AEROELASTIC SENSITIVITY ANALYSIS OF AIRLINER WING

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This paper describes the airliner wing flutter sensitivity analysis. The sensitivity coefficients define the influence of the structural parameter changes to the structure eigenvalue and flutter stability characteristics. Evaluated structural parameters represent the possible changes of the structure due to the installation of the smart high-lift devices at the leading and trailing edge region. In general, we can suppose the increasing of the mass and mass moment of inertia around the elastic axis and decreasing of the stiffness. Described effects are ordinarily considered destabilizing regarding the flutter. The main aim of the presented work is to evaluate the impact of components to the stability and to define the most critical regions or parameters.

Keywords: *aeroelasticity, flutter, eigenvalue, sensitivity, FNG wing*

1. Introduction

As a part of the 7th Framework Programme of the European Community, there was accomplished the project focused on research and development of the smart high-lift devices. These devices (e.g. [1]–[9]), which allow the smooth changing of the airfoil geometry can help to optimize aerodynamic characteristics of modernized wings. This can increase the operational efficiency of new generation airliners.

Smart high-lift devices are placed at the leading or trailing region of a wing, outside the main wing-box. Whereas we can expect a minor influence of their component to the wing integral stiffness, the mass of smart components placed far from the wing elastic axis may have some influence to a wing flutter characteristics.

The subject of the presented work is a sensitivity analysis. Sensitivity coefficients are defined as rate of change of a response parameter (e.g., eigenvalue or flutter one) with respect to change of a structural parameter (stiffness, inertia). The final aim of the task is evaluation of critical areas or parameters with respect to an aircraft structure flutter behavior, possibly also formulation of recommendations for a wing structural design and critical values for particular parameters.

2. Theoretical background

Design sensitivity analysis computes the rates of change of structural response quantities (r) (e.g. weight, strain, stress, modal frequency, dynamic response, flutter stability etc.) with respect to change of the design variables (x). Design variables are quantities which are changeable, related to the properties (p) which are included into the property entries of

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a structural model (e.g. thicknesses, area moments of inertia, spring constants, etc.). This relation may become a linear combination of design variables:

$$p_j = C_{j0} + \sum_i C_{ji}\, x_i \ , \tag{1}$$

where C_{j0} is constant term of relation and C_{ji} are multiplication terms of relation or a general, also non-linear function

$$p_j = f_j(\{x\}, \{C\}) \tag{2}$$

of design variables and constants (C). Sensitivity coefficients (λ) are evaluated at a particular design characterized by the vector of the design variables $\{x^0\}$, giving

$$\lambda_{ij} = \left(\frac{\partial r_j}{\partial x_j}\right)_{\{x^0\}} \ , \tag{3}$$

where subscripts are used to indicate i-th design variable and the j-th response. Eqn. (3) is just the slope of the response with respect to the design variable as shown in Fig. 1.

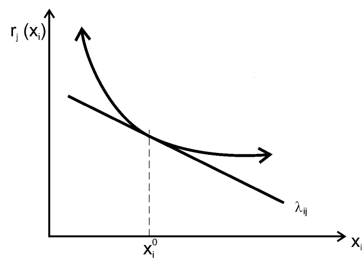

Fig.1: Graphical interpretation of the sensitivity coefficient

The optimization result quantities (design responses) may have the character of either ‘design constraints’ or ‘objective function’. There were used the following two types of the design response in the presented task.

2.1. Eigenvalue response sensitivity

The eigenvalue equation is

$$([K] - \lambda_n\,[M])\,\{\varphi_n\} = 0 \ , \tag{4}$$

where λ_n and $\{\varphi_n\}$ are the n-th eigenvalue and eigenvector respectively. $[K]$ is the structural stiffness and $[M]$ is the structural mass matrix. Eqn. (4) can be differentiated with respect to the i-th design variable x_i:

$$([K] - \lambda_n\,[M])\,\frac{\partial\{\varphi_n\}}{\partial x_i}\left(\frac{\partial[K]}{\partial x_i} - \lambda_n\,\frac{\partial[M]}{\partial x_i}\right)\{\varphi_n\} = \frac{\partial\lambda_n}{\partial x_i}\,[M]\,\{\varphi_n\} \ . \tag{5}$$

