UNSTEADY BOUNDARY LAYER FLOW DUE TO A STRETCHING POROUS SURFACE IN A ROTATING FLUID

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The induced unsteady flow due to a stretching porous surface in a rotating fluid, where the unsteadiness is caused by the suddenly stretched surface is studied in this paper. After a similarity transformation, the unsteady Navier-Stokes equations have been solved numerically using the Adams Predictor Corrector Method. It is found that there is a smooth transition from the small time solution to the large time or steady state solution.

Keywords: *stretching porous surface, rotating fluid, viscous flow, skin friction*

1. Introduction

The description of flow and heat transfer in the boundary layer induced by a stretching surface has many important applications in manufacturing processes in industry such as the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film and condensation processes, the cooling and/or drying of paper and textiles, and glass filer production, to name a just a few of these applications. In particular, in the extrusion of a polymer in a melt-spinning process, the extrusion from the die is generally drawn and simultaneously stretched into a sheet, which is then solidified throughout quenching or gradual cooling by direct contact with water. In all these cases, a study of the flow field and heat transfer can be of a significant importance because the quality of the final product depends to a large extent on the skin friction coefficient and the surface heat transfer rate. Recent papers by Wang et al. [1], Magyari and Keller [2], Chen [3] and Mahapatra and Gupta [4], and the book by Pop and Ingham [5] show considerable research activities in this area.

The fluid dynamics over a stretching surface is important in many practical applications such as extrusion of plastic sheets, paper production, glass blowing, metal spinning and drawing plastic films, to name just a few. The quality of the final product depends on the rate of heat transfer at the stretching surface. Since the pioneering study by Crane [6] who presented an exact analytical solution for the steady two-dimensional stretching of a surface in a quiescent fluid, many authors have considered various aspects of this problem and obtained similarity solutions.

The boundary later flow due to stretching vertical surface in a quiescent viscous and incompressible fluid when the buoyancy forces are taken into account have been considered

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only in the papers by Chen [3], and very recently by Ali [7], Abo – Eldahab [8], and Partha et al. [9]. A. Sri Ramulu et al. [10] studied Steady flow and heat transfer of a viscous incompressible fluid through porous medium over a stretching porous sheet. The studies carried out in these papers deal only with steady-state flow, but the flow and thermal fields may be unsteady due to either impulsive stretching of the surface of external stream and sudden changes in the surface temperature. However, to our best knowledge, only Kumari et al. [11] have studied the unsteady free convection flow over a continuous moving vertical surface in an ambient fluid. Both constant surface temperature and constant surface heat flux condition have been considered. However, only the assisting flow case has been considered in Kumari et al. [11]. When a flat surface is impulsively stretched in an ambient fluid, the inviscid (potential) flow is developed almost immediately, but the viscous flow within the boundary layer develops slowly and it becomes a fully developed flow after some time. The development of the boundary layer takes place in two stages. For small time t^- ($\ll 1$), the flow is dominated by the viscous forces and the convective acceleration plays only a minor role in the flow development. For large time ($t^- \to \infty$), it becomes steady. For the intermediate region, $1 < t^- < \infty$, there is a smooth transaction from unsteady to steady flow or the transition from the unsteady to steady flow takes place without a singularity or flow instability(or formation of cells), see Telionis [12].

Transient three dimensional electrical conducting viscous incompressible fluid fast an impulsively started horizontal porous plate relative to a rotating system taking into account the effect of hall current is studied by N. Ahmed and H. K. Sarmah [13].An Analytical solution to the problem of the MHD free and forced convection three dimensional flow of an incompressible viscous electrically conducting fluid with mass transfer along a vertical porous plate with transverse sinusoidal suction velocity studied by N. Ahmed [14]. The heat transfer and hydro magnetic boundary layer flow of an electrically conducting viscous incompressible fluid over a continues flat surface moving in a parallel free stream is investigated by Khem Chand [15]. Bisemini et al. [16] studied theoritical and numerical investigation on ship motion and propulsion in marine engineering.Lorenzini et al. [17] investigated that the geometric optimization of isothermal cavities according to Bejan's theory.

Nazar et al. [18] studied the unsteady flow due to a stretching surface in a rotating fluid, where the unsteadiness is caused by the suddenly stretched surface. In the present study we have considered the unsteady flow due to a stretching surface in a rotating fluid in the presence of porous media, where the unsteadiness is caused by the suddenly stretched surface. The transformed governing partial differential equations are solved numerically by using the Adams predictor-corrector method for some values of the physically governing the parameters.

