EVALUATION OF UNIAXIAL FATIGUE CRITERIA APPLIED TO MULTIAXIALLY LOADED UNNOTCHED SAMPLES

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Several multiaxial fatigue criteria have been developed and improved within the last couple of decades, but they are not very widely used in industrial applications. Many engineers and designers still use simple uniaxial criteria for multiaxial load cases. In order to test and validate/discard these uniaxial criteria on the basis of multiaxial load cases, the present work presents a comparison between several uniaxial criteria applied to a large set of experimental results for smooth unnotched samples tested under multiaxial loading. The effect of mean stress is also evaluated, in order to determinate how it affects the final results.

Keywords: *multiaxial stress, uniaxial fatigue criteria, mean stress, proportional loading*

1. Introduction

A vast range of methods for multiaxial fatigue prediction has been proposed in recent decades. They usually use a solution based on a description of the stress or strain state on a specific plane. The stress or strain parameters on this plane induced by the external loading and by the specific shape of the loaded structure are combined into a final damage parameter. The calculation methods differ in whether the maximum damage parameter found on a particular plane is looked for, or whether the spatial mean damage parameter is computed over all possible planes [1]. The only other major approach to the solution is to analyse the load path in a five-dimensional Ilyushin deviatoric space, which is represented later here by the Crossland method (Sec. 2.2.2).

All these methods have been developed mostly because of the need to cope in some way with non-proportional loading. In this kind of loading, the stress or strain tensors during the loading are not multiples of each other. Their individual components change independently. As a consequence, not only the principal stresses but also the principal directions change during the loading. Researchers have found that the utilization of a single load parameter on a specific plane (be it Mises stress on the octahedral plane, Tresca stress on the maximum shear stress range plane, or the maximum principal stress) cannot take into account the complexity of the loading induced when there is non-proportional loading. If the principal directions change during the loading, the positions of the maximum loaded planes clearly change as well. Relying on a single stress parameter does not provide an adequate response.

Any loading with two load channels, where there is at least a static non-zero load on one of the channels while the load on the other load channel is variable, can be classified as non-

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proportional. Here, the principal directions rotate during loading. Because this kind of load is far from rare in engineering practice, a good understanding of the potential calculation errors is necessary.

Utilization of a second parameter (affected by the potential phase shift) is the only solution recognized today in the research community. In practical terms, in order to determine the response of a material to fatigue under multiaxial stress we can choose among many various multiaxial methods specifically intended to work under such stress states: Dang Van, Crossland, Papadopoulus, Liu – Zenner, etc., see [1] for a closer description and evaluation, and also for a formulation of the currently most precise method – PCr by Papuga.

The multiaxial solution has a very important drawback, in addition to the tangle of numerous multiaxial methods, where few comparative studies have been published [1], [4], [5]. Applying a multiaxial solution requires good knowledge of the problem, together with either multiaxial fatigue software equipment or the ability to program the methods oneself. Even if this issue is solved, the final calculation takes more time than any simple uniaxial solution.

A quite distinct contrast to the application of multiaxial criteria in research practice therefore can be observed in industrial applications [2]. Simple uniaxial solutions often replace the application of multiaxial criteria, though the quality of this kind of simplification has not yet been properly validated. These methods are ready to apply even in the MS Excel spreadsheet, and no long preparations are needed before the real fatigue limit analysis is performed. However, for the reasons given above, there is some doubt as to whether these methods provide an adequate description of the real behaviour. The main objective of the present work is to validate or discard uniaxial fatigue criteria that are applied to multiaxial load cases. It is assumed, under these conditions [2], that the applicability of the methods should be limited to proportional loading. Then, in contrast to the non-proportional loading, the principal directions remain fixed during the loading. Because of the considerations raised above, the potential applicability of simplified solutions to other and more complex load cases will be evaluated in this paper.

2. Description of the problem, and technical background

In order to carry out the present evaluation, we used the results of 407 fatigue test series derived from the FatLim database [1] on uniaxial and multiaxial loading at the fatigue limit for various materials and loading conditions*. Various loading types were evaluated: with or without mean stress, proportional or non-proportional loading. The general condition that the fatigue limit for smooth unnotched specimens is evaluated was selected intentionally in order to minimize the impact of other potential degrees of freedom. The calculation of the final result can otherwise be burdened by many other uncertainties in processing the fatigue analysis – the way in which the load cycles are separated, partial damage summation, or the notch effect.

