

ESTIMATION OF RELIABILITY CHARACTERISTICS OF POWER OIL TRANSFORMERS

Miloš Hammer*, Jakub Ertl*, Ondřej Janda*

At present days, the requirements on reliability analysis of power oil transformers are increasing in technical practice. Properly estimated reliability characteristics inform the user how to service machine, and such estimation also saves the user's financial means. In the first part of this paper, the models for estimating reliability characteristics only from a set of times to failure of power oil transformers are described. Additionally, the Cox model, which enables for such machines to complete reliability analysis with diagnostic measurements, is introduced. In the last part of this paper, the Cox model is enhanced by factor analysis and moreover, the acquired results are compared with real behaviour of tested power oil transformers during operation.

Keywords: power oil transformer, reliability analysis, Cox model, factor analysis

1. Introduction

Power oil transformer is one of the most important components in electric distribution network. The different kinds of failures occur in transformers, they can interrupt electricity distribution, injure and kill operators, cause faults in other facilities, and finally, can lead to damages in industrial production and to economical losses. An emergency of transformers is very often accompanied with explosion, fire and other ecologic damages. In case of nuclear power plants, such failure can violate radiation safety. Different transformer emergencies are known all over the world when a scope of damages and consequences in individual cases achieves to an amount of hundreds of millions of CZK. For these reasons, it is necessary to monitor the state of power oil transformers and analyse their reliability. The most frequent causes of failures are presented in Table 1, see [1]. Power oil transformers are characterized by the declared lifetime (wear-out) which is specified by their producers. From this reason, their reliability and probable wear-out were not solved. However, at present days, high number of transformers which operate in the Czech Republic will approach to the end of their planned lifetimes, and a couple of scientific and research institutions are now interested in such problems. The user of such transformers calls for decision-making criteria according to which he will be able to make decision about the next operation and maintenance of machines. The decision-making criteria which are properly determined can save a huge amount of financial means. The technical diagnostics of such machines is necessary however such diagnostics will not give you satisfactory information about the next machine operation. For this purpose, it is necessary to analyse reliability and estimate reliability characteristics which can assist the user to stipulate the above mentioned decision-making criteria.

* doc. Ing. M. Hammer, CSc., Ing. J. Ertl, Ing. O. Janda, Brno University of Technology, FME, Institute of Production Machines, Systems and Robotics, Department of Electrical Engineering, Technická 2896/2, 61669 Brno, Czech Republic

Cause	System involved	Percentage
Failure of insulation	Insulation system	26 %
Error in production	Not specified	24 %
Unknown	Not specified	16 %
Short-circuit	Insulation system/Electrical circuit	7 %
Incorrect maintenance	Not specified	5 %
Overloading	Insulation system	5 %
Oil contamination	Insulation system	4 %
Overloading	Insulation system/Mechanical structural parts	4 %
Fire/explosion	Insulation system/Cooling vessel with accessories	3 %
Lightning	Insulation system	3 %
Flood	Cooling vessel with accessories	2 %
Moisture	Cooling vessel with accessories	1 %

Tab.1: Failure causes of power oil transformer

Additionally to the times to failure of machines, also the data from technical diagnostics must be involved into the reliability analysis of power oil transformers. Technical diagnostics is a discipline which deals with determining the conditions of technical facilities. Its output is a number of measured diagnostic quantities according to which other operability can be determined (operability of transformers in this case). For the detailed overview of such quantities see [2]. More precise estimates of reliability characteristics can be achieved by involving the values of diagnostic quantities into the reliability analysis of power oil transformers than taking into account only the times to failure of individual transformer.

2. Basic terms

A stochastic model of reliability of power oil transformers is based on the assumption that failure-free operation is a continuous random variable T which achieves only non-negative values. The following functional characteristics can be used for its description [3].

Distribution function $F(t)$ of random variable T is the function

$$F(t) = P(T < t) ,$$

which is defined for all $t \in (-\infty, \infty)$, and it expresses a probability of event for which the time of failure-free operation is shorter than t so that $F(t) = 0$ for $t \in (-\infty, 0)$.

Probability density of random variable T is non-negative function $f(t)$, for which

$$\int_0^t f(\tau) d\tau = F(t)$$

for all $t \in (-\infty, \infty)$.

