MODEL OF AVERAGED TURBULENT FLOW AROUND CYLINDRICAL COLUMN FOR SIMULATION OF THE SALTATION

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The paper presents a model of sand sediment transport in water stream in vicinity of a high cylindrical column. It is assumed that an averaged turbulent flow is horizontally layered in vicinity of the cylinder. In each layer the flow around the cylinder is modelled as a potential flow generated by a vertical dipole line. Flow in viscous sub-layer on the surface of the cylinder is neglected. The presented flow model is approximate; however it is simple for use. Trajectories of saltating particles near the cylinder were calculated. Further investigation of such flow will allow a determination of zones where solid particles will collide with the column. That might be useful for prevention of its damage or destruction.

Keywords: *sediment transport, flow around cylinder, logarithmic profile, dipole line, averaged turbulent flow*

1. Introduction

The effect of sand movement in vicinity of obstacles is very important for stability of various constructions. Often the constructions or their parts have a form of vertical cylinder. In water, for example, sand movement around cylindrical obstacles occurs in channels, rivers, lakes, sea, around various columns, building pillars, bridge legs, stands of pipes for boring, and breakwaters. For example, in desert when cylindrical column stands on a sand surface, blowing wind undermines the construction and as a result the column might decline or even fall.

Knowledge of flow of viscous fluid with particles around a cylinder is important for some engineering works both on land and in water. For instance, investigations of a sand movement around a cylindrical column are a subject of many up-to-day works [1–3].

In Kawamura et al. work, [2], the laminar flow of air around a circular cylinder standing on the sand is computed numerically and the movement of the sand is investigated. They used the MAC method for calculation of the flow field around the cylinder and for estimation of the sand transfer caused by the flow. In results they determined the shape of the ground near the cylinder, accumulation and dented regions, and the conditions at which the cylinder falls down.

Euler & Herget, [1], presented an extensive investigation of sand movement around spherical and cylindrical obstacles. A new approach was applied by conducting experiments in laboratory flume and validating against other laboratory and field data. The results of

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this work have shown a significant relationship between the morphometry of fluvial obstacle marks and obstacle Reynolds number.

Mochizuki et al., [3], experimentally studied sand movement in air around cylindrical obstacle standing on the sand. As a result they found zones of accumulation and erosion of sand bed.

The aim of the present investigation is to find a distribution of sand particles in water stream with turbulent velocity field affected by a cylinder standing up on the rough sand bed, see Fig. 1. The present model is based on model of sand saltation in wide open channel with rough bed, Kharlamova et al. [4], modified for turbulent flow around cylinder. A simple 3D model for calculation of averaged turbulent flow around a cylinder with possibility of its application to sand movement in vicinity of that cylinder was proposed.

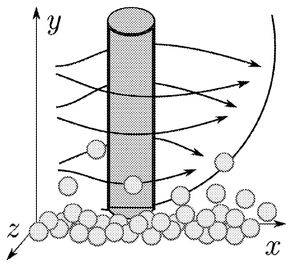


Fig.1: Scheme of average turbulent flow around cylindrical column with sand particles saltating above rough bed

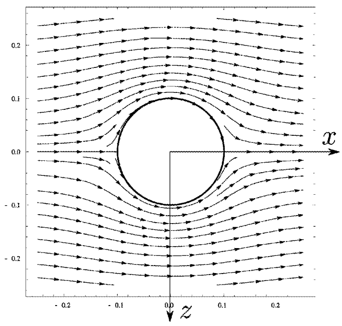


Fig.2: Potential flow in a horizontal plane around a vertical cylinder

2. The flow model

An averaged turbulent flow around cylindrical column with sand particles moving above rough bed is considered here. The sand movement occurs mainly in region near bed, Fig. 1. We assume that the averaged flow around the cylinder is horizontally layered. The tree-dimensional problem of turbulent flow above bed and around a cylindrical column is thus reduced to a two-dimensional problem of flow around a cylinder in each layer. In each horizontal layer the flow is approximated as potential flow of ideal fluid around cylinder, Fig. 2.