The uniaxial fatigue criteria utilized in this work are based on von Mises or Manson-McKnight equivalent stresses and their various modifications. In order to check them with the widely-used multiaxial counterparts, the Dang Van and Crossland criteria were selected as the criteria representing multiaxial solutions, mainly because of their popularity, though their popularity is not supported by their prediction quality [1]. It is expected that uniaxial

* The complete database can be found on www.pragtic.com/fatlim.php, while the exact set composition used here is explained in [1].

methods can hardly be better than the most enhanced multiaxial criteria (Papuga PCr [1]), while the selected multiaxial methods correspond to industrial trends in multiaxial analyses. The Dang Van method is the standard solution used by commercial fatigue solvers provided by HBM (nCode DesignLife), LMS (LMS.VirtualLab Component Durability), Safe Technology (FeSafe), and Ricardo (FEARCE).

Because the scope of the evaluated experiments is sufficient, it was possible to divide them into statistically acceptably large groups according to load mode and proportionality characteristics. Thus the methods could be analysed in terms of their suitability for defined conditions. The application of calculation procedures mentioned below transforms the input complex loading into the equivalent stress amplitude $\sigma_{\mathrm{a,eq}}$, which has to be equal to the fatigue limit under fully reversed axial loading f_{-1}. Because the experiments are related to the fatigue limit, equality of the load and material parameters is expected. Any deviation from equality can be documented by the fatigue index error:

$$\Delta FI = \frac{\sigma_{\mathrm{a,eq}} - f_{-1}}{f_{-1}}\, 100\,\% \;. \tag{1}$$

According to this relation, values of ΔFI below zero indicate that the criterion fails to predict the failure though it should do so, and the results will be referred to as 'non-conservative'. On the other hand, positive values of ΔFI, will be referred to as 'conservative' [1].

2.1. Uniaxial fatigue criteria

The application of uniaxial fatigue criteria relates to the computation of the equivalent stress that can be compared (and corrected in the presence of mean stress) with the fatigue limit of standard S-N curves.

For the case studied here, the equivalent stress is the von Mises equivalent stress, which can be obtained from the stress tensor components by applying the following formula:

$$\sigma_{\mathrm{eq,vM}} = \sqrt{\frac{1}{2}\left[(\sigma_{\mathrm{x}} - \sigma_{\mathrm{y}})^2 + (\sigma_{\mathrm{y}} - \sigma_{\mathrm{z}})^2 + (\sigma_{\mathrm{z}} - \sigma_{\mathrm{x}})^2 + 6\,(\tau_{\mathrm{xy}}^2 + \tau_{\mathrm{yz}}^2 + \tau_{\mathrm{zx}}^2)\right]}\;. \tag{2}$$

There are two basic variants, in which the amplitude and the mean value of the equivalent stress can be derived – computing the equivalent stress at every time instant, or computing it from the maximum range of stress tensor components during the loading cycle.

2.1.1. Signed von Mises stress (VMI1)

The solution described here as the signed von Mises stress (VMI1) analyses the equivalent stress signed by the sign of the first stress tensor invariant during loading. The maxima and minima of the current signed von Mises stress during loading serve to define the amplitude and the mean stress:

$$\begin{aligned} \sigma_{\mathrm{a}} &= \frac{\max\limits_t[\sigma_{\mathrm{eq,vM}}\ \mathrm{sgn}(I_1)] - \min\limits_t[\sigma_{\mathrm{eq,vM}}\ \mathrm{sgn}(I_1)]}{2}\;, \\ \sigma_{\mathrm{m}} &= \frac{\max\limits_t[\sigma_{\mathrm{eq,vM}}\ \mathrm{sgn}(I_1)] + \min\limits_t[\sigma_{\mathrm{eq,vM}}\ \mathrm{sgn}(I_1)]}{2}\;. \end{aligned} \tag{3}$$

2.1.2. Manson-McKnight method (MMK)

By contrast, the various variants of the **Manson-McKnight method** (MMK) [2] compute the amplitude and mean stresses from the maxima and minima of every stress tensor component during the loading cycle:

$$\begin{aligned}
\sigma_{i,\mathrm{a}} &= \frac{\max\limits_t(\sigma_i) - \min\limits_t(\sigma_i)}{2} \,, \qquad & \tau_{ij,\mathrm{a}} &= \frac{\max\limits_t(\tau_{ij}) - \min\limits_t(\tau_{ij})}{2} \,, \\
\sigma_{i,\mathrm{m}} &= \frac{\max\limits_t(\sigma_i) + \min\limits_t(\sigma_i)}{2} \,, \qquad & \tau_{ij,\mathrm{m}} &= \frac{\max\limits_t(\tau_{ij}) + \min\limits_t(\tau_{ij})}{2} \,.
\end{aligned} \tag{4}$$