Reliability function $S(t)$ of random variable T is the function

$$S(t) = 1 - F(t) = P(T \geq t)$$

for all $t \in (-\infty, \infty)$. The function $S(t)$ describes the reliability of an object, and it is the probability for which the time of failure-free operation will exceed the time t . Additionally, we will describe in our paper how such characteristics can be obtained by means of the

measured data [4]. Let us assume that there is available a population of n times to failure of some machine t_1, t_2, \dots, t_n as an independent realization of random variable T . So the function

$$S_n(t) = \frac{1}{n} \sum_{i=1}^n I_{(t, \infty)}(t_i) ,$$

where

$$I_A(t_i) = \begin{cases} 1 , & \text{when } t_i \in A , \\ 0 , & \text{otherwise} \end{cases}$$

is called empirical reliability function. The value $S_n(t)$ gives a ratio of the number of observations with a value higher than t to the number of all observation. It gives the first information about a shape of reliability function which can be used for a proper selection of reliability model.

The other functional characteristic which describes the random variable T is called failure rate $h(t)$:

$$h(t) = \frac{f(t)}{S(t)} .$$

Cumulative failure rate $H(t)$ of random variable T is given by the following equation

$$H(t) = \int_0^t h(\tau) d\tau .$$

And finally, the characteristic called the mean residual lifetime $r(t)$ tries to find an answer to the question how many lifetimes on average remains for a given machine at the time t . It is defined in the following way

$$r(t) = E(T - t | T > t) .$$

By modification of the formula, we obtain

$$r(t) = \frac{\int_t^\infty S(\tau) d\tau}{S(t)} .$$

Clearly defined formulae exist among the above described characteristics [4]. Some examples follow for the situation when failure rate $h(t)$ of random variable T is known; this function is used as the output of models described below. The following formulae are valid:

$$\begin{aligned} f(t) &= h(t) \exp \left[- \int_0^t h(\tau) d\tau \right] , \\ F(t) &= 1 - \exp \left[- \int_0^t h(\tau) d\tau \right] , \\ S(t) &= \exp \left[- \int_0^t h(\tau) d\tau \right] . \end{aligned}$$

In the next part, some procedures how to estimate the reliability characteristics will be described.

2.1. Model with constant failure rate (KIT)

As evident from the name of this model, the constant failure rate of random variable T is assumed, thus

$$h(t) = \lambda, \quad \lambda > 0.$$

From the above mentioned it follows that

$$f(t) = \lambda e^{-\lambda t},$$

which is the formula for the probability density of random variable with exponential probability distribution and with the parameter λ . It is evident that this model can be especially used in cases when the random variable T is governed by such exponential distribution. To estimate the parameter λ , it is possible to use the function `expfit` which is implemented in the MATLAB system. The KIT model describes very well the reliability of objects in which failures occur due to random causes and not as a consequence of wear-out.

2.2. Model with power failure rate (MIT)

The positive constants α and β are assumed. Failure rate of observed random variable in accordance with the assumption of power time function can be described by the following formula

$$h(t) = \frac{\alpha}{\beta^\alpha} t^{\alpha-1}.$$

It is possible to derive that

$$f(t) = \frac{\alpha}{\beta} \left(\frac{t}{\beta} \right)^{\alpha-1} \exp \left[- \left(\frac{t}{\beta} \right)^\alpha \right].$$

It is the probability density of random variable described by the Weibull probability distribution with parameters α and β [3]. The Weibull probability distribution is sometimes called as three-parametric distribution [4], [5], in our case the third parameter will be equal to zero. The parameters α and β can be estimated by using the MATLAB (`wblfit`).

The MIT model is suitable for the situations when the monitored random variable is described by the Weibull distribution. It is evident that if $\alpha = 1$, we will obtain the KIT model.

3. Estimate of reliability characteristics

The models which were described in chapter 2 were applied to the data file containing 30 times to failure of selected power oil transformers. Firstly, the empirical reliability function was determined. By means of the equations described above, other empirical characteristics were calculated.

Furthermore, the KIT and MIT models were used to estimate the mentioned reliability characteristics. As part of modelling, the hypotheses were tested if the used data are governed by the exponential distribution or the Weibull distribution. In this case the

Kolmogorov-Smirnov Goodness-of-Fit test was used, for details see [5]. The hypothesis on the exponential data distribution was refused whereas the Weibull distribution was not refused. The resulting estimates using all three procedures (empirical characteristics ERC, characteristics estimated by KIT and MIT models) are plotted in individual figures. The depicted reliability characteristics are defined in the previous chapter, time t is given in operational years.