Averaged turbulent flow

Consider a uniform turbulent flow above rough plane in semi-infinite space, with flow direction parallel to the x-axis. Consider a vertical distribution of the velocity. According to the Prandl mixing length theory and bed shear stress theory, the averaged velocity of turbulent flow is according to logarithmic law:

$$u_{\mathrm{tx}} = \frac{u_*}{\kappa} \ln\left(\frac{y}{y_0}\right) , \tag{1}$$

where u_* is shear velocity, κ – Karman's constant, y – distance from the bed plane, y_0 – a constant depending on hydraulic roughness, Schlichting & Gersten [5]. A use of such approximation (logarithmic law) neglects turbulent fluctuations of the flow, because we consider already averaged turbulent flow.

The logarithmic law of average velocity distribution was confirmed in many experiments, in pipes and in open channels, and for medium and large Reynolds numbers, see for example Nikuradse, [6]. In open channels the logarithmic law is valid only in the region of 30–40 % of the depth; and it is not valid near water surface and also close to the bottom.

Whereas we consider an averaged turbulent flow, we will suppose that at each fixed height (distance from the bed, y) the velocity of flow is constant. So we suppose that the average turbulent flow is horizontally layered around cylinder. Hence, all turbulent fluctuations in each layer will be neglected; each particle of the averaged fluid flow will stay in the same horizontal layer and there will no mixing of layers neither in front of nor past cylinder. The assumption might be true for an infinitely long cylinder or for an enough long cylinder with a plane ceiling on its top.

Later for description of layered flow (or two-dimensional flow in each horizontal layer) we will use the potential flow theory.

Two-dimensional inviscid potential flow around a cylindrical obstacle

Consider now an irrotational flow of ideal (incompressible, inviscid) fluid around a cylinder of radius R_c, at point with radius vector $\vec{r}_c = (x_c, z_c)$, oriented so that its axis is normal to the flow. The magnitude of unperturbed fluid velocity is u, and it is directed parallel to the x-axis, $\vec{u} = (u, 0)$. If the reference frame is changed so that the fluid velocity at infinity is zero, then the cylinder moves with velocity $-u$. Flow around the cylinder under above mentioned assumptions can be held as a uniform potential flow around a dipole located at the centre of the cylinder. A dipole is defined as a source and a sink of the same strength, when distance between them tends to zero; their strengths tend to infinity. Potential of the dipole is, Milne-Thomson [7],

$$\varphi(\vec{r}) = -\frac{R_c^2\,\vec{u}\,(\vec{r}-\vec{r}_c)}{|\vec{r}-\vec{r}_c|^2}\ , \tag{2}$$

where μ is strength of the dipole, $\mu = R_c^2\,u$. Velocity field generated by the dipole (or by moving cylinder) is $\vec{u}_c = -\vec{\nabla}\,\varphi$. The velocity field generated by a stationary dipole/cylinder with external flow is

$$\vec{u}_{\text{tot}} = \vec{u} - \vec{u}_c\ . \tag{3}$$

The flow on Fig. 2 is example of a potential flow of ideal fluid around circular cylinder in two-dimensional case.

The use of ideal fluid approximation of turbulent flow follows from the fact that horizontal profile of averaged turbulent flow in pipes or channels is very similar to the flow profile of ideal fluid. Near the wall the velocity has approximately the same magnitude as at the centreline, see Fig. 3.

To describe the two-dimensional flow around a cylinder the Prandtl's theory is used (see for instance Oertel [8]). The flow of viscous incompressible fluid around a streamlined body can be approximately represented as a combination of a potential flow of ideal fluid around body and a flow in viscous sub-layer on its surface. The thickness of viscous sub-layer on surface, $\delta \sim l/\sqrt{Re}$, depends on Reynolds number of the flow, Re, and characteristic length of the body, l. The Reynolds number in our case $Re = u\,D_c/\nu$ is determined by diameter of the cylinder, $D_c = 2\,R_c$, flow velocity at infinity, u, and ν – kinematic viscosity of the fluid. So the thickness of viscous sub-layer on cylinder surface is $\delta \sim D_c/\sqrt{u\,D_c/\nu}$.

