UNSTEADY HARTMANN FLOW WITH HEAT TRANSFER OF A VISCOELASTIC FLUID UNDER EXPONENTIAL DECAYING PRESSURE GRADIENT

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The unsteady Hartmann flow of a conducting incompressible non-Newtonian viscoelastic fluid between two parallel horizontal insulating porous plates is studied with heat transfer. A uniform pressure gradient which decays exponentially is imposed in the axial direction. An external uniform magnetic field and uniform suction and injection through the surface of the plates are applied in the vertical direction. The two plates are kept at different but constant temperatures while the Joule and viscous dissipations are considered in the energy equation. Numerical solutions for the governing momentum and energy equations are obtained using finite differences. The effect of the magnetic field, the parameter describing the non-Newtonian behavior, and the velocity of suction and injection on both the velocity and temperature distributions is investigated.

Keywords: *MHD flow, heat transfer, non-Newtonian, viscoelastic, electrically conducting fluids, suction and injection*

1. Introduction

The Hartmann flow of an electrically conducting viscous incompressible fluid between two parallel plates in the presence of a transversely applied uniform magnetic field has attracted the attention of many researchers due to its interesting applications in many areas such as magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and fluid droplets-sprays [1–8]. The effect of uniform suction and injection through the parallel plates on unsteady Hartmann flow of a conducting Newtonian fluid was given by Attia [9, 13]. The hydrodynamic flow of a non-Newtonian viscoelastic fluid was handled by many authors [14–17] as it has important industrial applications.

In the present paper, the flow of an electrically conducting non-Newtonian viscoelastic fluid is studied with heat transfer in the presence of a uniform magnetic field and an imposed exponential decaying pressure gradient. The flow is subjected to a uniform suction from above and a uniform injection from below, and an external uniform magnetic field perpendicular to the plates. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number [4]. The two plates are kept at two different but constant temperatures whereas the Joule and viscous dissipations are taken into account in the energy

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equation. This configuration is a good approximation to some practical situations such as heat exchanger, flow meters, and pipes that connect system components. The governing momentum and energy equations are solved numerically using finite differences and the effect of the magnetic field, the non-Newtonian fluid characteristics as well as the velocity of suction and injection on both the velocity and temperature fields is reported.

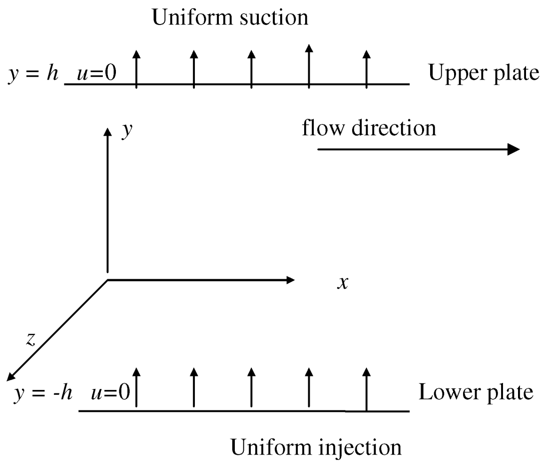

Fig.1: The geometry of the problem

2. Formulation of the Problem

The fluid is incompressible, viscoelastic and flows between two infinite horizontal parallel insulating porous plates located at the $y = \pm h$ planes and extend in the infinite x and z directions, as shown in Fig. 1. The upper and lower plates are kept at two constant temperatures T_2 and T_1 respectively, with $T_2 > T_1$. The flow is driven by a uniform and exponential decaying pressure gradient $\mathrm{d}P/\mathrm{d}x$ in the x-direction, and a uniform suction from above and injection from below which are impulsively applied at $t = 0$. An external uniform magnetic field B_o is applied in the positive y-direction which is assumed to be also the total magnetic field, as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number which is the ratio of the fluid flux to the magnetic diffusivity and is one of the more important parameters in MHD [4]. The plates are assumed to be infinite in the x and z-directions which makes the physical quantities do not change in these directions. Thus, the velocity vector of the fluid, in general, is given by