2. Mathematical description

Consider the two-dimensional stretching of a surface in a rotating fluid. At time $t = 0$, the surface at $z = 0$, is impulsively stretched in the x-direction in a rotating fluid. Due to the Coriolis force, the fluid motion is three-dimensional. Let (u, v, w) be the velocity components in the direction of Cartesian axes (x, y, z), respectively, with the axes rotating at an angular velocity Ω in the z direction.

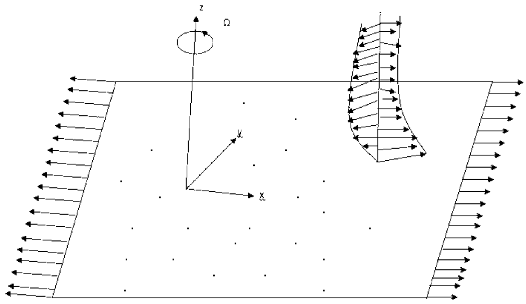

Fig.1: Physical model and co-ordinate system

The unsteady Navier-Stokes equations governing the flows are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \ , \tag{1}$$

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} + w\,\frac{\partial u}{\partial z} - 2\,\Omega\,v - \frac{\nu}{k}\,u = -\frac{1}{\varrho}\,\frac{\partial p}{\partial x} + \nu\,\nabla^2 u \ , \tag{2}$$

$$\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} + w\,\frac{\partial v}{\partial z} - 2\,\Omega\,u - \frac{\nu}{k}\,v = -\frac{1}{\varrho}\,\frac{\partial p}{\partial y} + \nu\,\nabla^2 v \ , \tag{3}$$

$$\frac{\partial w}{\partial t} + u\,\frac{\partial w}{\partial x} + v\,\frac{\partial w}{\partial y} + w\,\frac{\partial w}{\partial z} = -\frac{1}{\varrho}\,\frac{\partial p}{\partial z} + \nu\,\nabla^2 w \ , \tag{4}$$

where p is the pressure, ϱ is the density, ν is the kinematic viscosity, k is the gyro viscosity and ∇^2 denotes the three-dimensional Laplacian. Let the surface be impulsively stretched in the x-direction such that the initial and boundary conditions are

$$\begin{aligned} &t < 0 : u = v = w = 0 \quad \text{for any} \quad x, y, z \ , \\ &t \geq 0 : u = a\,x, \ v = w = 0 \quad \text{at} \quad z = 0 \ , \\ &u \to 0, \ v \to 0, \ w \to 0, \quad \text{as} \quad z \to \infty \ , \end{aligned} \tag{5}$$

where a $(a > 0)$ has the dimension of $[t^{-1}]$ and represents the stretching rate. We now introduce the following similarity variables

$$\begin{gathered} u = a\,x\,f'(\xi,\eta) \ , \quad v = a\,x\,h(\xi,\eta) \ , \quad \eta = \left(\frac{a}{\nu}\right)^{1/2} \xi^{1/2}\,z \ , \quad w = -(a\,\nu)^{1/2}\,\xi^{1/2}\,f(\xi,\eta) \ , \\ \xi = 1 - \mathrm{e}^{-\tau} \ , \quad \tau = a\,t \ . \end{gathered} \tag{6}$$

Then, equations (2)–(4) become

$$f''' + \frac{1}{2}\,(1-\xi)\,\eta\,f'' + \xi\,(f\,f'' - f'^2 + 2\,\lambda\,h + K\,f') = \xi\,(1-\xi)\,\frac{\partial f'}{\partial \xi} \ , \tag{7}$$

$$h'' + \frac{1}{2}\,(1-\xi)\,\eta\,h' + \xi\,(f\,h' - f'\,h - 2\,\lambda\,f' + K\,h) = \xi\,(1-\xi)\,\frac{\partial h}{\partial \xi} \ , \tag{8}$$

where $\lambda = \Omega/a$ and $K = \nu/k$ porosity parameter. The boundary conditions (5) become

$$f(\xi,0) = 0 \ , \quad f'(\xi,0) = 1 \ , \quad f'(\xi,\infty) = 0 \ , \quad h(\xi,0) = 0 \ , \quad h(\xi,\infty) = 0 \ . \tag{9}$$