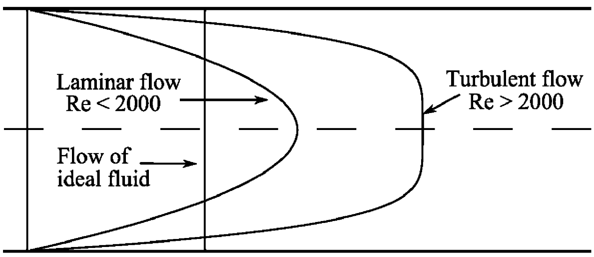


Fig.3: Schematic velocity profiles of laminar, averaged turbulent flow, and flow of ideal fluid

For neglecting flow in viscous sub-layer we suppose that sub-layer is thin. In that case, for saltation purposes let the thickness of viscous sub-layer δ is less than or of the order of the diameter of saltating particle, d. As a consequence, the saltating particle remains in the sub-layer near cylinder surface relatively short time. So, if $\delta \leq d$, then $D_c/\sqrt{u\,D_c/\nu} \leq d$. This relation can be considered as a restriction of applicability of saltation model in the context of given flow model. It is evident that in the given 3D flow model of averaged turbulent flow around cylinder, thickness of viscous sub-layer depends not only on angle θ, like in 2D case, but also on vertical position of the given point, Fig. 4. In region near bed flow velocity $u(y)$ is less then it is on the top, therefore the thickness of viscous sub-layer is larger at bed and thinner on top: $\delta = f(\theta)\,D_c/\sqrt{u(y)\,D_c/\nu}$. In stagnation point ($\theta = 0$) thickness of layer is minimal $f(\theta) = 2.65$, in separation point ($\theta \approx 104$) it is maximal $f(\theta) = 6.23$, Fitzpatrick [9], Figs. 4, 5. So a restriction for flow velocity in the each layer, or at each y-coordinate, can be found: $\delta_{\mathrm{stag}} \leq d \Rightarrow u(y) \leq 7\,D_c\,\nu/d^2$.

For Reynolds numbers greater than 40 the Karman's vortex street appears in flow after the cylinder, Fitzpatrick [9]. At Reynolds number greater than 2000, as illustrated in Fig. 5, the boundary layer separates and a vortex wake forms behind the cylinder. For calculation of the separation point Prandtl assumed an existence of uniform transverse flow around a circular cylinder which illustrated in Fig. 2 and described by Eq. (3). However such flow assumes no separation of the boundary layer. Since the results derived by Prandtl are in good accordance with experiments, one can conclude that Eq. (3) describes flow sufficiently accurately at least in front of the cylinder, i.e. for $x < x_c$, where x_c is a point of separation of the flow.

In further, it will be shown how this two-dimensional potential flow model will be used for description of three-dimensional averaged turbulent flow around cylinder.

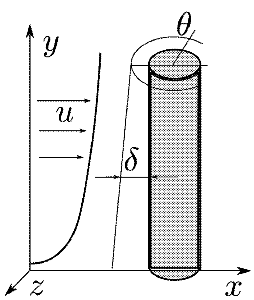


Fig.4: Viscous sub-layer on cylinder surface in 3D flow model

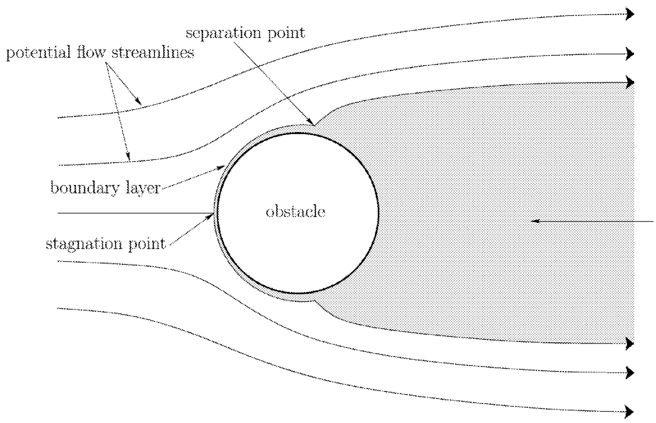


Fig.5: A flow separation on the surface of a circular cylinder, illustration by Fitzpatrick [9]