$$\mathbf{v}(y,t) = u(y,t)\,\mathbf{i} + v_\mathrm{o}\,\mathbf{j} \;.$$

The velocity component in y-direction is assumed to have a constant value v_o because of the uniform suction. The fluid motion starts impulsively from rest at $t = 0$, that is $u = 0$ for $t \leq 0$. The no-slip condition at the plates implies that $u = 0$ at $y = \pm h$. It is also assumed that the initial temperature of the fluid is T_1, thus the initial and boundary conditions of temperature are $T = T_1$ at $t = 0$, $T = T_1$ at $y = -h$, $t > 0$ and $T = T_2$ at $y = h$, $t > 0$. The fluid motion is governed by the momentum equations [18]

$$\varrho\left(\frac{\partial u}{\partial t} + v_\mathrm{o}\,\frac{\partial u}{\partial y}\right) = -\frac{\mathrm{d}P}{\mathrm{d}x} - \sigma\,B_\mathrm{o}^2\,u + \frac{\partial \tau_\mathrm{xy}}{\partial y}\;, \qquad (1)$$

where ϱ and σ are, respectively, the density and the electric conductivity of the fluid. The second term in the right side represents electromagnetic force and $\tau_{\rm xy}$ is the component of the shear stress of the viscoelastic fluid given in Ref. [14] as,

$$\tau_{\rm xy} = \mu \frac{\partial u}{\partial y} - \frac{\mu}{\alpha} \frac{\partial \tau_{\rm xy}}{\partial t} , \tag{2}$$

where μ is the coefficient of viscosity and α is the modulus of rigidity. In the limit α tends to infinity or at steady state, the fluid behaves like a viscous fluid without elasticity. Solving Eq. (2) for $\tau_{\rm xy}$ in terms of the velocity component u we obtain

$$\frac{\partial \tau_{\rm xy}}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{1}{\alpha}\frac{\partial}{\partial y}\left(\mu \frac{\partial}{\partial t}\left(\mu \frac{\partial u}{\partial y}\right)\right) \approx \mu \frac{\partial^2 u}{\partial y^2} - \frac{\mu^2}{\alpha}\frac{\partial^3 u}{\partial t\, \partial y^2} , \tag{3}$$

where the term $(1/\alpha^2)\, \frac{\partial}{\partial y}(\mu \frac{\partial}{\partial t}(\mu \frac{\partial \tau_{\rm xy}}{\partial y}))$, which is proportional to $(1/\alpha^2)$ is neglected. Substituting Eq. (3) in the momentum Eq. (1) yields

$$\varrho\left(\frac{\partial u}{\partial t} + v_{\rm o}\frac{\partial u}{\partial y}\right) = -\frac{{\rm d}P}{{\rm d}x} - \sigma B_{\rm o}^2\, u + \mu \frac{\partial^2 u}{\partial y^2} - \frac{\mu^2}{\alpha}\frac{\partial^3 u}{\partial t\, \partial y^2} . \tag{4}$$

The temperature distribution is governed by the energy equation [18]

$$\varrho\, c_{\rm p}\left(\frac{\partial T}{\partial t} + v_{\rm o}\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \sigma B_{\rm o}^2\, u^2 + \mu\left(\frac{\partial u}{\partial y}\right)^2 , \tag{5}$$

where $c_{\rm p}$ and k are, respectively, the specific heat capacity at constant pressure and the thermal conductivity of the fluid which are assumed constants. The second and third terms on the right side represent the Joule and viscous dissipations respectively. Introducing the following dimensionless variables and parameters

$$\hat{y} = \frac{y}{h} , \qquad \hat{t} = \frac{\mu\, t}{h^2 \varrho} , \qquad \hat{u} = \frac{\varrho\, h\, u}{\mu} , \qquad p = \frac{P\, \varrho\, h^2}{\mu^2} , \qquad \hat{T} = \frac{T - T_1}{T_2 - T_1} ,$$

the suction parameter:

$$S = \frac{\varrho\, h\, v_{\rm o}}{\mu} ,$$

the Hartmann number:

$$Ha = B_{\rm o}\, h \sqrt{\frac{\sigma}{\mu}} ,$$

the magnetic Reynolds number ($\mu_{\rm e}$ is the magnetic viscosity):

$$R\sigma = \mu_{\rm e}\, \sigma\, \frac{\mu}{\varrho} ,$$

the Prandtl number:

$$Pr = \frac{\mu\, c_{\rm p}}{k} ,$$

the Eckert number:

$$Ec = \frac{\mu^2}{\varrho^2\, h^2\, c_{\rm p}\, (T_2 - T_1)} ,$$

the viscoelastic parameter:

$$K = \frac{\mu^2}{\varrho\,\alpha\,h^2} .$$

Equations (4) and (5) may be written as, (all hats are dropped for convenience)

$$\frac{\partial u}{\partial t} + S\,\frac{\partial u}{\partial y} = -\frac{\mathrm{d}P}{\mathrm{d}x} - Ha^2\,u + \frac{\partial^2 u}{\partial y^2} - K\,\frac{\partial^3 u}{\partial t\,\partial y^2} , \tag{6}$$

$$\frac{\partial T}{\partial t} + S\,\frac{\partial T}{\partial y} = \frac{1}{Pr}\,\frac{\partial^2 T}{\partial y^2} + Ha^2\,Ec\,u^2 + Ec\left(\frac{\partial u}{\partial y}\right)^2 . \tag{7}$$

The initial and boundary conditions for the velocity and temperature in the dimensionless form are written as

$$u(y,0) = 0 , \qquad u(-1,t) = u(1,t) = 0 , \tag{8}$$

$$T(y,0) = 0 , \qquad T(-1,t) = 0 , \qquad T(1,t) = 0 , \tag{9}$$

whereas the pressure gradient is assumed in the form $\mathrm{d}P/\mathrm{d}x = C\,\mathrm{e}^{-\alpha t}$. It should be pointed out that we are dealing with dimensionless physical variables such as velocities and temperatures which can be fitted to the common physical ranges in different physical applications.

3. Numerical solution

Equations (6) and (7) represent a system of partial differential equations which is solved numerically under the initial and boundary conditions (8) and (9), using the finite difference method. The Crank-Nicolson implicit method [19] is used at two successive time levels. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximation in the y-direction. The diffusion terms are replaced by the average of the central differences at two successive time-levels. Finally, the resulting block tridiagonal system is solved using the generalized Thomas-algorithm [19]. We define the variables $v = \partial u/\partial y$ and $H = \partial T/\partial y$ to reduce the second order differential Eqs. (6) and (7) to first order differential equations. The finite difference representations for the resulting first order differential Eqs. (6) and (7) take the form

$$\begin{aligned} &\frac{u_{i+1,j+1} - u_{i,j+1} + u_{i+1,j} - u_{i,j}}{2\,\Delta t} + S\,\frac{v_{i+1,j+1} + v_{i,j+1} + v_{i+1,j} + v_{i,j}}{4} = -\frac{dP}{dx} + \\ &+ \frac{(v_{i+1,j+1} + v_{i,j+1}) - (v_{i+1,j} + v_{i,j})}{2\,\Delta y} - Ha^2\,\frac{u_{i+1,j+1} + u_{i,j+1} + u_{i+1,j} + u_{i,j}}{4} - \\ &\qquad - K\,\frac{(v_{i+1,j+1} - v_{i,j+1}) - (v_{i+1,j} - v_{i,j})}{\Delta t\,\Delta y} , \end{aligned} \tag{10}$$

$$\begin{aligned} &\frac{T_{i+1,j+1} - T_{i,j+1} + T_{i+1,j} - T_{i,j}}{2\,\Delta t} + S\,\frac{H_{i+1,j+1} + H_{i,j+1} + H_{i+1,j} + H_{i,j}}{4} = \\ &\qquad = \frac{1}{\mathrm{Pr}}\,\frac{(H_{i+1,j+1} + H_{i,j+1}) - (H_{i+1,j} + H_{i,j})}{2\,\Delta y} + DISP , \end{aligned} \tag{11}$$