The primes denote the differentiation with respect to η. The wall shear stress $\tau_\mathrm{w}^\mathrm{x}$ and $\tau_\mathrm{w}^\mathrm{y}$ in the x and y directions are given by $\tau_\mathrm{w}^\mathrm{x} = \mu\,[\partial u/\partial z]_{z=0}$ and $\tau_\mathrm{w}^\mathrm{y} = \mu\,[\partial v/\partial z]_{z=0}$ and related to the non-dimensional skin friction coefficient in x and y directions. $C_{\mathrm{f}}^{\mathrm{x}}$ and $C_{\mathrm{f}}^{\mathrm{y}}$, respectively, according to

$$\begin{aligned} C_{\mathrm{f}}^{\mathrm{x}} &= \frac{\tau_{\mathrm{w}}^{\mathrm{x}}}{\varrho\,(a\,x)^2} = \frac{\nu}{(a\,x)^2}\left(\frac{\partial u}{\partial z}\right)_{z=0} , \\ C_{\mathrm{f}}^{\mathrm{y}} &= \frac{\tau_{\mathrm{w}}^{\mathrm{y}}}{\varrho\,(a\,x)^2} = \frac{\nu}{(a\,x)^2}\left(\frac{\partial v}{\partial z}\right)_{z=0} . \end{aligned} \tag{10}$$

Using variable (6), we obtain

$$C_{\mathrm{f}}^{\mathrm{x}}\,\mathrm{Re}_{\mathrm{x}}^{1/2} = \xi^{-1/2}\,f''(\xi,0)\ , \qquad C_{\mathrm{f}}^{\mathrm{y}}\,\mathrm{Re}_{\mathrm{x}}^{1/2} = \xi^{-1/2}\,h'(\xi,0)\ , \tag{11}$$

where $\mathrm{Re}_{\mathrm{x}} = (a\,x)\,x/\nu$ is the local Reynolds number.

3. Method of solution

The terms is $\xi\,(1-\xi)\,\partial f/\partial\xi$ replaced by

$$\left(\xi + \frac{\Delta\xi}{2}\right)\left(1 - \xi - \frac{\Delta\xi}{2}\right)\frac{f'(\xi+\Delta\xi,\eta) - f'(\xi,\eta)}{\Delta\xi}$$

as an approximation to find the variables variable $\xi + \Delta\xi$ knowing at ξ as a function of η. Similarly $\xi\,(1-\xi)\,\partial h/\partial\xi$ is replaced by

$$\left(\xi + \frac{\Delta\xi}{2}\right)\left(1 - \xi - \frac{\Delta\xi}{2}\right)\frac{h(\xi+\Delta\xi,\eta) - h(\xi,\eta)}{\Delta\xi} .$$

We thus have one third order another second order ordinary differential equations for each ξ. Needs five conditions to be prescribed on f and h.

At $\xi = 0$, the equations (7) and (8) reduced to

$$f''' + \frac{1}{2}\,\eta\,f'' = 0 \qquad \text{and} \qquad h'' + \frac{1}{2}\,\eta\,h = 0\ .$$

By solving these conditions we know $f(0,\eta)$, $f'(0,\eta)$, $f''(0,\eta)$, $f'''(0,\eta)$, $h(0,\eta)$, $h'(0,\eta)$. The method of solving these equations for $\xi = 0, \Delta\xi, 2\Delta\xi$ etc is now described below.

We assume values for missing initial conditions namely $\alpha = f''(0)$ and $\beta = h'(0)$ we find α and β by solving $G(\alpha,\beta) = f''(\xi,\infty,\alpha,\beta) = 0$ and $P(\alpha,\beta) = h(\xi,\infty,\alpha,\beta) = 0$ for each ξ.