In this approach, the tensors of amplitudes can be transformed to the equivalent stress amplitude and the equivalent mean stress by the following formulas:

$$\sigma_\mathrm{a} = \sqrt{\frac{1}{2}\left[(\sigma_{\mathrm{x,a}} - \sigma_{\mathrm{y,a}})^2 + (\sigma_{\mathrm{y,a}} - \sigma_{\mathrm{z,a}})^2 + (\sigma_{\mathrm{z,a}} - \sigma_{\mathrm{x,a}})^2 + 6\left(\tau^2_{\mathrm{xy,a}} + \tau^2_{\mathrm{yz,a}} + \tau^2_{\mathrm{zx,a}}\right)\right]} \,, \tag{5}$$

$$\sigma^*_\mathrm{m} = \sqrt{\frac{1}{2}\left[(\sigma_{\mathrm{x,m}} - \sigma_{\mathrm{y,m}})^2 + (\sigma_{\mathrm{y,m}} - \sigma_{\mathrm{z,m}})^2 + (\sigma_{\mathrm{z,m}} - \sigma_{\mathrm{x,m}})^2 + 6\left(\tau^2_{\mathrm{xy,m}} + \tau^2_{\mathrm{yz,m}} + \tau^2_{\mathrm{zx,m}}\right)\right]} \,, \tag{6}$$

The individual variants of the Manson-McKnight method presented here differ mainly in the determination of the final equivalent mean stress value, labelled here as σ_m. This value is computed from the σ^*_m by some of the signing procedures. The original Manson-McKnight method also used the first stress invariant for signing, but in its maximum value during the loading:

$$\sigma_\mathrm{m} = \sigma^*_\mathrm{m}\ \mathrm{sgn}(I_{1,\mathrm{d}}) \,, \tag{7}$$

where $I_{1,\mathrm{d}}$ is the value of the first stress tensor invariant at the moment when reaches the greatest distance from zero during the load cycle.

2.1.3. Modified Manson-McKnight methods (MMMK)

The modified variants MMMK were introduced mainly because of the understanding that very slight changes in the loading can have an inappropriately pronounced impact [2] on the final sign of σ_m. We have found two different variants, where multiplication is used instead of signing. The first variant links to the NASALIFE report [2] and is therefore labelled as the modified Manson-McKnight method, variant NASALIFE (MMMK-N):

$$\sigma_\mathrm{m} = \sigma^*_\mathrm{m} \frac{\sigma_{1,\max} + \sigma_{3,\min}}{\sigma_{1,\max} - \sigma_{3,\min}} \,. \tag{8}$$

Here the maxima and minima are set during the load cycle with the expectation that σ_1 is the highest principal stress, while σ_3 corresponds to the lowest principal stress.

Though the second variant seems to differ only slightly, it is a distinct solution:

$$\sigma_\mathrm{m} = \sigma^*_\mathrm{m} \frac{\Sigma\sigma_1 + \Sigma\sigma_3}{\Sigma\sigma_1 - \Sigma\sigma_3} \,. \tag{9}$$

Because we found it in the paper by Filippini et al. [3], it is labelled as MMMK-F. In order to understand the meaning of the Σ characters, the following formulas are necessary:

$$\Sigma\sigma_1 = \sigma_{1,\max} + \sigma_{1,\min} \,, \qquad \Sigma\sigma_3 = \sigma_{3,\max} + \sigma_{3,\min} \,. \tag{10}$$

Because of the composition of the two multiplicators, it can be expected that the most pronounced difference between the two methods will be found for dominant torsion loading.

2.1.4. Mean stress correction

The examined load cases of many evaluated experiments result in non-zero mean stresses σ_m, so the mean stress has to be included in the evaluation. For that purpose, we will use the Walker correction factor for mean stress. Its use is proposed in [2] :

$$\sigma_\mathrm{a,eq} = \sigma_\mathrm{a}^\gamma\, (\sigma_\mathrm{m} + \sigma_\mathrm{a})^{1-\gamma} \ . \tag{11}$$

It gave better results than other corrections (Goodman, Gerber, ...), when we tested it. Usually, the value of parameter γ has to be derived by fitting to the experimental data. Thanks to the experimental data sets from the FatLim database [1] that is used here, the information on the fatigue limit in repeated axial loading f_0 could be used for transforming Eq. 11 to :

$$\gamma = 1 - \frac{\log\left(2\,\frac{f_{-1}}{f_0}\right)}{\log(2)} \ . \tag{12}$$

2.2. Multiaxial Fatigue Criteria

Because of space limitations, only the popular Dang Van and Crossland criteria will be evaluated. While the Dang Van criterion represents the critical plane criteria, the Crossland criterion looks for the minimum hyperball circumscribed to the load path in the five-dimensional Ilyushin deviatoric space (see [1], [4]). The Papuga PCr method referred to in [1] as the optimum solution, is not included in the comparison for two reasons. Firstly, it is not used in the industrial sector today, and, secondly, it is so distinctly better in all categories evaluated here, that it would not make sense to use it for checking the applicability of simplified uniaxial methods.