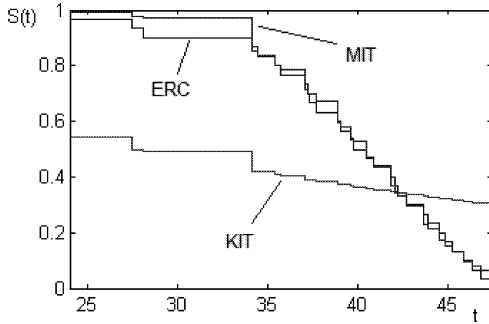


Fig.1: Reliability functions

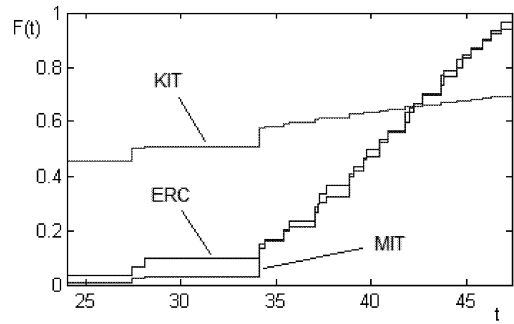


Fig.2: Distribution functions

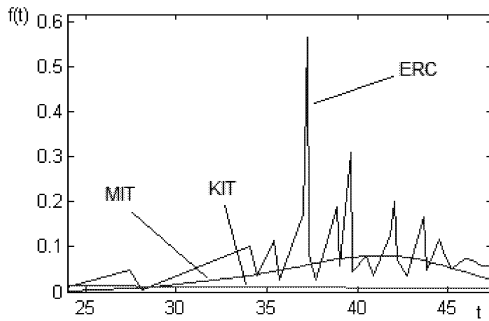


Fig.3: Probability densities

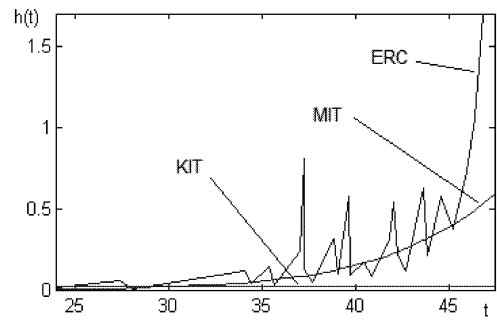


Fig.4: Failure rates

The estimates of reliability functions are shown in Fig. 1; it is evident that the estimate based on MIT model is strongly close to the empirical reliability function compared to KIT model estimate. The corresponding curves of individual distribution functions are shown in Fig. 2. The estimated probability densities (Fig. 3) differ each other more remarkably for corresponding models, and it is not possible to make decision which of the models more approaches to the estimate based on the empirical reliability function. It is evident in Fig. 4, in which failure rates are shown, that even in this case the MIT model is more appropriate, that means the power failure rate is assumed. This fact is shown in Fig. 5.

The function of sum of operational time and mean residual lifetime $r(t) + t$ were estimated as the last reliability characteristics. Its estimates are shown in Fig. 6, which are in fact the probable wear-out years of machine.

In this case, the estimate based on the empirical reliability function is more close to the estimate of MIT model. This can be explained by the fact that the hypothesis of Weibull probability distribution was not refused. The mentioned data file, from which the reliability characteristics were estimated, also involved the diagnostic quantities measured in the wear-out year of individual transformers.

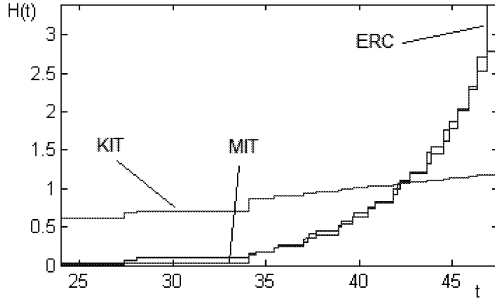


Fig.5: Cumulative failure rates

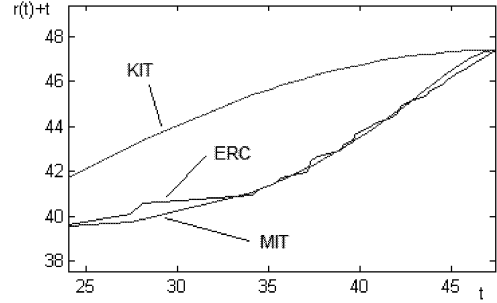


Fig.6: Sum of operational time and mean residual lifetime

In the next part of the paper, it will be shown that the diagnostic measurements contribute to more precise estimates of reliability characteristics.