These values of α, β are found by Newton's method for this we need derivatives of G and P with respect to α and β at $\eta = \infty$. These are obtained by solving the following two linear equations

$$\begin{aligned} &F''' + \frac{1}{2}\,(1-\xi)\,\eta\,F'' + \xi\,(f\,F'' - F\,f'' - 2\,f'\,F' + 2\,\lambda\,H + K\,F') = \\ &\qquad = \left(\xi + \frac{\Delta\xi}{2}\right)\left(1 - \xi - \frac{\Delta\xi}{2}\right)\frac{F'(\xi+\Delta\xi,\eta) - F'(\xi,\eta)}{\Delta\xi} = 0\ , \\ &H'' + \frac{1}{2}\,(1-\xi)\,\eta\,H' + \xi\,(f\,H' - F\,h' - f'\,H - F'\,h - 2\,\lambda\,F' + K\,H) = \\ &\qquad = \left(\xi + \frac{\Delta\xi}{2}\right)\left(1 - \xi - \frac{\Delta\xi}{2}\right)\frac{H(\xi+\Delta\xi,\eta) - H(\xi,\eta)}{\Delta\xi} = 0 \end{aligned}$$

where

$$F = \frac{\partial f}{\partial \alpha}\ , \quad F' = \frac{\partial f'}{\partial \alpha}\ , \quad F'' = \frac{\partial f''}{\partial \alpha}\ , \quad F''' = \frac{\partial f'''}{\partial \alpha}\ , \quad H = \frac{\partial h}{\partial \alpha}\ , \quad H' = \frac{\partial h'}{\partial \alpha}$$

with the initial conditions

$$F(\xi,0) = 0\ , \quad F'(\xi,0) = 0\ , \quad F''(\xi,0) = 1\ , \quad H(\xi,0) = 0\ , \quad H'(\xi,0) = 0\ .$$

This solution gives derivatives of G and P with respect to α.

If we use the initial conditions $F(\xi,0) = 0$, $F'(\xi,0) = 0$, $H(\xi,0) = 0$, $H'(\xi,0) = 1$ to obtain the derivative G and P with respect to β.

α and β are connected for every time we are found that three or four iteration were enough to find α and β.

4. Results and disscussion

The computations are carried out for $0 \le \xi \le 1$, various values of some values of parameter λ and porosity parameter K. To validated our results, we have compared the values of the reduced skin frictions $f''(\xi,0)$ and $h'(\xi,0)$ when $\xi = 1$ (final steady-state flow) with those of Nazar (2004). The results are found to be in excellent agreement. The comparison is shown in Table 1, for various values of λ. It can be seen that the values are negative for values of λ considered in this study. The values of $C_\mathrm{f}^\mathrm{x}\,\mathrm{Re}_\mathrm{x}^{1/2}$ increases with λ as can be observed from Table 1.

λ	$f''(0)$ Nazar 2004	$f''(0)$ present	$h'(0)$ Nazar 2004	$h'(0)$ present
0	-1	-1	0	0
0.5	-1.1384	-1.1389	-0.5128	-0.5123
1	-1.3250	-1.3255	-0.8371	-0.8368
2	-1.6523	-1.6523	-1.2873	-1.2873

Tab.1

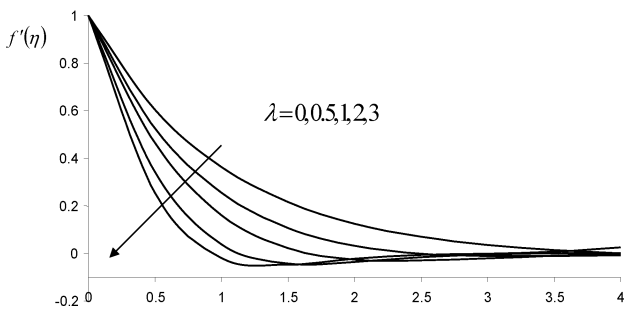

Fig.2: Similarity velocity profile in x-direction for $\xi = 1$

Fig. 2 shows that the evolution of the similarity velocity profiles $f'(\eta)$ at the final steady state flow at $\xi = 1$ in the x-direction. It is observed that the velocity decay monotically and exponentially for $\lambda = 0$ and small values of λ, while the decay is oscillatory for large values of λ. The velocity profiles decrease with increase of λ.