where $DISP$ represents the Joule and viscous dissipation terms which are specified in terms of the velocities and their gradients and accordingly, are known from the solution of Eq. (10)

and can be evaluated at the mid point $(i + 1/2, j + 1/2)$ of the computational cell. Computations are made for $C = 5$, $\alpha = 1$, $Pr = 1$, and $Ec = 0.2$. Grid-independence studies show that the computational domain $0 < t < \infty$ and $-1 < y < 1$ can be divided into intervals with step sizes $\Delta t = 0.0001$ and $\Delta y = 0.005$ for time and space, respectively. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when the values of every one of the unknowns u, T, and their gradients differ by less than 10^{-6} for the last two time steps for all values of y.

4. Results and Discussion

Figures 2–3 present the profiles of the velocity u and the temperature T respectively with time for different values of time t and for $K = 0$, 0.5, and 1 and for $Ha = 1$, $S = 1$. As shown in the figures, the profiles are asymmetric about the $y = 0$ plane because of the suction. Figures 2–3 show that, when $K = 0$, u and T reach the steady state monotonically with time. On the other hand, increasing K decreases u and T at small time but increases them at large time. Figures 2b and 2c indicate the effect of K in decreasing the temperature as a result of decreasing the velocity u and its gradient which decreases the dissipations.

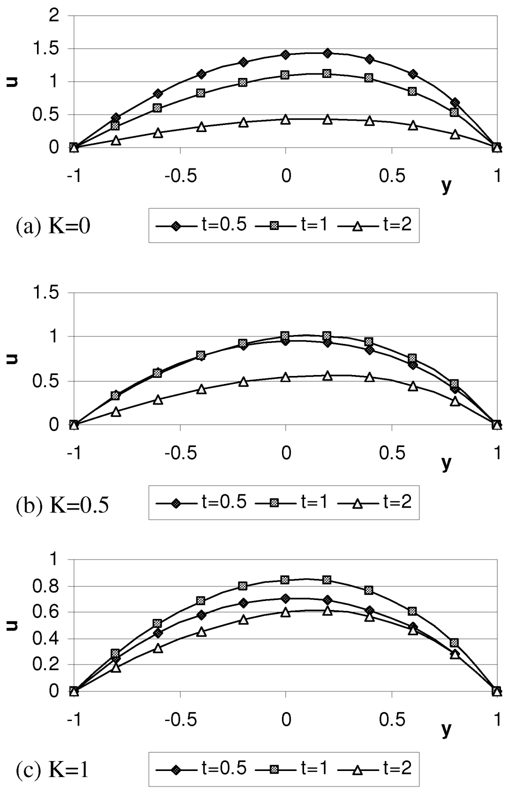

Fig.2: Time variation of the profile of u for various values of K ($Ha = 1$ and $S = 1$)

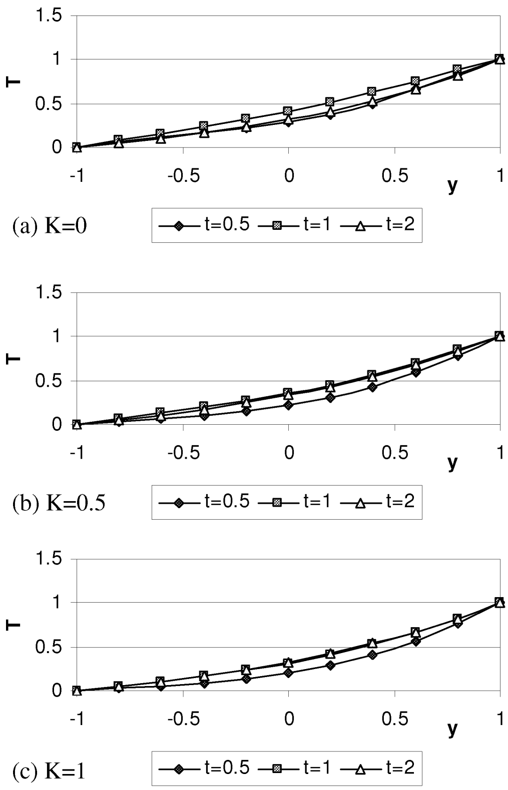

Fig.3: Time variation of the profile of T for various values of K ($Ha = 1$ and $S = 1$)