4. Cox model for reliability characteristics modelling

Unlike the models described in chapter 2, the model which will be introduced now can estimate the reliability characteristics depending on other measured quantities. The diagnostic quantities can be combined into a random vector $\mathbf{X}^T = (X_1, X_2, \dots, X_p)$. The realization of random vector is then vector $\mathbf{x}^T = (x_1, x_2, \dots, x_p)$. During the reliability analysis of power oil transformers, the vector \mathbf{x}^T represents the diagnostic quantities which are measured during failure of machine. The symbol $T_{\mathbf{x}}$ indicates random variable T for a given realization of random vector $\mathbf{X} = \mathbf{x}$. Let us remind that the random variable T represents the time to failure of power oil transformer.

The Cox model works with the time to failure as well as the models described in chapter 2, however, the effect of diagnostic quantities represented by the realization \mathbf{x}^T of random vector \mathbf{X}^T is involved into the reliability characteristics which are used as output from this model. The model which is described in this paper assumes so called proportional failure rate $h_{\mathbf{x}}(t)$ of random variable T . In fact, it is failure rate of random variable T during realization \mathbf{x}^T . The proportional failure rate is described by means of the following formula:

$$h_{\mathbf{x}}(t) = h_0(t) g_1(\mathbf{x}) ,$$

where $g_1(\mathbf{x})$ is a positive function \mathbf{x} and $h_0(t)$ represents the initial failure rate, that is, failure rate in the case when $g_1(\mathbf{x}) = 1$. The function $h_0(t)$ is only estimated by means of the times to failure; it is the case when $x_1 = x_2 = \dots = x_p = 0$. Select in the Cox model

$$g_1(\mathbf{x}) = e^{\beta^T \mathbf{x}} ,$$

then

$$h_{\mathbf{x}}(t) = h_0(t) e^{\beta^T \mathbf{x}} .$$

This selection of the function $g_1(\mathbf{x})$ is justified because for $\mathbf{x}^T = (0, 0, \dots, 0)$ is

$$g_1(\mathbf{0}) = e^{\beta^T \mathbf{0}} = e^{\mathbf{0}} = 1 .$$

The parameters of model $h_{\mathbf{x}}(t) = h_0(t) e^{\beta^T \mathbf{x}}$, i.e. the vector $\beta^T = (\beta_1, \beta_2, \dots, \beta_p)$ and initial failure rate $h_0(t)$ are estimated by using of the maximum likelihood; you can find similar

approach in [3]. It is easy to derive that the reliability function can be expressed in the following way:

$$S(t) = [S_0(t)]^{g_1(\mathbf{x})} ,$$

where

$$S_0(t) = e^{-\int_0^t h_0(u) du} .$$

The Cox model is implemented in the MATLAB system, its output is primarily used to estimate the vector β and cumulative initial failure rate $H_0(t)$. From the formula

$$h_x(t) = h_0(t) e^{\beta^T \mathbf{x}}$$

it is evident that

$$h_0(t) = \frac{dH_0}{dt} .$$

Proportional failure rate $h_x(t)$ and reliability function $S(t)$ can be determined by means of $h_0(t)$. According to [4] the other functional reliability characteristics can be easily calculated, for example:

- distribution function $F(t)$,
- probability density $f(t)$,
- cumulative failure rate $H(t)$,
- mean residual lifetime $r(t)$.

The Cox model can be used for censored and non-censored data. Censoring is used during monitoring n transformers over a certain time t . It is supposed that k transformers fail to the time t and $n - k$ transformers works failure-free to the time t . Thus, k times to failure and $n - k$ times t will be entered into the Cox model. Because the Cox model does not use the censored data during estimating parameters β , it is necessary to mark the censored and non-censored times to failure in used software. Usually, 0 and 1 are used for such purpose.

Before using the Cox model, it is necessary to normalize the data from diagnostic measurements [3]. Let there is available population of n times to failure together with diagnostic measurements $\mathbf{x}_j^T = (x_{1j}, x_{2j}, \dots, x_{pj})$.