Model of tree-dimensional turbulent flow around a cylinder

Now consider a tree-dimensional case, where, how was mentioned above, the averaged turbulent flow is horizontally layered. The velocity of each layer in absence of the cylinder is $\vec{u}_{\mathrm{t}} = (u_{\mathrm{tx}}, 0, 0)$, see Eq. (1). The two-dimensional velocity field in each layer (for the same vertical coordinates) in presence of cylinder is calculated by Eq. (3). Thus, the averaged turbulent velocity field in three-dimensional case around the cylinder is defined as a combination of vertical velocity distribution of averaged turbulent flow and horizontal distribution of velocity of ideal fluid around cylinder

$$\vec{u}_{\mathrm{tot}} = \vec{u}_{\mathrm{t}} - \vec{u}_{\mathrm{c}} \ , \tag{4}$$
$$\vec{u}_{\mathrm{c}} = \left(\frac{\partial\varphi}{\partial x}, 0, \frac{\partial\varphi}{\partial z}\right) \ , \quad \varphi = -\frac{R_{\mathrm{c}}^2\, u_{\mathrm{tx}}\,(x - x_{\mathrm{c}})}{(x - x_{\mathrm{c}})^2 + (y - y_{\mathrm{c}})^2} \ .$$

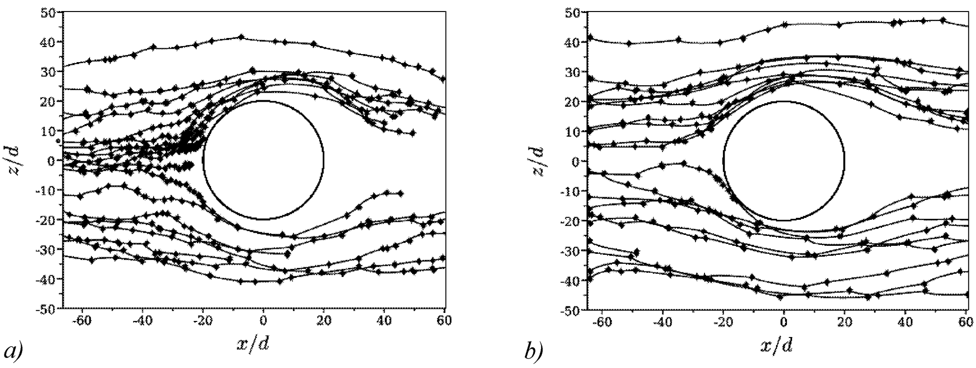
Later we will consider a three-dimensional saltation model in averaged three-dimensional turbulent flow defined by Eq. (4), $\vec{u}_{\mathrm{tot}} = (u(x, y, z), 0, u(x, y, z))$, where particles also saltate, as in the recent model of particle saltation over rough bed, Kharlamova et al. [4].

3. Saltation in turbulent flow around cylinder

The model of sand saltation in turbulent flow in wide open channel with rough bed was described by Kharlamova et al., [4]. This model was modified for new conditions, i.e. for turbulent flow around cylinder.

The saltation of sand is simulated in velocity field defined by Eq. (4) in semi-infinite space, where the height of flow is indefinite. Though the flow particles remain in the same horizontal planes, the sand particles move in 3 dimensions in consecutive jumps over bed. The sand particles are subject to gravity, buoyancy, drag, added-mass, Magnus forces and a moment of viscous force. The expressions for drag, added-mass, Magnus forces, and moment of viscous forces, which are defined in Kharlamova et al., [4], are depended on the liquid velocity (see Eq. 4).