When eqn. (5) is premultiplied by $\{\varphi_n\}^{\mathrm{T}}$, the first term become zero and eqn. (5) can be then solved for the eigenvalue derivatives

$$\frac{\partial\lambda_n}{\partial x_i} = \frac{\{\varphi_n\}^{\mathrm{T}}\left(\dfrac{\partial[K]}{\partial x_i} - \lambda_n\,\dfrac{\partial[M]}{\partial x_i}\right)\{\varphi_n\}}{\{\varphi_n\}^{\mathrm{T}}\,[M]\,\{\varphi_n\}} \ . \tag{6}$$

In practice the solution of eqn. (6) is based on the semi-analytical approach. The derivatives of the mass and stiffness matrices are approximated using the finite differences. Equation is solved for each retained eigenvalue referenced in the design model and for each design variable.

2.2. Aeroelastic flutter response sensitivity

Aeroelastic flutter stability equation is given by:

$$\left[M_{\mathrm{hh}}\, p^2 + \left(B_{\mathrm{hh}} - \frac{1}{4}\,\varrho\,\bar{c}\,V\,\frac{Q^{\mathrm{Im}}_{\mathrm{hh}}}{k}\right) p + \left(K_{\mathrm{hh}} - \frac{1}{2}\,\varrho\,V^2\,\frac{Q^{\mathrm{Re}}_{\mathrm{hh}}}{k}\right)\right]\{u_{\mathrm{h}}\} = 0\ . \tag{7}$$

Eqn. (7) represents the modified PK-method of the flutter solution [12]. The PK-method was first proposed by Irwin and Guyett [13] in 1965. It is approximate method of finding a rate-of-decay type solution. At present, it is primary method widely used for the flutter solutions [14]. The method is applicable also for the design optimization purposes because the method performs the flutter analysis at user-specified velocities and the damping values obtained by the method are the appropriate design responses. M_{hh}; B_{hh} and K_{hh} are modal mass, damping and stiffness matrices respectively, which are a function of Mach number (M) and reduced frequency (k). $Q^{\mathrm{Re}}_{\mathrm{hh}}$ and $Q^{\mathrm{Im}}_{\mathrm{hh}}$ are real and imaginary part of a complex aerodynamic matrix, which is also a function of parameters M and k. ϱ is the air density, $\bar{c}$ is a reference length and u_{h} is a modal amplitude vector. In the iteration procedure the eigenvalue p is expected in the form

$$p = \omega\,(\gamma \pm j)\ , \tag{8}$$

where γ is a transient decay rate coefficient. Note that structural damping coefficient is:

$$g = 2\,\gamma\ . \tag{9}$$

Flutter sensitivity computes the rates of change of this transient decay rate coefficient γ with respect to changes of the design variables. Eqn. (7) is differentiated with respect to the design variables for the quantity $(\partial\gamma/\partial x_i)$.

The solution is semi-analytic in nature with derivatives approximated using either forward differences or central differences.

3. Analytical procedure

The proposed solution methodology [10], [11] is based on FE analysis. The FE model must include separate elements for the wing-box, leading edge and trailing edge regions, thus the detailed FE model is envisaged. Considering the dynamic analysis, such a model includes local modes which do not affect the global dynamics and the structure stability. Such modes make the analysis unclear. Also, the effect of a computational time and disk-space saving may be significant. Therefore the model is reduced by means of the standard Guyan reduction (also called static condensation) [15]–[17]. Obviously, there is a minor difference between the full and reduced model modes, since the reduction is based on the partition of the stiffness matrix.