Then, the values z_{ij} instead of x_{ij} will be entered into the Cox model in the following form:

$$z_{ij} = \frac{x_{ij} - \bar{x}_i}{s_i} ,$$

where

$$\bar{x}_i = \frac{1}{n} \sum_{j=1}^n x_{ij} ,$$

$$s_i = \sqrt{\frac{1}{n} \sum_{j=1}^n (x_{ij} - \bar{x}_i)^2} ,$$

for $i = 1, \dots, p$ and $j = 1, \dots, n$.

The Cox model was applied to a data of power oil transformers identical to those in the previous chapter. The Cox model was used for calculation of censored and non-censored data. Reliability function and distribution function of times to failure are depicted in the figure for each group (censored and non-censored data). Both functions are shown depending

on the time t which is again given in operational years for a given machine. Furthermore, the most predicating functional reliability characteristic was estimated, that is, the function which is given by the sum of operational time and mean residual lifetime. Unfortunately, calculation of this characteristic did not succeed for both group, and so the misrepresented results were obtained. This fact was caused by a high number of input diagnostic quantities. A too large vector \mathbf{x}^T (in some cases up to 60 values for single measurement) was entered into the Cox model, and the model failed and showed division by zero in individual calculation steps. The ideas how to eliminate such unpleasant fact and the procedure how to estimate successfully the characteristics can be found in the following chapter 5.

4.1. Cox model for non-censored data

It is evident in Fig. 7 that reliability of tested transformers was significantly reduced after 28-year operation, and reliability function is approximately zero from the year 29. We can say that the transformers have been operated since the year 29 at the end of residual lifetime. Described conclusions correspond to the reality, that is, the reliability function properly reveals failure of tested power oil transformers. Calculation of the other reliability characteristic (the function of the sum of operational time and mean residual lifetime) failed in this case.

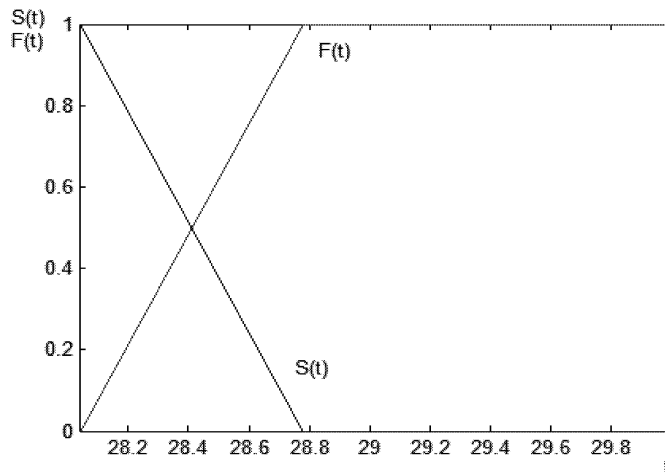


Fig.7: Reliability function and distribution function of times to failure

4.2. Cox Model for Censored Data

It is evident in Fig. 8 that the Cox model for censored data gives the same results as the Cox model for non-censored data. The curve of reliability function $S(t)$ and distribution function $F(t)$ for machine is again shown depending on the time t (in operational years). Calculation of the other reliability characteristic (the function of the sum of operational time and mean residual lifetime) also failed in this case. The ideas how to eliminate this fact can be found again in the following chapter 5.

As the Cox model for non-censored and censored data gives the same results, the effect of factor analysis will be described only for non-censored data in the next chapter.

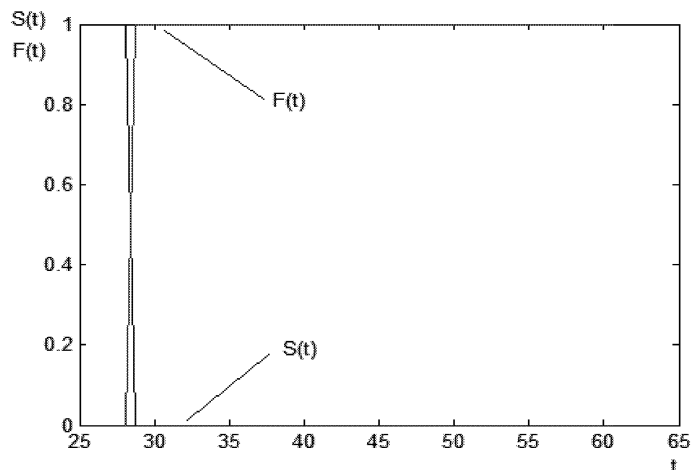


Fig.8: Reliability function and distribution function of times to failure

5. Factor analysis and Cox model

The factor analysis ranks among multi-dimensional statistical methods, and it is focused on search analysis and data file description. It also finds its applications in cases when a large number of mutually linked quantities are monitored. From the mathematical point of view, the real dimension of the problem is lower than the dimension of the investigated data file. It is the mathematical statistical method which, to a certain extent, eliminates the disadvantages of the Cox model directly for diagnostic measurements of power oil transformers.