2.2.1. Dang Van method

The equivalent stress amplitude can be described as a mix of the maximum shear stress amplitude C_a encountered on any plane during loading and the maximum value of the first invariant of the stress tensor I_1 found during the load cycle :

$$\sigma_\mathrm{a,eq} = a_\mathrm{DV}\, C_\mathrm{a} + b_\mathrm{DV}\, I_1 \ . \tag{13}$$

The material parameters a_DV and b_DV can be derived from two pure uniaxial fatigue limits in fully reversed push-pull (f_{-1}) and torsion (t_{-1}) loading :

$$a_\mathrm{DV} = \kappa = \frac{f_{-1}}{t_{-1}} \ , \qquad b_\mathrm{DV} = 1 - \frac{\kappa}{2} \ . \tag{14}$$

The fatigue limit ratio κ is also defined in Eq. (14).

2.2.2. Crossland method

According to Papadopoulos et al. [4], the square root of the amplitude of the second invariant of stress tensor deviator $\sqrt{J_{2,\mathrm{a}}}$ relates to the radius of the minimum circumscribed hyperball in the five-dimensional Ilyushin deviatoric space defined by the following five parameters transformed from stress tensor deviator components s_{ij} :

$$s_1 = \sqrt{\frac{3}{2}}\, s_{\mathrm{xx}} \ , \qquad s_2 = \frac{1}{\sqrt{2}}\,(s_{\mathrm{yy}} - s_{\mathrm{zz}}) \ , \\ s_3 = \sqrt{2}\, s_{xy} \ , \qquad s_4 = \sqrt{2}\, s_{xz} \ , \qquad s_5 = \sqrt{2}\, s_{yz} \ . \tag{15}$$

The criterion itself follows this formula:

$$\sigma_{\mathrm{a,eq}} = a_{\mathrm{C}} \sqrt{J_{2,\mathrm{a}}} + b_{\mathrm{C}}\, I_{1,\max} \ . \tag{16}$$

The analyses of the criterion separately under fully reversed push-pull and torsion loads enable the material coefficients to be derived:

$$a_{\mathrm{C}} = \kappa \ , \qquad b_{\mathrm{C}} = 1 - \frac{\kappa}{\sqrt{3}} \ . \tag{17}$$

Though the minimization problem of finding the minimum enclosed hyperball is not simple, the Crossland criterion has the asset that the minimization is run only once for the evaluated load path. The solution is thus much more straightforward and the computation is quicker than the solution with the Dang Van method, which operates on a multitude of discrete planes in the search for the maximum shear stress amplitude.

3. Results

Figs. 1–3 compare the overall statistic values of ΔFI obtained for individual criteria. In cases where the particular bar in the chart exceeds the scope of the graph, the number attached to it marks the final value.

The analysed experimental data was regrouped according to the type of loading, the presence of mean stress, and proportional or non-proportional loadings.

The experiments cover the following groups:
- **All** – all 407 experiments defined by Papuga in [1].
- **MS, Ax** – experiments with loading on the axial load channel only and non-zero mean stress.
- **MS, To** – experiments with loading on the torsion channel only with non-zero mean stress.
- **MS, Ax+Ax, IP** – experiments with bi-axial tensile loading without any phase shift. A typical representative might be e.g. a pressurized vessel, which is loaded proportionally because of the inner pressure. The group can nevertheless contain both proportional and non-proportional load cases, depending on the relation between the cycle asymmetry factors R on the individual load channels. However, the number of experiments gathered in this group is low, and separating proportional and non-proportional experiments would lead to numbers that are not statistically useful.
- **MS, Ax+Ax, OP** – obviously non-proportional loading, where the effect of out-of-phase loading mixes with the non-zero mean stress at one load channel at least.