The next step is a flutter analysis. The aerodynamic model for the simulation of the unsteady aerodynamic forces is based on the Doublet-Lattice Subsonic Lifting Surface Theory. The theory was presented by Albano and Rodden in 1969 [18], the theoretical basis is linearized aerodynamic potential theory. The lifting surfaces are modeled by the trapezoidal flat panels which are parallel to the flow. Each of aerodynamic macroelements is divided into small trapezoidal lifting elements (boxes) in strips parallel to the free stream with the surface edges, fold lines and hinge lines on the box boundaries. The flutter stability is calculated by eqn. (7).

The purpose of the flutter analysis is to find the target flutter instability. The flutter speed and frequency as well as the flutter shape and contributing modes are evaluated. The flutter calculation has a character of the non-matched analysis. In the non-matched analysis, the aerodynamic forces are given using one reference Mach number. Therefore the analysis velocities do not match the Mach number and the results have a character of artificial states (e.g., the application example presented in section 4 uses the subsonic aerodynamic theory, although the flutter state was found at a supersonic speed). Such an approach is frequently used in the flutter analysis because it allows to evaluate the rate of reserve in the flutter stability with respect to the specific velocity (e.g., certification velocity). Also, it allows to perform a sensitivity analysis.

The final step is the sensitivity analysis. Sensitivity coefficients are calculated for a specific property of elements with respect to: 1) frequencies of the flutter major contributing modes and 2) damping of the target flutter mode. We used the following types of the design variables:

1) stiffness characteristics: E – Young's modulus; G – shear modulus; E and G values linked using the Poisson's number;
2) inertia characteristics: ϱ – density;
3) geometry characteristics (influencing both stiffness and inertia): T – shell element thickness.

Design variables were connected to the elements at the local level, it means that each element (with own property and material input) was specified as a separate design variable. The elements of the wing part out of the wing box were used as design variables. Further examples of the aeroelastic sensitivity analysis and optimization can be found in the referenced papers [19]–[23].

4. Application example – airliner wing / engine component model

The application example represents the narrow body airliner wing with an engine on a pylon (FNG wing). The structural model is shown in the fig. 2.

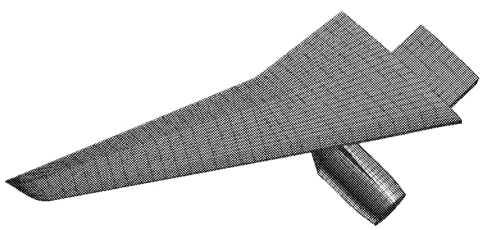

Fig.2: Airliner wing – structural model

The structural model includes main load carrying structural elements modeled by means of beam and plate elements. The residual structure inertia characteristics are included by means of concentrated mass elements. The aerodynamic model consists of a wing, pylon, engine and splitter. The wing is modeled by means of seven macroelements in order to hold the wing planform shape with enough accuracy. The pylon is modeled by one macroelement. The engine is modeled by means of cross-surface model including two horizontal and two vertical macroelements with the root chord at the engine centerline. The splitter avoiding the boundary effect at the wing root is modeled via one macroelement. The density of panelization is increasing spanwise from the root to the tip, and also at the leading or trailing edge regions. The aerodynamic model is presented in fig. 3.