The objective of the factor analysis is to create new (artificial) variables – *factors* so that the maximum amount of information contained in the original file is revealed by means of the minimum number of such factors. The dimension of new data approaches to the real value, which is important assumption for the next processing by the regression analysis. The factor analysis can be used as a tool which reduces the data file dimension. For more details about the theory of factor analysis see in [6].

5.1. Factor analysis and Cox model for non-censored data

The theory of factor analysis was applied to the database of diagnostic quantities measured at the selected group of power oil transformers. Primarily, the aim was to reduce the data. Then, the factor scores were used as input into the Cox model instead of the measured diagnostic quantities. The same reliability analysis was carried out using the Cox model as in the previous chapter with only difference that the factor scores were applied instead of the measured diagnostic quantities. To extract the factors and estimate the factor scores, the STATISTICA software was applied which involves even the algorithm calculating the number of factors. The default setting was not changed, and the factor model parameters were revealed by the principal component method, and the factors were rotated by a simple varimax method [6].

If the factor analysis was applied to the database of diagnostic measurements for all selected power oil transformers, even the second reliability characteristic, that is the function of sum of operational time and mean residual lifetime, was properly estimated. It can be

seen in Fig. 9 that applying of the factor analysis did not change the reliability function and the distribution function of times to failure. Fig. 10 shows the function which is given by the sum of operational times and mean residual lifetime. This reliability characteristic has probably the most predicting value for the reliability analysis and for the prediction of next machine lifetime.

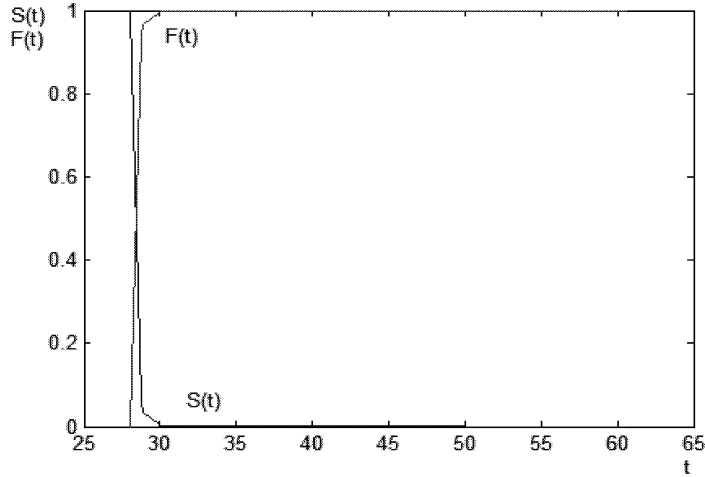


Fig.9: Reliability function and distribution function of times to failure

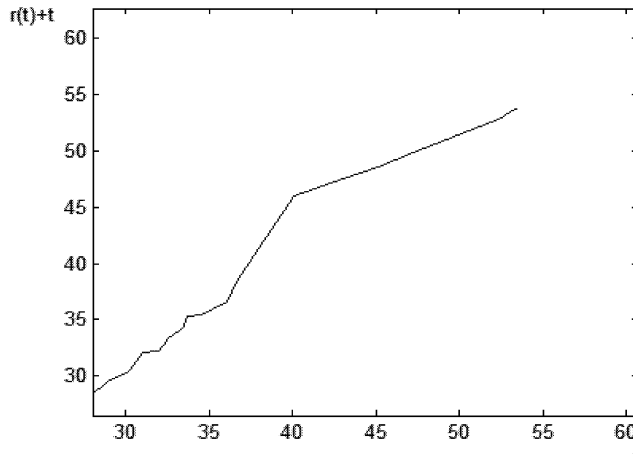


Fig.10: Function which is given as sum of operational time and mean residual lifetime

Time t in both figures again indicates the operational time of machine with the unit of one operational year.