- **MS, Ax+To** – the loads cover both in-phase and out-of-phase loading, nevertheless the loading is mostly non-proportional because of the static stresses that are included. The proportionality of the loading would be ensured only for the same cycle asymmetry coefficients R on both load channels. The group is further divided into:
  - **MS-Ax, Ax+To** – an axial static load is accompanied by a torsion amplitude, and an axial amplitude can also be involved.
  - **MS-To, Ax+To** – a torsion static load is accompanied by an axial load amplitude, and periodical torsion loading can also be involved. A typical structure of this type is an unbalanced rotating shaft for power transmission.
- **nMS, Ax+To, OP** – the perfect adepts of typical non-proportional loading caused by out-of-phase loading on axial and torsion load channels without any mean stress effect.
- **nMS, Ax+To, IP** – typical proportional multiaxial loading. Unfortunately, the other load combination of this type (nMS, Ax+Ax, IP) could not be evaluated because there is no data of this kind in the database. The probable reason is the limited availability of experimental facilities able to implement this load combination – specimens providing the bi-axial tensile load combination are usually pressurized tubes with an intrinsic mean stress effect due to the typical repeated loading. This group of experiments further divides into several groups according to the type of material:
  - **brittle** – materials with fatigue limit ratio $\kappa < 1.25$.
  - **ductile** – materials with $\kappa > 1.25$.

4. Discussion

4.1. Overall analysis

The results can analysed separately for each group of experiments. The Crossland and Dang Van methods provide similar standard deviations for most of the groups, but they differ in range and mean value parameters. The steadier output of the Crossland solution in these two parameters improves the credibility of the method, if the intrinsic non-conservativeness of the output is understood and is included in the safety factors. Papuga [1] criticizes the Crossland method because of its shift of the mean values of the fatigue index error to the non-conservative side, and also because its mean values under out-of-phase loading (nMS, Ax+To, OP here) and in-phase loading (nMS, Ax+To, IP) differ substantially. This type of behaviour, which can also be observed on a slightly lesser scale in the Dang Van method, is not an acceptable property for a multiaxial criterion.

An interesting overall comparison can be made between the original Manson-McKnight (MMK) and Dang Van methods. The Dang Van method is only slightly better. Since it is known to have appeared in many papers, applications and manuals of commercial fatigue solvers, it is no wonder that many engineers and companies see no reason to use multiaxial solution under such conditions. MMK fails in computation on brittle materials, which also affects its performance in the whole nMS, Ax+To, IP group.

The general failure of the VMI1 method is obvious. Because the most marked problems are related to problems of the mean stress inclusion if axial and torsion loads are active, it can be assumed that the signing procedure results in load cycles that do not reflect reality because there are abrupt changes of sign.

The difference between the modified MMMK methods is unimportant. The MMMK-N formula is simpler for calculation, and should therefore be preferred.

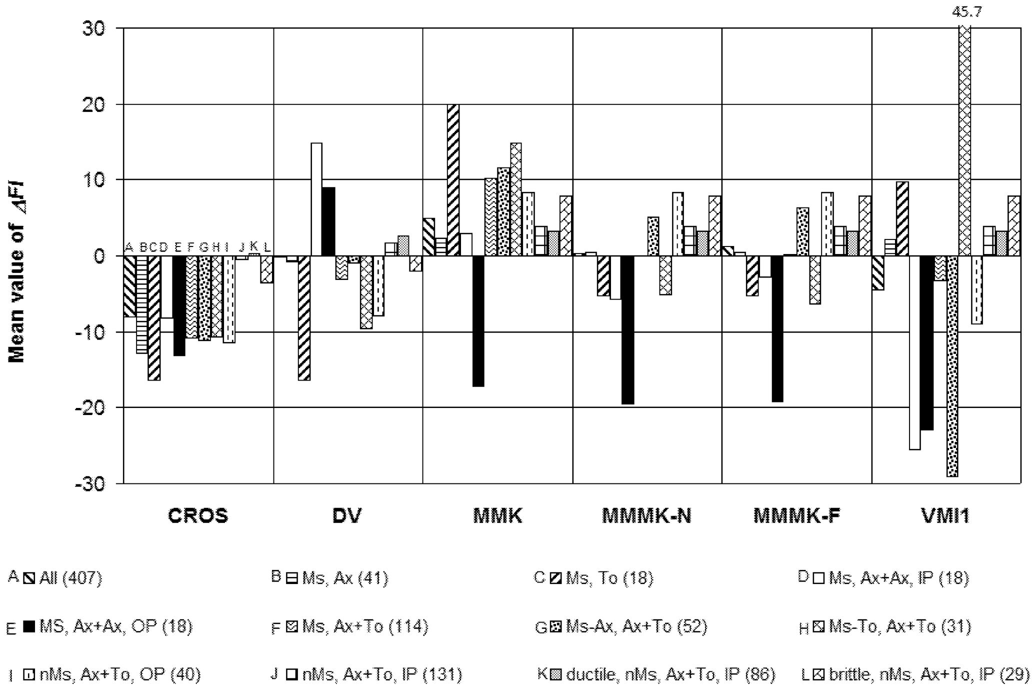

Fig.1: Mean values of ΔFI; the order of the columns corresponds to the legend read row by row from left to right