Fig. 3 depicts the similarity velocity profile for steady state ($\xi = 1$) for different values of λ. From the Fig. 3, it is noticed that decrease in the fluid velocity with in the boundary

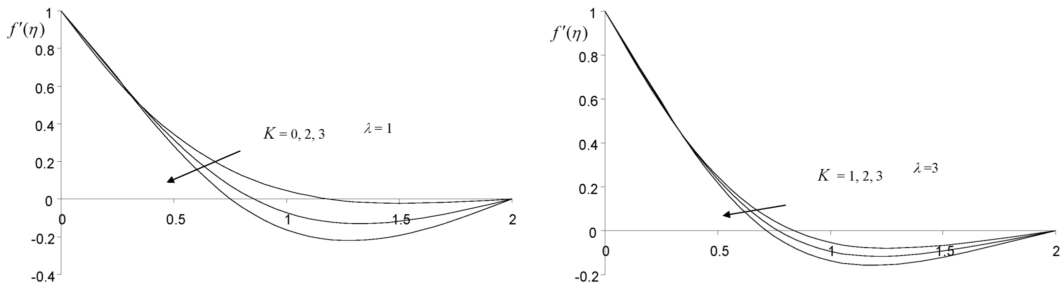

Fig.3: Similarity velocity profiles in x-direction for $\xi = 1$

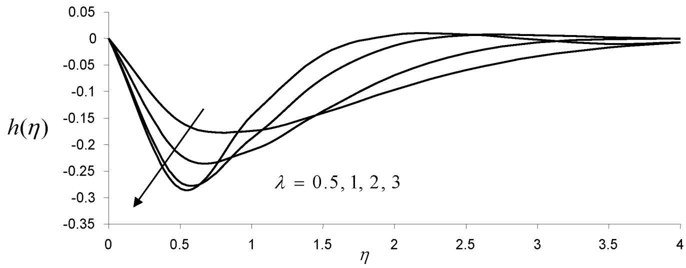

Fig.4: Similarity velocity profile in y-direction for $\xi = 1$

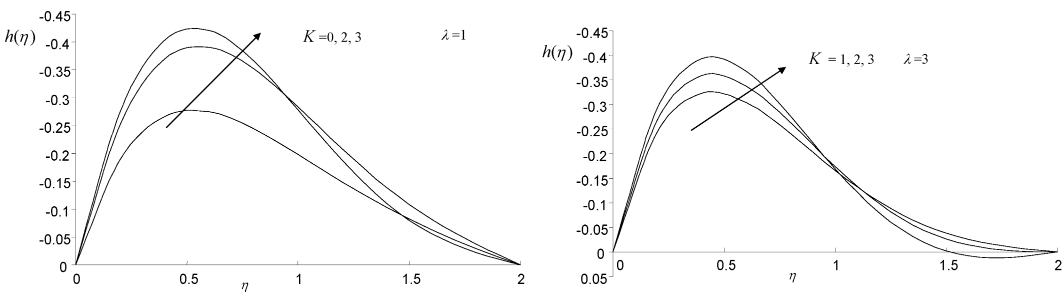

Fig.5: Similarity velocity profiles in y-direction for $\xi = 1$

layer due to the porosity parameter K. It is interesting to note that the influence of porosity parameter is more when the $\lambda = 3$ compared with $\lambda = 1$.

The similarity velocity profiles $h(\eta)$ in y-direction for $\xi = 1$ is shown in Fig. 4 for some values of λ. It can be seen for zero and small values of λ that the velocity decays monotonically and exponentially, while for large value of λ the decay is oscillatory.

Figure 5 shows the influence of the porosity parameter K on similarity velocity profile h for steady state ($\xi = 1$) when $\lambda = 1$ and $\lambda = 3$. It is noticed that increase in the porosity parameter K leads to increase in the similarity velocity profiles $h(\eta)$. It is note worthy to note that the effect of porosity parameter is more important when $\lambda = 1$ then $\lambda = 3$.

The variations of the skin friction coefficient with ξ in x-direction and in y-direction are shown in Fig. 6 and Fig. 7 respectively for different values of λ. It is noticed that the value of $C_{\mathrm{f}}^{\mathrm{x}} \mathrm{Re}_{\mathrm{x}}^{1/2}$ and $C_{\mathrm{f}}^{\mathrm{y}} \mathrm{Re}_{\mathrm{x}}^{1/2}$ decreases with the increases of λ.