5.2. Factor analysis and cox model for non-censored data divided into three types

From the previous chapter of this paper, it is evident that the Cox model mostly fails due to a high number of input diagnostic quantities. This problem can be partly eliminated by means of the factor analysis, and it is also possible to divide the diagnostic quantities

into 3 types and apply the Cox model and the factor analysis to each type separately. The diagnostic quantities can be divided into:

- quantities which characterize the oil fill condition,
- quantities of gas chromatography,
- quantities of winding insulation condition.

The classification of diagnostic quantities into individual types can be found in [7]. Because the problems of the previous models described in this paper are based on estimate of the function which is given as the sum of operational time and mean residual lifetime, only results related to this reliability characteristic are recorded. Only results which represent the application of the Cox model and the factor analysis for non-censored data are presented; the results were nearly identical for the censored data.

1) Quantities characterizing oil fill condition

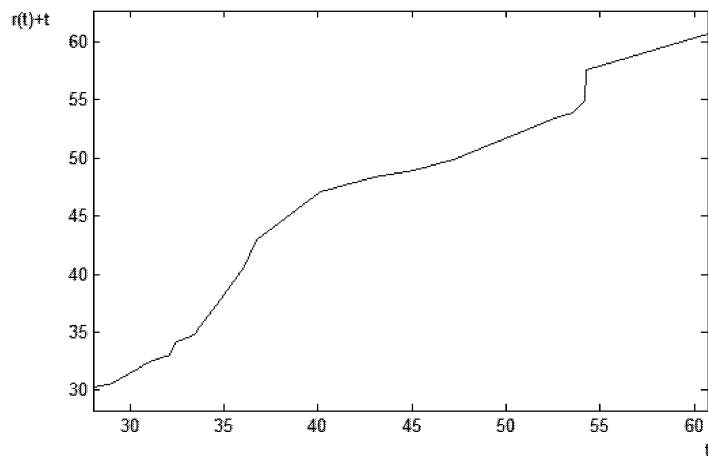


Fig.11: Function of the sum of operational time and mean residual lifetime estimated from the quantities which characterize oil fill condition

2) Quantities of gas chromatography

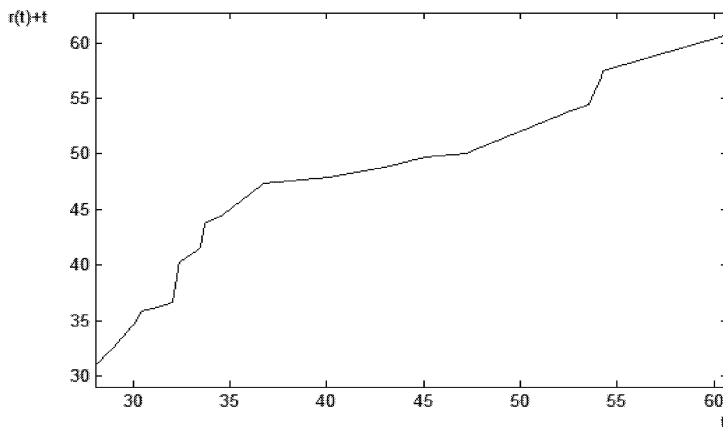


Fig.12: Function of the sum of operational time and mean residual lifetime estimated from the quantities of gas chromatography

3) Quantities of winding insulation condition

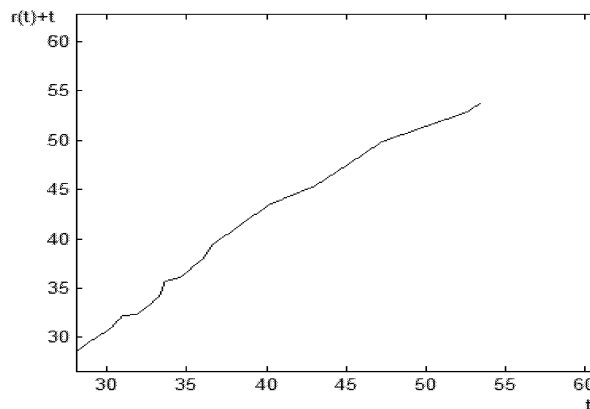


Fig.13: Function of the sum of operational time and mean residual lifetime estimated from the quantities of winding insulation condition

From Fig. 11 and Fig. 12, it is evident that the mean residual lifetime determined on the basis of the quantities of both oil fill condition and gas chromatography condition is described by the almost similar curves. Both reliability characteristics differ at most between the operational years 30 and 37 when the gas chromatography quantities gave more optimized results in view of the residual lifetime. According to the estimated mean residual lifetime determined from the quantities of winding insulation condition (Fig. 13), it is evident that the machine will fail after 55 years of operation. This fact corresponds to the reality, the selected power oil transformers ceased to operate between the years 54 and 56.