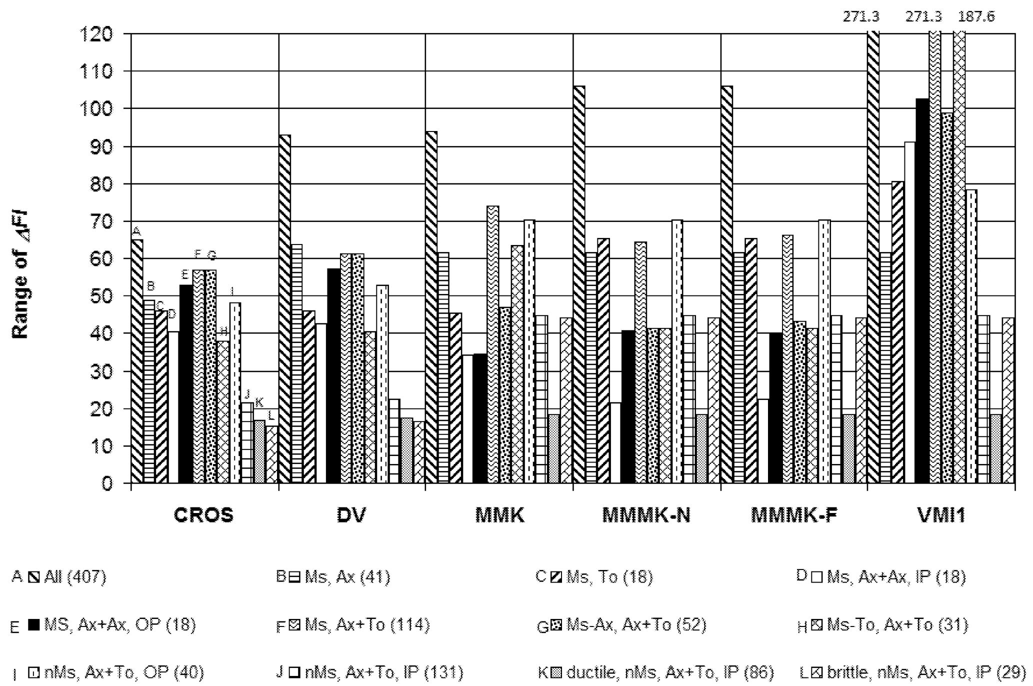

Fig.2: Range of ΔFI fatigue index error for the same sets of experiments as in Fig. 1

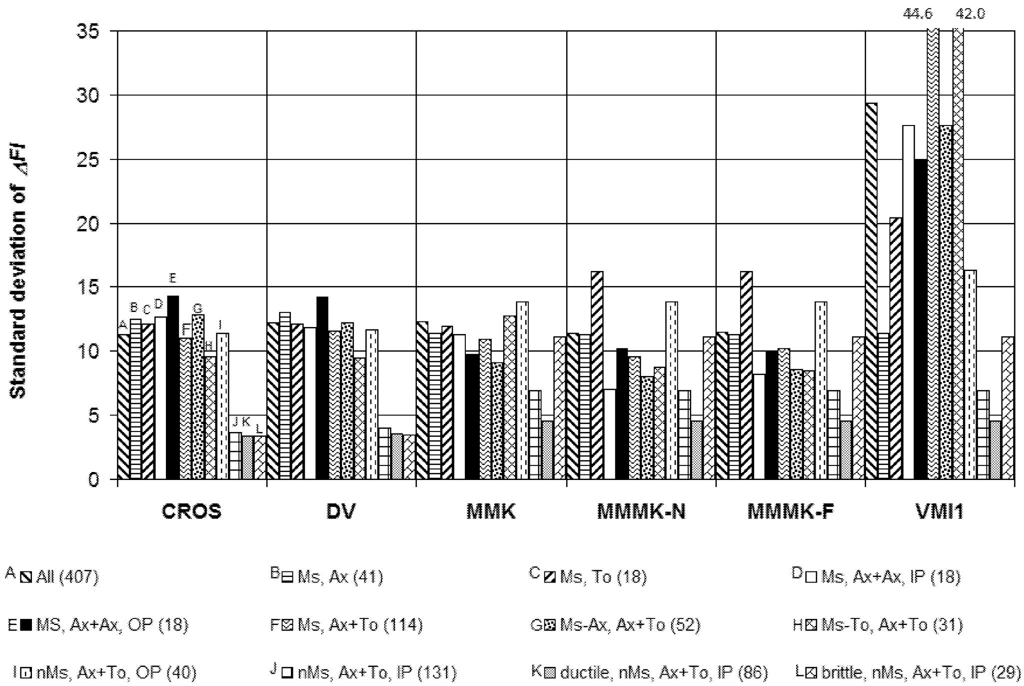

Fig.3: Standard deviations of ΔFI fatigue index error for the same sets of experiments as in Fig. 1