Figures 4–5 depict the time progression of u and T at the centre of the channel ($y = 0$), respectively for different values of the parameter Ha and for $K = 0$, 0.5, and 1 and for $S = 0$. Increasing the parameter Ha decreases u and the time of approaching its steady state for all values of K because the magnetic resistive force on u increases with the increment in Ha. It is clear from Fig. 4 that the parameter K has a marked effect on the time of approaching its steady state of u and this effect becomes more pronounced for smaller values of Ha.

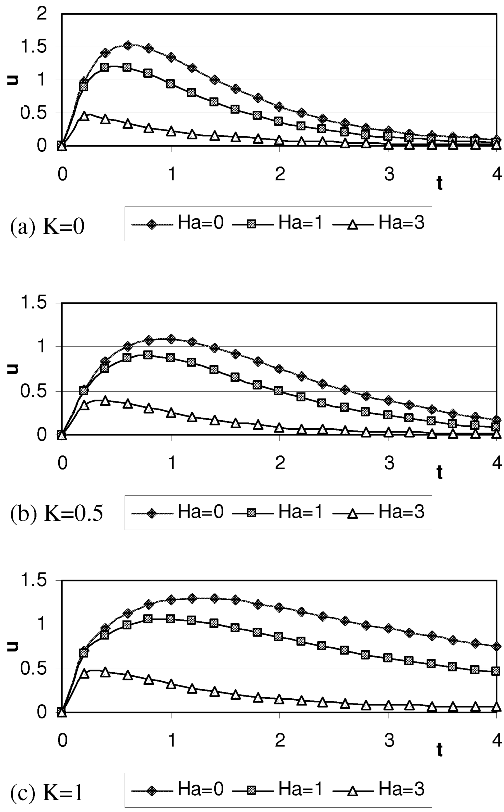

Fig.4: Effect of the parameter Ha on u for various values of K (S = 0)

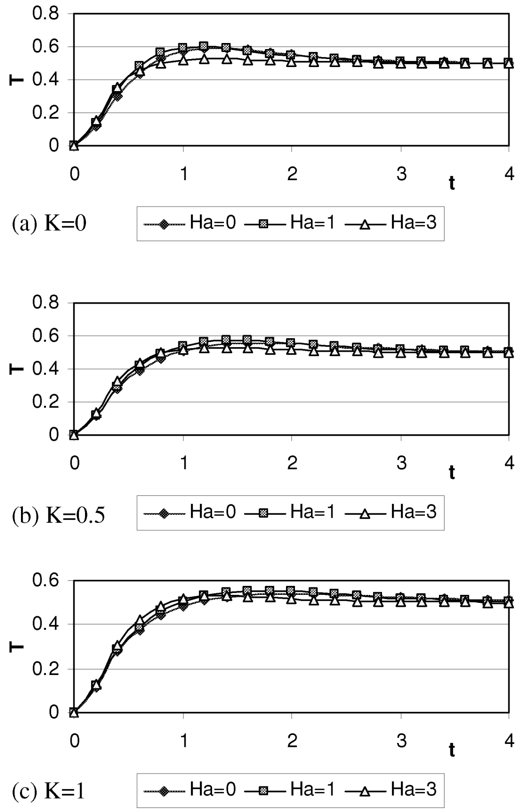

Fig.5: Effect of the parameter Ha on T for various values of K (S = 0)

Figure 5 tells that the effect of the parameter Ha on T depends on time. Increasing Ha increases T at small times but decreases it at large times which can be attributed to the fact that, for small times, an increment in Ha increases the Joule dissipation which is proportional to Ha^2. For large times, increasing Ha decreases u significantly and, consequently, decreases the Joule and viscous dissipations which accounts for crossing $T(t)$ curves for all values of K. Figures 5b and 5c indicate that such a crossover becomes more pronounced for higher K. The effect of changing K on the steady state time of T may be neglected for all values of Ha.