It can be stated that the quantities must be divided into 3 types within the detailed reliability analysis, and the Cox model and the factor analysis must be applied for better accuracy of reliability characteristics.

6. Conclusion

The data file which contained the wear-out times and the diagnostic measurements of 30 selected power oil transformers was primarily used in this paper to estimate reliability characteristics based only on the recorded times to failure. The KIT and MIT models were compared with the characteristics determined by means of the empirical reliability function. The MIT model described the data nature better, which means that the final reliability characteristics obtained in this model better approach to the empirical characteristics. This can be attributed to the fact that the Weibull probability distribution of times to failure was not refused. The mentioned models which operate only with the times to failure can be more specified using the data from diagnostic measurements of individual transformers. It was proved that the additional information is not negligible for the high-quality reliability analysis of power oil transformers. The reliability characteristics were estimated by means of the Cox model, and it is evident that such results predict the behaviour of machines more precisely.

The reliability of a group of power oil transformers at a water power plant in the Czech Republic was estimated using the Cox model. The model for censored and non-censored data was used. The estimated functional reliability characteristics were identical for both

models. Results did not depend on the fact if censored or non-censored data were used. The function which is given as the sum of operational time and mean residual lifetime could not be estimated. This was caused by a high number of diagnostic quantities which entered into the Cox model.

The problems related to a high amount of diagnostic quantities which enter into the Cox model can be eliminated by means of the factor analysis. The factor analysis is applied to a set of data which is acquired by diagnostic measurements, and the resulting factor scores enter into the Cox model instead of the diagnostic quantities. Such input data reduction can properly estimate all considered reliability characteristics of power oil transformers including the questionable function which is given as the sum of operational time and mean residual lifetime.

The diagnostic quantities can be divided into 3 types to obtain more precise estimate of reliability characteristics, and the Cox model and the factor analysis can be calculated for each type of quantities separately. In most cases, all reliability characteristics can be obtained in this way. The characteristics for each type of quantities were similar, and it is possible to select only one type of quantities, and perform other reliability analysis accordingly. Due to high purchase price of power oil transformers, it is suitable to select only such type of quantities which gives at least favourable results in respect to the residual lifetime.

According to reliability characteristic shown in Fig. 13 the tested machines are found in failure condition after 55 years of operation. This finding corresponds to the reality; the tested power oil transformers stopped operation between the operational years 54 and 56.

Acknowledgements

These results were obtained with the financial support of BUT, especially of the specific research program No. FSI-S-11-5 'Environmental and Safety Aspects of the Development, Manufacture and Operation of Machines'.

References

- [1] Šimková M.: Příspěvek k diagnostice výkonových olejových transformátorů, Brno: FME, Brno University of Technology, Institute of Production Machines, Systems and Robotics, 2010, 154 p., PhD. thesis supervisor: doc. Ing. Hammer Miloš, CSc.
- [2] In-company standard ČEZ, a.s. reg. no. 00/05: Profylaktika elektrických strojů netočivých – výkonové transformátory, Praha: ČEZ, a.s., 2006, 93 p.
- [3] Smith P.J.: Analysis of Failure and Survival Data, Chapman and Hall/CRC, 2002, ISBN-13: 978-1584880752
- [4] Karpíšek Z.: Teorie spolehlivosti – metody a aplikace, [PDF soubor], available at <http://www.math.fme.vutbr.cz/default.aspx?catalog=3&catsrtext=1&catsrfield=38>
- [5] Anděl J.: Matematická statistika, Praha: SNTL, 1978
- [6] Überla K.: Faktorová analýza, Bratislava: ALFA, 1974
- [7] Hammer M., Ertl J., Barvenčík O., Kutálek D.: Využití časových řad v diagnostice výkonových olejových transformátorů – 1. část, [PDF soubor], available at <http://elektrorevue.cz/cz/clanky/energetika-vykonova-elektronika-elektrotechnologie/15/vyuziti-casovych-rad-v-diagnostice-vykonovych-olejovych-transformator----1--cast/>

Received in editor's office: February 3, 2012

Approved for publishing: March 5, 2012