4.2. MS, Ax

With the exception of the Crossland method, the results for this load combination are very close for all methods. Any of the uniaxial solutions can be used, and it will provide results with similar prediction quality as the Dang Van method.

The Crossland solution provides a better range, but the pronounced shift to the non-conservative mean value of the fatigue index error should be noted.

4.3. MS, To

The results of the Crossland and Dang Van method are distinctly shifted to the non-conservative side for this load combination. The MMK method provides the same range, and the deviation from the zero mean ΔFI value is also substantial, but to the conservative side. This method appears to be safer for use than other evaluated methods.

The modified MMMK methods are markedly worse. Though the mean value of ΔFI is better than in other methods, the output in range and standard deviation parameters shows that the modifications implemented to fix the behaviour under torsion loads do not work well.

4.4. MS, Ax+Ax, IP

For this type of loading, the MMMK methods are the best choice. Multiaxial methods are worse here, and even MMK provides a better solution than they do. Though we noted in Section 3 that the experiments also include the mean stress effect and result in non-proportional loading generally, the uniaxial methods are better here.

The VMI1 method should be abandoned in such cases, because it provides the worst results.

4.5. MS, Ax+Ax, OP

Out-of-phase loading and the additional influence of mean stress are quite stringent conditions. The multiaxial methods provide predictions closer to in-phase loading in Section 4.4, while the mean ΔFI values of the (M)MMK methods largely differ. The decrease in the mean quality of the predictions is very distinct, and the mean value approaches $\Delta FI = -20\,\%$. These methods should therefore be abandoned in cases of this kind, though they provide quite a narrow band of results otherwise. The failure of the VMI1 method is obvious.

4.6. MS, Ax+To

The overall failure of the VMI1 method is obvious, even if the separate sub-groups are checked. If the evaluation is done over a whole group, the differences among results of other criteria are not very pronounced. Both multiaxial methods provide better ranges of ΔFI results, but MMK and its clones are very close, and are not shifted to the non-conservative side as the results of the Crossland criterion are. The subgroups discussed in Sections 4.7 and 4.8 greatly affect the final outcome, when mixed together. Thus the positive behaviour of the MMMK methods noted in Sections 4.7 and 4.8 is not wholly reflected in the range parameter of ΔFI here, and these methods appear inferior to multiaxial solutions. This is mostly the outcome of the 10 % shift between the mean values of ΔFI here in the two following groups. Anyway, the standard deviation of ΔFI shows that the MMMK methods behave best.

4.7. MS-Ax, Ax+To

The differences between the two multiaxial methods are minor – only the better-placed mean value of ΔFI of the Dang Van method can be noted. All (M)MMK methods provide better prediction than multiaxial counterparts. In comparison to the basic MMK, the modified MMMK methods form a narrower band of results, as can be seen from the standard deviation of ΔFI value. They are the optimum solution for use.

4.8. MS-To, Ax+To

Both multiaxial methods provide a mean ΔFI value shifted pronouncedly (by approximately 10 %) to the non-conservative side. Though the MMK method provides worse results for the MS-To, Ax+To group, the overall mean ΔFI error value is shifted to the conservative side, so it can be accepted as a safe (and a not substantially worse) solution, if necessary. For this load combination, the optimum way is to apply the modified MMMK solution, because it results in a narrower band than the multiaxial solutions provide. The VMI1 method fails completely in all aspects.

4.9. nMS, Ax+To, OP

The results for cases where there are no mean stresses are almost the same for all uniaxial methods as regards the overall ranges, though the VMI1 results are worse and slightly shifted to the non-conservative side. The application of multiaxial methods leads to more positive outcomes, and it is apparent that multiaxial methods deserve their position.

4.10. nMS, Ax+To, IP

If we take this group as a unit, we note a marked difference between uniaxial methods and multiaxial methods. When the analyses continue to a detailed evaluation for individual groups of materials, it becomes evident that the applicability of the methods and the expected quality of their results are more similar when ductile materials are examined. When brittle materials are admitted into the analysis, the results of all uniaxial methods deteriorate substantially.