Moving particles rebound from bed particles, see Fig. 6. The mutual collisions of conveyed particles are not taken into account. Collision of saltating particle with cylinder surface is a new phenomenon in this model of saltation, and it is inelastic with the same coefficients of restitution and friction as those for collision of saltating particle with bed particles. For simulation, particles were randomly discharged upstream of the cylinder, at distance $x = 1.5\,D_c$.



a) *b)*

Fig.6: Examples of saltation trajectories in water in vicinity of vertical cylinder; lines show trajectories of individual jumps, the cross signs denote places of rebounds from bed; diameter of particles – $d = 0.75$ mm, diameter of the cylinder – $D_c = 3$ cm, a) $u_* = 2$ cm/s, $u_{av} = 26$ cm/s, $L_s = 6\,d$, $H_s = d$; b) $u_* = 2.5$ m/s, $u_{av} = 33$ cm/s, $L_s = 12\,d$, $H_s = 1.5\,d$

Fig. 6 shows trajectories of saltating sand particles in water streams above rough bed and around cylinder, where L_s and H_s is averaged saltation length (length of particle jump) and height (height of particle jump), respectively, Nino & Garcia [10]. The sizes of particles, cylinder and flow velocities were found in experimental work of Euler & Herget, [1]. The parameters of water are: $\varrho = 1\,000\,\mathrm{kg/m^3}$, $\nu = 10^{-6}\,\mathrm{m^2/s}$; sand particles: $d = 0.75\,\mathrm{mm}$, $\varrho_s = 2\,650\,\mathrm{kg/m^3}$; cylinder: $D_c = 3\,\mathrm{cm}$. Diameter of saltating and bed particles are the same. Mean flow velocity – $u_{av} = 26-33\,\mathrm{cm/s}$, shear velocity – $u_* = 2-2.5\,\mathrm{cm/s}$. Bed roughness was assigned as $k_s = 1.5\,d$, thus, a bed constant in velocity profile is $y_0 = 1.5\,d/30$. Reynolds number of the cylinder based on mean flow velocity is $Re = D_c\,u_{av}/\nu$: $Re = 7800-9900$.

Fig. 6 illustrates that particles moving in flow with greater velocity reached longer and higher jumps (b) than particles in flow with lower velocity (a). At lower velocity the particles collided with bed more frequently than at higher, moreover, they collided with cylinder not only on its front side, but also on lateral sides, especially at lower velocity (a). It's worth noting that at higher velocity the saltating particles moved mostly round the cylinder and did not touch it (b). The trajectories of the particles trace the streamlines; and particles avoid the zone past cylinder, where turbulent wake should take place.

4. Discussion

The main advantages of the presented model of averaged turbulent flow around circular cylinder are in its simplicity and simplicity of its application. Calculation of trajectories of saltating particles in such complicated flow pattern can be conducted more easily and faster with this flow model. However, it can better describe the flow in front of cylinder and worse behind it. The model presents a rough estimation of the flow, and further, a subject of our next research will be its verification.

Fig. 6 shows an example of joint application of the present flow model and a model of sand saltation in turbulent flow around circular cylinder.

Later, the theory of potential flow around of several cylinders, developed by Kharlamov & Filip, [11], allows us extend the investigation of the saltation in vicinity of several cylinders located in a row perpendicular to the flow.

5. Conclusion

The present research deals with saltation of sand particle in averaged turbulent flow around a cylindrical column. The investigation offers a combination of two models : 3D model of saltation in turbulent flow and 3D model of averaged turbulent flow around a cylindrical column. The 3D model of turbulent flow around cylinder represents combination of vertical averaged turbulent flow, described by logarithmic law, and horizontal layered flow around cylinder, described as a potential flow of ideal fluid.

The 3D model of flow is controversial and it represents now as a first approximation of the problem of turbulent flow around cylinder. However, it has some advantages: the model allow fast estimation of flow velocity in each given point of flow; it can be easily combine with other models where distribution of velocity is needed, e.g. saltation of sand; and last, given model can be extended to the research of flow past several cylinders.

Combination of two models : turbulent flow around cylinder and saltation, allows study of sediment transport in new type of flow geometry. It allows simulate movement of sand around cylindrical obstacle and estimate movement's parameters, as well. It also can be used for estimation of zones where solid particles collide with column most frequently. Thereby, the given research can help prevent damage of cylindrical constructions in turbulent flow.

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