Figures 6–7 show the time progression of u and T at the centre of the channel, respectively for different values of the suction parameter S and for $K = 0$, 0.5, and 1 and for $Ha = 1$. Figure 6 indicates that increasing the suction decreases u due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed. The parameter K has a marked effect on the steady state time of u for all values of S. Figure 7 presents that increasing S decreases the temperature at the centre of the channel for all values of K and t. This is due to the effect of convection in pumping the fluid from the cold lower half towards the centre of the channel. The effect of S on T is clear for all K.

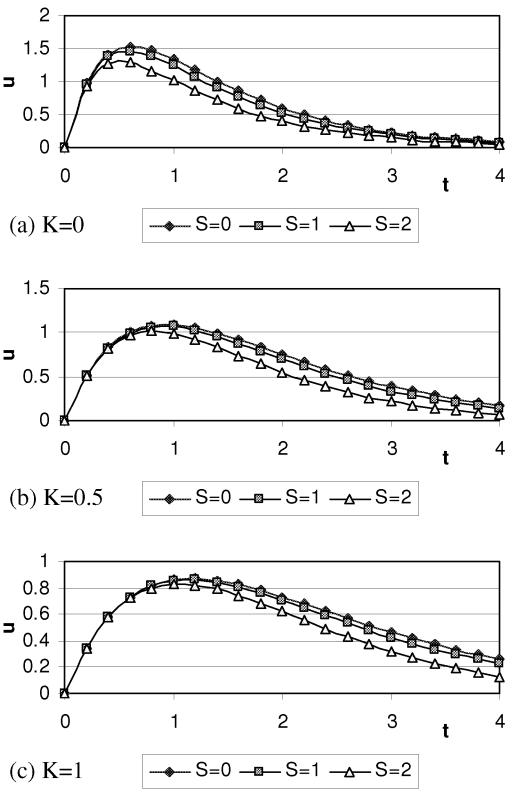

Fig.6: Effect of the parameter S on u for various values of K ($Ha = 0$)

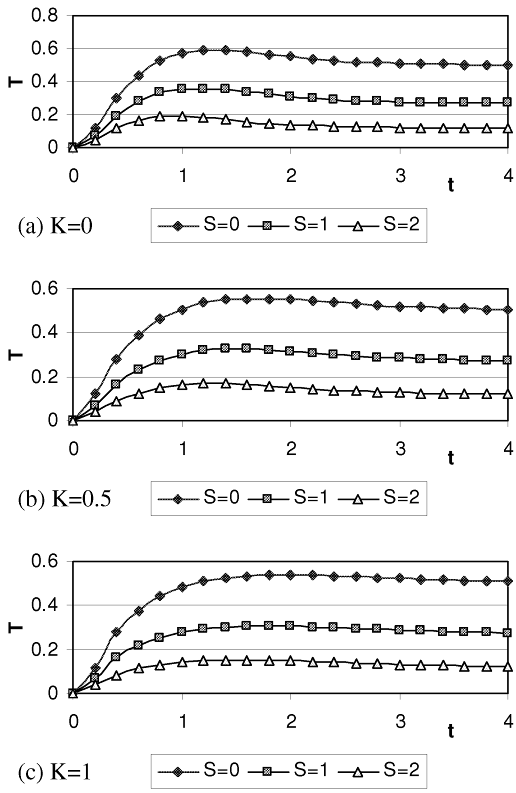

Fig.7: Effect of the parameter S on T for various values of K ($Ha = 0$)

5. Conclusions

The unsteady Hartmann flow and heat transfer of an electrically conducting viscoelastic fluid were studied in the presence of a uniform magnetic field as well as a uniform suction and injection. The influence of the viscoelastic parameter, the magnetic field, and the suction and injection velocity on the velocity and temperature distributions was investigated. The viscoelastic parameter has a clear effect on the velocity and temperature distributions and their steady state times for all values of the magnetic field and the suction velocity. The variation of the velocity and temperature with the viscoelastic parameter was detected to be time dependent. Also, it is of interest to find that the dependence of the temperature on the magnetic field vary with time for all values of the viscoelastic parameter.

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