There is no pronounced difference among the results for each of the uniaxial methods.

4.11. Overall

If the focus is set only on proportional multiaxial loading without any mean stress effect, it is apparent that any uniaxial method can be used and the results will not be substantially worse than for any of the popular multiaxial methods examined here, provided that the material is not brittle. The MS, Ax and MS, To groups can also be qualified as proportional loading, though there is no multiaxial loading. Here, the original MMK provides better quality than other uniaxial methods and approaches the results of the Dang Van solution.

The out-of-phase loading group shows the domain where multiaxial methods achieve better results. Out-of-phase loading is not correctly reflected by (M)MMK methods, and this results in the obvious shift of the ΔFI mean value to a non-conservative prediction. The output of the whole MS, Ax+Ax group is not shown here, but the shift of the ΔFI mean value to the non-conservative solution is so extreme that the multiaxial methods are obviously superior here. On the other hand, the (M)MMK methods provide better predictions for MS, Ax+To load cases, probably because of the better implemented mean stress effect, even when the out-of-phase experiments are included.

Among the uniaxial methods, the (M)MMK criteria provide much better quality than the VMI1 method. However, potential users should note that only periodical and static loads were admitted into this analysis. This limitation completely avoided the necessity to find a way to separate the individual cycles from the load history and to sum the partial damages from every load cycle. If the evaluated load history is more complex, the use of VMI1 allows immediate load decomposition, but the (M)MMK methods are designed to be run on already defined cycles. The additional step of load decomposition has to be performed by some yet undefined solution.

5. Conclusion

A comparison has been made of the prediction quality of two multiaxial methods (by Dang Van and Crossland) and several uniaxial solutions based on the von Mises stress (various variants of the Manson-McKnight criterion and direct processing of the instantaneous signed von Mises stress – VMI1). The test set comprised data on experimentally set multiaxial fatigue limits on unnotched smooth samples from various materials, with a total of 407 experiments.

1. The starting assumption that uniaxial methods should provide an adequate solution for multi-channel proportional loading is confirmed for the current set, with the exception of brittle materials, where the range of relative error of these criteria increases substantially. For these proportional load cases, all evaluated uniaxial solutions provide similar results. It is preferable to use either of the two multiaxial criteria for brittle materials.
2. Instantaneous signing of the von Mises stress by the actual value of the first stress invariant and subsequent decomposition to individual cycles provided by the VMI1 method leads to similar results as by other methods for proportional multiaxial load cases without mean stress, but for any more complicated loading it provides results burdened by disastrous errors.
3. For simple torsion loading with mean stress involved, the MMK method is simpler than the other methods and provides acceptably safe results.
4. Biaxial loading with mean stress involved and in-phase loading (e.g. a pressurized vessel) is optimally covered by MMMK methods, while MMK results are acceptable. Both multiaxial representatives lead to worse scatter of the results in this case.
5. The positive properties of multiaxial methods are reflected above all in cases with out-of-phase loading, where the uniaxial methods tend to provide results that are either excessively non-conservative or excessively scattered. In the case of in-phase loading, even load combinations that are more complex (and potentially lead to non-proportional loading) can be satisfactorily and more quickly solved by (M)MMK methods.
6. If (M)MMK methods are to be preferred for any application with a more complex load history, it will first be necessary to find a suitable method for decomposing the load history to separate cycles.

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Nomenclature :

a_{C}, b_{C}	material parameters of the Crossland method
a_{DV}, b_{DV}	material parameters of the Dang Van method
Ax, To	axial loading, torsion loading
C	shear stress on the examined plane
ΔFI	fatigue index parameter
f_{-1}	fatigue limit in fully reversed axial loading
f_0	fatigue limit in repeated axial loading
I_n	n-th invariant of the stress tensor
IP, OP	In-Phase loading, Out-of-Phase loading
κ	fatigue limit ratio ($\kappa = f_{-1}/t_{-1}$)
J_n	n-th invariant of the stress tensor deviator
MS	experiment includes mean stress
nMS	experiment does not include any mean stress
R	coefficient of the cycle asymmetry ($R = \sigma_{\min}/\sigma_{\max}$)

s_{ij}	components of the stress tensor deviator
$\sigma_{\mathrm{a,eq}}$	equivalent stress amplitude
σ_i, τ_{ij}	normal and shear stress tensor components
t_{-1}	fatigue limit in fully reversed torsion loading

Indices :

a	amplitude
eq	equivalent value
m	mean
1,2,3	index of the principal stress, invariants, ...
max	maximum value
min	minimum value

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