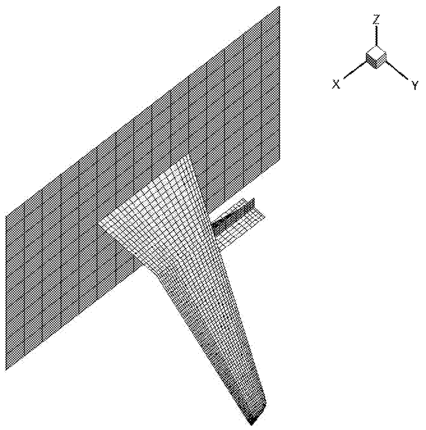

Fig.3: Airliner wing – aerodynamic model

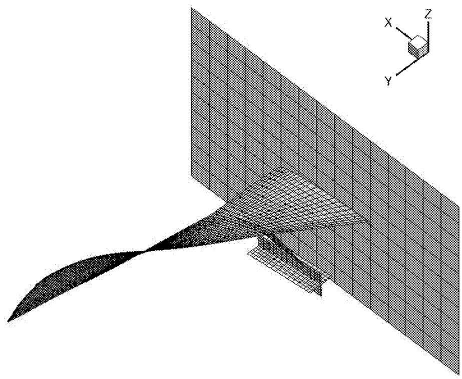

Fig.4: Airliner wing – flutter shape

#	title	f_0 [Hz]
1	Wing 1st vertical bending	2.216
2	Engine vertical vibrations (y-axis)	2.551
3	Engine horizontal vibrations (z-axis)	3.756
4	Wing 2nd vertical bending	5.472
5	Wing 1st horizontal bending	8.884
6	Wing 3rd vertical bending	12.447
7	Wing 1st torsion	17.795
8	Wing 4th vertical bending	22.785

Tab.1: Airliner wing natural frequencies

The interpolation between the structural and aerodynamic model was realized by means of the surface splines. The spline function transforms the aerodynamic loads into the structural model and structural deformations into the aerodynamic model. The surface spline is based on the infinite plate. The spline surface function $w(x, y)$ is a smooth function based on the discrete set of known points.

The modal characteristics were calculated by means of standard Lanczos method. The summary of the 8 lowest natural frequencies is given in the tab. 1.

The flutter analysis of the reference state was performed by means of the PK method. The analysis included 14 modes. The structural damping was included via common value

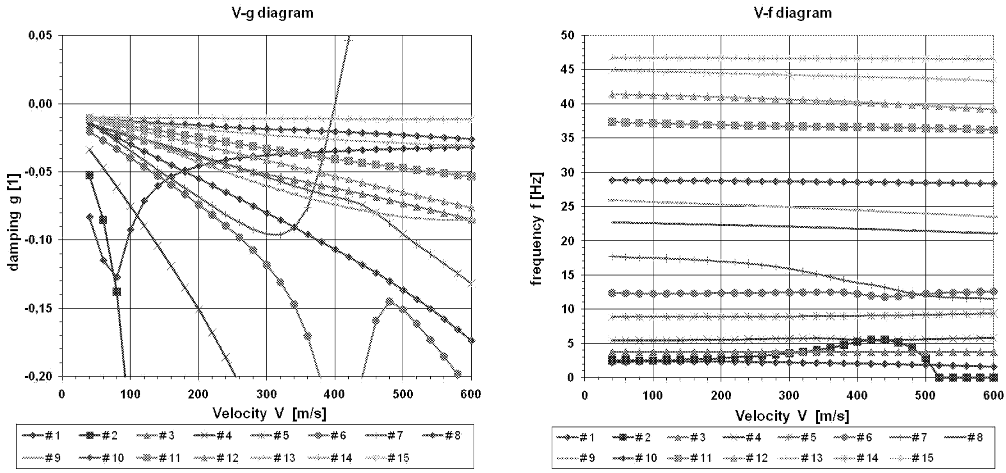

Fig.5: Airliner wing flutter – V-g-f diagram (legend denotes mode-numbers)

of the damping ratio of 1 %. The density was considered $\varrho = 1.225\,\mathrm{kg\,m^{-3}}$ (ISA value for altitude $H = 0$). There was found the flutter state of the bending torsional flutter. The flutter velocity was $V_{\mathrm{FL}} = 400.04\,\mathrm{m\,s^{-1}}$; the flutter frequency was $f_{\mathrm{FL}} = 13.87\,\mathrm{Hz}$. The flutter shape is presented in fig. 4, the primary flutter mode was #7 (1^{st} wing torsion), the critical combination of the modes was: 1^{st} and 3^{rd} wing bending and 1^{st} wing torsion (#1; #6; #7). The V-g-f diagram is presented in the fig. 5.

The resulting sensitivities are presented as the normalized values. The normalization was performed with respect to the maximum value within the same type of the design variables (stiffness, inertia, geometry) and the same type of design response (eigenvalue, flutter).