Figs. 8 and 9 show the variation of skin friction coefficient $C_{\mathrm{f}}^{\mathrm{x}} \mathrm{Re}_{\mathrm{x}}^{1/2}$ and $C_{\mathrm{f}}^{\mathrm{y}} \mathrm{Re}_{\mathrm{x}}^{1/2}$ with τ for some values of λ. It can be seen that as λ increases the skin friction coefficient decreases. It is noticed that there is agreement between the results when we solved fully unsteady

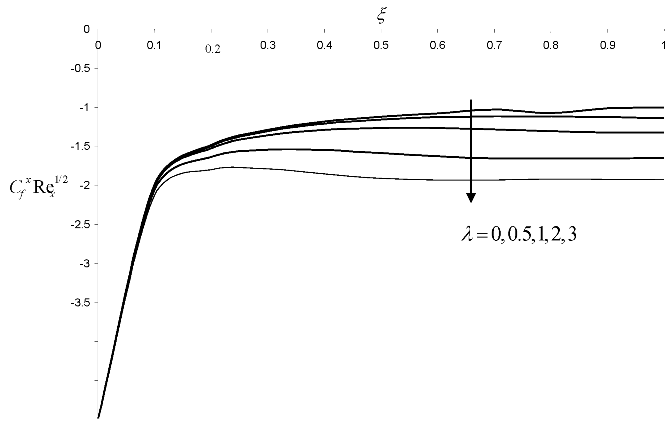

Fig.6: Variation with of ξ the skin friction coefficient in x-direction for some value of λ

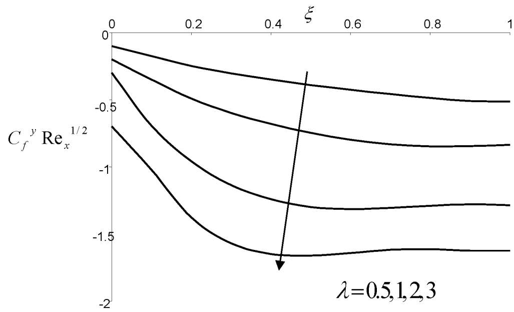

Fig.7: Variation with of ξ the skin friction coefficient in y-direction for some value of λ

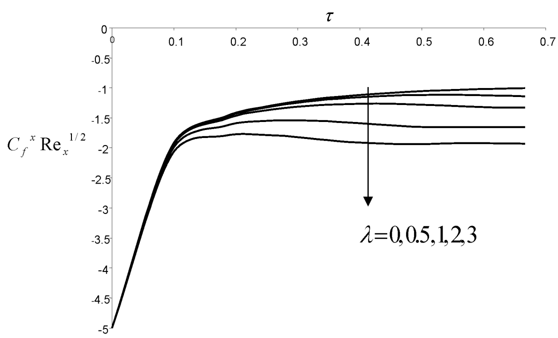

Fig.8: Variation with of τ the skin friction coefficient in x-direction for some value of λ

boundary layer equations and final steady-state equations. It is observed that due to impulsive motion, the skin friction coefficient as large magnitude (absolute values) for small time ($\tau \approx 0$) after the start of the motion, and decreases monotonically and reaches the steady

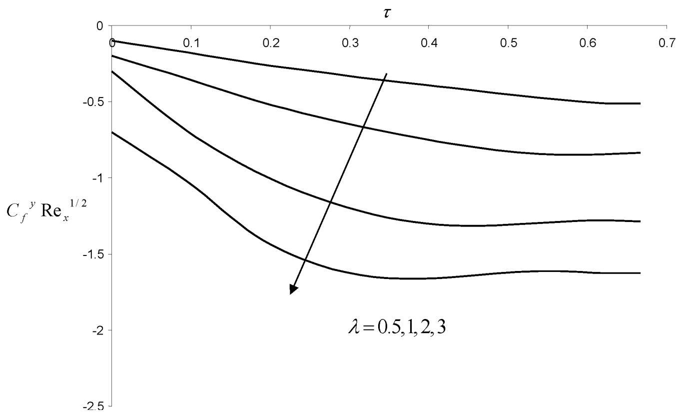

Fig.9: Variation with of τ the skin friction coefficient in y-direction for some value of λ

state values at $\xi = 1$ ($\tau \to \infty$). There is, therefore, a smooth transisation from the small time solution to the large time solution.

Nomenclature

u, v, w	velocity components in x, y, z directions
Ω	angular velocity
p	pressure
ϱ	density
k	gyro viscosity
K	porosity parameter
ν	kinematic viscosity
$\mathrm{RE_x}$	local Reynolds number
τ	shear stress
C_f	skin friction coefficient
∇^2	Laplace transform operator
a	stretching rate
f'	similarity velocity profile in x-direction
h	similarity velocity profile in y-direction
f	similarity velocity profile in z-direction

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