf{const}$ , where the angle  $\lambda_B$  is analogous to the angle  $\lambda_T$  and the angle  $\alpha_B$  is analogous to the angle  $\alpha_T$ . So we omit influences of the platform vibrations on the motion of the point  $B$  (Fig. 3, 4, 5).

The horizontal trajectory of the point  $B$  is

$$x_B = v_B t_A \cos \lambda_B . \quad (25)$$



If the position of the point  $B$  in the time  $t_A = 0$  is given in the coordinate system UTM,  $B = (E_{B0}, N_{B0}, H_{B0})$ , then the change of position of the point  $B$  towards its initial position  $B_0$  is

$$\begin{aligned}\Delta E_B &= E_B - E_{B0} = x_B \sin \alpha_B, \\ \Delta N_B &= N_B - N_{B0} = x_B \cos \alpha_B, \\ \Delta H_B &= H_B - H_{B0} = x_B \tan \lambda_B.\end{aligned}\tag{26}$$

We will assume that the vector of the target speed  $\mathbf{v}_T = (v_T, \alpha_T, \lambda_T)$  is the vector of the target relative speed towards the moving point  $B$ , accordingly  $\mathbf{v}_T = \mathbf{v}_{TA} - \mathbf{v}_B$ , where  $\mathbf{v}_{TA}$  is the vector of the target absolute speed towards the ground,  $\mathbf{v}_{TA} = (v_{TA}, \alpha_{TA}, \lambda_{TA})$ . This vector can be determined from the vector equation  $\mathbf{v}_{TA} = \mathbf{v}_B + \mathbf{v}_T$ . This assumption means that the vector of the relative speed of the target  $\mathbf{v}_T$  is of crucial importance for simulations of aiming to the target and therefore it is suitable to assign it as the basic speed, while the size of the vector  $\mathbf{v}_{TA}$  is only of secondary significance for simulations.

It holds true for coordinates of the point  $T$  towards the point  $B_0$

$$\begin{aligned}\Delta E_{TA} &= E_{TA} - E_{B0} = \Delta E_T + \Delta E_B, \\ \Delta N_{TA} &= N_{TA} - N_{B0} = \Delta N_T + \Delta N_B, \\ \Delta H_{TA} &= H_{TA} - H_{B0} = \Delta H_T + \Delta H_B.\end{aligned}\tag{27}$$

where the relative coordinates of the point  $T$  towards the point  $B$  are given by relations (1) and (2).

## 5. Conclusions

The traditional model had been developed gradually since the end of 19<sup>th</sup> century and it was finalized in the second half of 1930s [5, 6, 7]. The model does not differentiate between angles  $\varepsilon_S$  and  $\varepsilon_{TB}$ . As it is obvious from given relations and from the graphs in Fig. 7, this simplification represents a serious defect, which fully takes effect during simulations of elevation motions in systems with unlimited elevation. Our contribution consists in recognition of this problem and its elimination.

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