The fig. 6 and 7 show the normalized sensitivities over the wing spanwise direction. The wing is divided into 32 sections numbered from the root to the tip. Fig. 6 shows the normalized sensitivities of the leading edge upper skin thickness to the eigenvalue responses represented by the normalized sensitivities of frequencies of flutter contributing modes (#1,

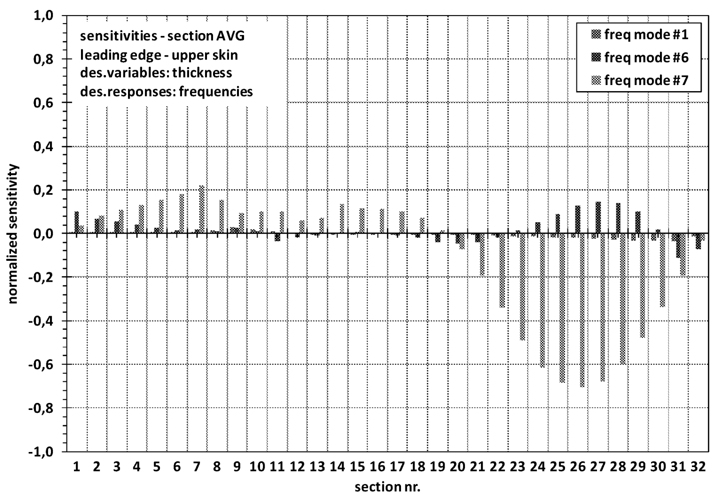

Fig.6: Airliner wing – eigenvalue response normalized sensitivities

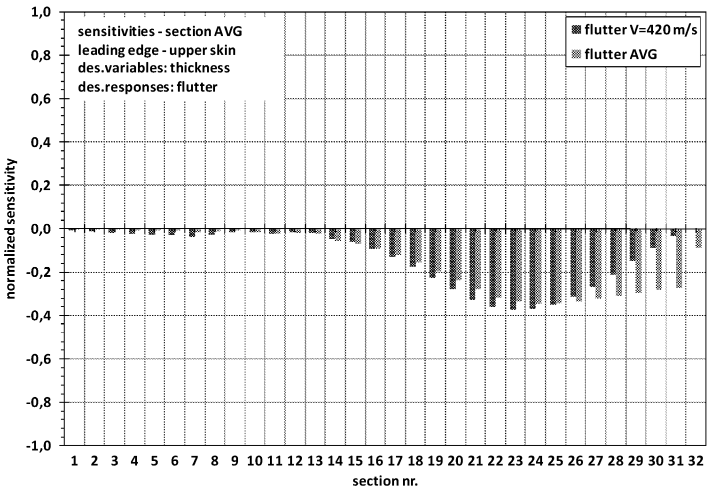

Fig.7: Airliner wing – flutter response normalized sensitivities

#6 and #7). Fig. 7 shows the sensitivities to the flutter responses represented by the damping at the velocity of 420 m s^{-1} and averaged damping sensitivity over the whole velocity range (flutter AVG).

The inertia design variables have much higher sensitivities then stiffness ones. As apparent from fig. 7, flutter sensitivities are negative for the design variables of the leading edge region. This means that the increasing of the design variable have a stabilizing effect. Contrary to this the flutter sensitivities for the design variables of the trailing edge region are positive, it means that the increase of the design variable has destabilizing effect. The maximal destabilizing effect has the increasing of the mass at the trailing edge region around the spanwise section 23, which is approximately at 70 % of the wing half-span.

5. Conclusion

Submitted paper presents the aircraft wing flutter sensitivity analyses. The eigenvalue and flutter sensitivity coefficients are calculated for the leading and trailing edge region structural parameters. Structural stiffness and inertia parameters represent the possible structural changes due to the installation of the smart high-lift devices. The sensitivity coefficients are calculated with respect to the natural frequencies of the flutter major modes and the flutter stability responses. The procedure is demonstrated on the narrow-body airliner wing example. The most sensitive are inertia parameters, the critical region is the trailing edge region. In terms of the spanwise direction the critical area is around 70–75 % of the half-span.

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